



Modeling and Control of a Hybrid Soft–Rigid Space Manipulator System

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Space manipulators are a key enabling technology for future on-orbit servicing and active debris removal missions, yet conventional rigid designs can limit overall system dexterity. This work introduces a novel hybrid soft–rigid space manipulator system (HSMS), which integrates a tendon-driven continuum manipulator with a traditional rigid robot. A strain parameterization technique known as geometric variable strain (GVS) is employed to derive a closed-form, minimum-order dynamic model of the HSMS by leveraging the principle of virtual work. Thrusters and variable-speed control moment gyros are incorporated to activate the spacecraft base and enhance the realism of the description. Two inverse-dynamics control designs are then proposed: one to bring the system to rest from a nonzero momentum condition and another one to track a three-dimensional trajectory in the end-effector task space. Extensive numerical simulations validate both the dynamic model and the proposed controllers, demonstrating the potential of the proposed HSMS concept for advanced space-manipulation tasks.

Nomenclature

$\mathbf{g}_{A/B}$	=	homogeneous transformation matrix from frame $\mathcal{A}$ to frame $\mathcal{B}$
$\mathbf{I}_n$	=	identity matrix of size n
$\mathbf{J}^A$	=	body Jacobian of frame $\mathcal{A}$
$\mathbf{M}_j$	=	inertia matrix of the j th rigid body
$\bar{\mathbf{M}}_{\mathcal{X}}$	=	cross-section inertia matrix of the soft link expressed in frame $\mathcal{X}$
$\mathbf{0}_{n \times m}$	=	null matrix of size $n \times m$
$\mathbf{q}$	=	configuration vector of the hybrid space manipulator
$\mathbf{q}_B$	=	vector of spacecraft-base coordinates
q_i	=	coordinate of the i th rigid manipulator joint
$\mathbf{q}_S$	=	vector of strain coordinates in the geometric variable strain parameterization
$\mathbf{R}_{A/B}$	=	orthogonal matrix expressing the rotation from frame $\mathcal{A}$ to frame $\mathcal{B}$
$\mathbf{r}_{A/B}^C$	=	position vector of frame $\mathcal{B}$ with respect to frame $\mathcal{A}$ expressed in frame $\mathcal{C}$
s	=	arc-length coordinate of the soft manipulator link
$\mathbf{u}$	=	collection of all actuation inputs
$u_{\text{cab},i}$	=	control tension applied to the i th cable actuator
$u_{\text{rev},i}$	=	control torque applied to the i th rigid joint
$u_{\text{thr},i}$	=	i th thruster force
δ_i	=	gimbal angle of the i th control moment gyro
$\boldsymbol{\eta}$	=	vector of velocity field coordinates
$\boldsymbol{\xi}$	=	vector of strain field coordinates
$\hat{\cdot}$	=	isomorphism $\hat{\cdot}: \mathbb{R}^6 \rightarrow \mathfrak{se}(3)$
$\tilde{\cdot}$	=	isomorphism $\tilde{\cdot}: \mathbb{R}^3 \rightarrow \mathfrak{so}(3)$
Φ_{ξ}	=	strain basis in the geometric variable strain parameterization
$\mathcal{X}(s)$	=	cross-section reference frame of the soft link at coordinate s
Ω_i	=	spin rate of the i th control moment gyro
$\mathbf{w}_j$	=	body wrench acting on the j th rigid body

$\bar{\mathbf{w}}_{\mathcal{X}}$ = distributed wrench acting on the soft link at frame $\mathcal{X}$

I. Introduction

THE expansion and long-term sustainability of space infrastructure necessitate advanced on-orbit robotic capabilities to service aging spacecraft, perform active debris removal, and facilitate the manufacturing and assembly of large orbital structures [1]. Space manipulators, defined as spacecraft equipped with one or more robotic arms mounted on the primary platform, will play a crucial role in addressing these challenges [2,3], as evidenced by numerous space missions designed as technology demonstrators for such platforms. A seminal example is the Engineering Test Satellite VII (ETS-VII), launched in 1997 by Japan's National Space Development Agency (NASDA, now part of JAXA), which pioneered the first in-orbit deployment of a spacecraft-mounted robotic arm [4]. In 2007, DARPA executed the Orbital Express mission, validating foundational technologies for autonomous on-orbit servicing [5]. More recently, the European Space Agency (ESA) initiated Clear-Space-1, an in-orbit demonstration mission scheduled for launch in 2026. This mission will advance robotic debris removal by capturing and deorbiting a defunct satellite using a multimanipulator capture system [6].

Autonomous operation of orbiting space manipulators poses significant challenges, arising from the inherent complexity of their dynamics and the unstructured nature of the space environment. To address these challenges, a substantial body of research has focused on system modeling, control design, trajectory planning, sensor integration, and mission architecture optimization, as outlined in several surveys [2,3,7].

From a modeling and control design perspective, most studies to date focus on rigid space manipulators—systems comprising rigid bodies (the spacecraft base and manipulator links) interconnected by rigid joints [8–11]. A notable extension to this framework integrates thrusters and momentum exchange devices into the rigid spacecraft-manipulator model [12]. However, to comply with payload constraints, spacecraft-mounted manipulators are often designed as lightweight structures [13], rendering their flexible modes susceptible to excitation during high-amplitude/high-speed maneuvers or interactions with massive payloads [14]. Unlike terrestrial manipulators, these systems exhibit exacerbated dynamic coupling due to spacecraft base motion, thereby inducing accelerations in the manipulator appendages. Consequently, several studies have incorporated flexibility into system analysis and control design methodologies.

A prevalent approach treats the base as a rigid body while modeling elastic deformations in slender components, such as manipulator links [15], flexible joints [14], and solar panels [16].

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To ensure stability and achieve accurate end-effector positioning in the presence of flexibility, a variety of methods have been proposed. A popular strategy is to employ active vibration damping [17–19], which depends heavily on the availability of vibration measurements and can lead to actuator saturation. More complex approaches, such as H_∞ -based designs [15] or sliding-mode control [20], simultaneously account for flexibility effects and unknown disturbances.

Recent studies have proposed an alternative paradigm to traditional rigid or rigid-flexible space manipulator architectures. The seminal work by Doggett et al. [21,22] describes the development and prototyping of the Tendon-Actuated Lightweight In-Space Manipulator (TALISMAN), a novel long-reach, general-purpose robotic system powered by tendon-actuated joints. In this architecture, joint motion is produced via cables actuated by tension at the cable base, emulating the biomechanical behavior of human tendons. Compared to heritage designs such as the Canadarms [23], TALISMAN's tendon-driven actuation substantially reduces the mass associated with motors and gearboxes, since these components need not be located at the joints.

Building on the TALISMAN concept, subsequent research focuses on developing comprehensive kinematic and dynamic models for tendon-actuated space manipulators. For example, Li et al. [24] model a four-degree-of-freedom (4-DoF) tendon-actuated system as an equivalent rigid multibody system and design a controller to simultaneously stabilize the spacecraft base attitude and track end-effector trajectories. They show that the joint torque required to actuate the tendon-driven system is significantly lower than that of similarly sized space manipulators with conventional revolute joints. A potential drawback of the pulley configuration proposed in [21,22,24], however, is the increased size and complexity of the robotic arm.

To address these issues, Kernot and Ulrich [25] investigated a cable-sheath transmission mechanism (CSTM) for a multilink space manipulator. While the CSTM design reduces the system's mass and volume, it suffers from friction-induced losses and hysteresis. Consequently, they develop two adaptive controllers to capture a non-cooperative space target: a direct one based on $\mathcal{L}_1$ theory and an indirect one extending the approach in [26] to the multilink case. They validated their methodology through laboratory experiments and verified that the $\mathcal{L}_1$ controller yields the best performance when tracking a dynamic reference signal.

In this paper, we build on the advantages of tendon-driven actuation for robotic arms. However, rather than using cable actuators to drive a chain of rigid links, we route them along the backbone of a soft component, which serves as the distal link. This approach draws on recent advances in continuum (or “soft”) manipulators—a class of robotic systems that can deform continuously to emulate the natural motions found in humans and animals [27]. In contrast to the rigid designs in [21,22,24,25], our continuum, highly compliant interface is more amenable to environmental interaction, opening new possibilities for grasping, in-space manipulation, and complex maintenance operations.

The first tendon-driven continuum spacecraft-mounted manipulator was introduced in [28], where the authors describe the dynamic model of a novel Continuum-Manipulator Space Robot (CMSR). The CMSR in [28] consists of two tendon-driven continuum manipulators, modeled as Euler–Bernoulli beams, mounted on a satellite base that also supports two flexible solar panels. The multibody dynamics of the coupled base–manipulator–panel system are derived using a Lagrangian framework, and the resulting model comprises a set of implicit differential–algebraic equations (DAEs). To mitigate the beating phenomenon in the solar arrays, the authors propose tuning the physical parameters of the system, such as the diameter and bending stiffness of the manipulators. In this work, the manipulator is discretized into a set of nodes, and an interpolation scheme reconstructs the position and orientation of the beam cross-sections between consecutive nodes. Notably, this approach yields a high-dimensional, nonminimal set of equations of motion, wherein the continuum manipulator's state is represented by the global coordinates of its discrete nodes.

The CMSR model is used in [29] to develop an integrated controller to track a reference motion for the base and the manipulators while simultaneously suppressing disturbances on the solar arrays induced by the platform's motion and accounting for actuator saturation. The original nonlinear optimal control problem is first transformed into a mixed complementarity problem (MiCP) [30] to enforce actuator limits and subsequently reformulated as a set of continuous, differentiable equations solved via a nonlinear root-finding scheme. Notably, the desired manipulator trajectory is not specified in terms of end-effector task-space coordinates and velocities but rather through prescribed time-varying curvature values for the manipulator segments.

The CMSR-based control design introduced in [29] was further developed in [31]. In that work, the DAE model is first discretized into a set of nonlinear algebraic equations using the generalized- α algorithm [32], and a state transition matrix (STM) is then obtained through implicit differentiation of such equations with respect to the system state. A nonlinear MPC formulation leveraging the derived STM is subsequently proposed to track two task-space trajectories with the tips of the continuum manipulators while maintaining a steady base and suppressing vibrations in the solar arrays. Although this approach successfully achieves trajectory tracking with the continuum manipulators, it remains computationally intensive due to the required discretization, implicit differentiation, and use of the MPC framework.

Our work differs from [28,29,31] in two main ways. First, from a configuration standpoint, we study a hybrid manipulator rather than a purely soft one by mounting a tendon-driven soft link on a conventional 2-DoF rigid spacecraft manipulator. This design leverages the advantages of both rigid and soft architectures. On one hand, it provides a distal compliant link specifically designed to interact with the environment. On the other hand, it mitigates the energy losses and accuracy degradation commonly associated with longer continuum segments, which are caused by friction between cable actuators and routing disks. Second, in terms of mathematical formulation, our approach yields a minimal set of ordinary differential equations in local coordinates (e.g., joint displacements), rather than a high-dimensional set of DAEs in global coordinates, greatly simplifying the analysis and control design activity.

A. Contributions

The main contribution of this paper is the introduction, modeling, and preliminary control design of a novel hybrid soft–rigid space manipulator system (HSMS). In particular, we investigate a manipulator system in which a continuum appendage is affixed to the distal end of a multilink rigid arm. By integrating a tendon-driven continuum segment with a rigid robotic arm, the proposed design aims to harness the compliance and lightweight attributes of soft robotics while preserving the maneuvering accuracy commonly associated with rigid arms—thus offering the potential to exploit the advantages of both purely soft and purely rigid manipulator designs. To the best of our knowledge, this is the first instance of a hybrid soft–rigid satellite-mounted manipulator in the literature.

Specifically, we build on the state-of-the-art in reduced-order modeling of hybrid multibody systems [33] to provide, for the first time, a minimal-order, closed-form expression of the equations of motion of an HSMS. In contrast to existing approaches for spacecraft-mounted continuum manipulators, our model is easy to work with and thus is particularly suitable for control design. To substantiate this claim, we augment the multibody description with a set of spaceborne actuators, such as thrusters and single-gimbal variable-speed control moment gyros (VSCMGs), and propose two baseline inverse-dynamics controllers. These controllers are formulated in both task space and configuration space, wherein the control inputs are computed in terms of low-level variables such as gimbal rates, wheel accelerations, and thruster forces. Despite the overall system complexity and the controllers' relative simplicity, numerical simulations demonstrate excellent performance, indicating that our approach constitutes a promising candidate for preliminary studies of space missions involving hybrid soft–rigid manipulators.

II. Problem Formulation

In this section, we present the kinematic and dynamic model of an HSMS. We specifically consider an HSMS composed of $R + 1$ rigid bodies—namely, the spacecraft base and R rigid manipulator links—connected by R revolute joints, and a tendon-driven continuum appendage rigidly attached to the tip of the distal rigid link. A representative configuration for the case $R = 2$ is illustrated in Fig. 1. The spacecraft base is fully actuated by thrusters and momentum exchange devices that exert a wrench on its 6 DoFs. The control inputs for the hybrid manipulator consist of torques applied to the R rigid joints and tensions applied to the C cable actuators. It should be noted that, in this paper, although we assume a specific configuration, the proposed formulation is sufficiently general to be adapted to any spacecraft supporting multiple manipulators that combine both soft and rigid elements.

A. Hybrid Soft–Rigid Kinematics

1. Preliminaries

We define a global inertial reference frame $\mathcal{I}$; a spacecraft body frame $\mathcal{B}$ whose axes are aligned with the spacecraft base principal inertia axes; manipulator link frames $\mathcal{R}_i$ ($i = 1, \dots, R$), where each $\mathcal{R}_i$ is attached to the center of mass of the i th rigid manipulator link; and frame $\mathcal{X}$ is associated with a cross-section of the soft link, where $s \in [0, L]$ is the corresponding curvilinear abscissa. The base frame $\mathcal{F}$ of the soft link is defined as $\mathcal{F} \triangleq \mathcal{X}(s = 0)$; likewise, the end-effector frame $\mathcal{E}$ is attached to the tip of the soft element, $\mathcal{E} \triangleq \mathcal{X}(s = L)$. The above-defined frames are illustrated in Fig. 1.

The configuration of the rigid part of the HSMS is characterized by the pose of the base together with the R joint coordinates, and it is denoted by $\mathbf{q}_R \triangleq [\mathbf{q}_B^T, q_1, \dots, q_R]^T \in \mathbb{R}^{6+R}$, where $\mathbf{q}_B$ is a six-dimensional vector expressing the degrees of freedom of an ideal free joint between frame $\mathcal{I}$ and the spacecraft frame $\mathcal{B}$, and $q_1, \dots, q_R$ are the joint angles. The relative transformations between subsequent rigid frames are composed as follows:

$$\mathbf{g}_{\mathcal{I}/\mathcal{F}}(\mathbf{q}_R) = \mathbf{g}_{\mathcal{I}/\mathcal{B}}(\mathbf{q}_B) \mathbf{g}_{\mathcal{B}/\mathcal{R}_1}(q_1) \cdots \mathbf{g}_{\mathcal{R}_{R-1}/\mathcal{R}_R}(q_R) \mathbf{g}_{\mathcal{R}_R/\mathcal{F}} \quad (1)$$

where $\mathbf{g}_{\mathcal{Y}/\mathcal{Z}} \in \text{SE}(3)$ is the pose of frame $\mathcal{Z}$ with respect to frame $\mathcal{Y}$, and $\mathbf{g}_{\mathcal{R}_R/\mathcal{F}}$ is a constant pose offset between the R th link frame $\mathcal{R}_R$ and the root of the soft link $\mathcal{F}$.

The configuration of the soft appendage is described by $\mathbf{g}_{\mathcal{F}/\mathcal{X}}(\cdot) : [0, L] \mapsto \text{SE}(3)$, parameterized by the arc length coordinate s such that $\mathbf{g}_{\mathcal{F}/\mathcal{E}} = \mathbf{g}_{\mathcal{F}/\mathcal{X}}(s = L)$. The hybrid forward kinematics for the HSMS stems from the composition of the rigid and soft kinematics:

$$\mathbf{g}_{\mathcal{I}/\mathcal{E}} = \mathbf{g}_{\mathcal{I}/\mathcal{F}}(\mathbf{q}_R) \mathbf{g}_{\mathcal{F}/\mathcal{X}}(s = L) = \mathbf{g}_{\mathcal{I}/\mathcal{F}}(\mathbf{q}_R) \mathbf{g}_{\mathcal{F}/\mathcal{E}} \quad (2)$$

While the configuration of the rigid subsystem is described by the pose of the base and the manipulator joint coordinates, the continuum appendage is defined by its strain field $\hat{\xi}(s) \in \mathfrak{se}(3)$,

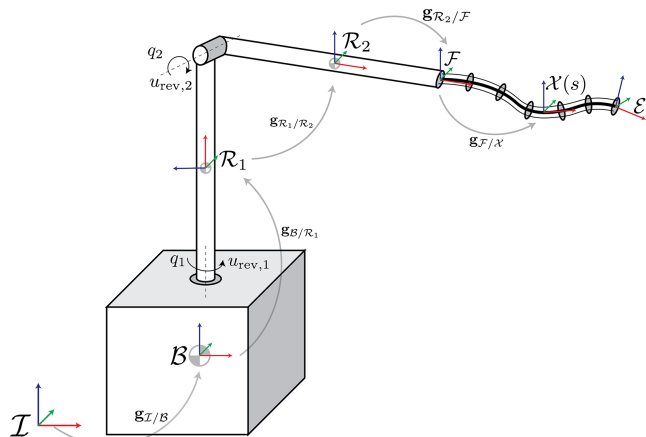


Fig. 1 Schematic of a hybrid space manipulator system with three rigid bodies ($R = 2$).

$$\hat{\xi}(s) = \mathbf{g}^{-1}(s) \dot{\mathbf{g}}'(s) \quad (3)$$

where $(\cdot)' \triangleq \partial/\partial s(\cdot)$. The velocity field $\hat{\eta}(s) \in \mathfrak{se}(3)$, also referred to as body twist or, simply, velocity, is defined as

$$\hat{\eta}(s) = \mathbf{g}^{-1}(s) \dot{\mathbf{g}}(s) \quad (4)$$

where $(\dot{\cdot}) \triangleq \partial/\partial t(\cdot)$.

We introduce the “hat” symbol $\hat{\cdot}$ to denote the isomorphism $\hat{(\cdot)} : \mathbb{R}^6 \mapsto \mathfrak{se}(3)$, and, analogously, the “tilde” symbol to denote the isomorphism $\tilde{(\cdot)} : \mathbb{R}^3 \mapsto \mathfrak{so}(3)$, where $\mathfrak{so}(3)$ is the vector space of the skew-symmetric matrices in $\mathbb{R}^{3 \times 3}$.

Therefore, given $\mathbf{k}, \mathbf{p}, \boldsymbol{\omega}, \mathbf{v} \in \mathbb{R}^3$, the vector of strain components is $\xi = [\mathbf{k}^T, \mathbf{p}^T]^T \in \mathbb{R}^6$, the velocity vector is $\eta = [\boldsymbol{\omega}^T, \mathbf{v}^T]^T \in \mathbb{R}^6$, and the following expressions hold:

$$\hat{\xi} = \begin{bmatrix} \tilde{\mathbf{k}} & \mathbf{p} \\ \mathbf{0}_{1 \times 3} & 0 \end{bmatrix} \in \mathfrak{se}(3), \quad \hat{\eta} = \begin{bmatrix} \tilde{\boldsymbol{\omega}} & \mathbf{v} \\ \mathbf{0}_{1 \times 3} & 0 \end{bmatrix} \in \mathfrak{se}(3) \quad (5)$$

In Eq. (5), $\mathbf{k}$ and $\mathbf{p}$ represent the angular and linear components of the strain in the cross-sectional $\mathcal{X}$ -frame, respectively, while $\boldsymbol{\omega}$ and $\mathbf{v}$ are the angular and linear components of the body velocity of frame $\mathcal{X}(s)$.

The kinematic relationships in Eqs. (2) and (4) can be combined to yield the end-effector velocity:

$$\hat{\eta}_{\mathcal{I}/\mathcal{E}}^{\mathcal{E}} = \mathbf{g}_{\mathcal{I}/\mathcal{E}}^{-1} \dot{\mathbf{g}}_{\mathcal{I}/\mathcal{E}} \quad (6)$$

where, in general, vector $\eta_{\mathcal{X}/\mathcal{Y}}^{\mathcal{Z}} \in \mathbb{R}^6$ is read as “the velocity components of frame $\mathcal{Y}$ with respect to frame $\mathcal{X}$, expressed in frame $\mathcal{Z}$ ”. In what follows, when referring to the so-called “body velocities,” i.e., when $\mathcal{Z} \equiv \mathcal{Y}$, we will omit the superscript for brevity. Formally, we write $\hat{\eta}_{\mathcal{X}/\mathcal{Y}} \triangleq \hat{\eta}_{\mathcal{X}/\mathcal{Y}}^{\mathcal{Y}}$. Expanding the terms in Eq. (6) and introducing the Adjoint representation $\text{Ad}_{\mathbf{g}}$ —which transforms twists between different coordinate frames—leads to

$$\hat{\eta}_{\mathcal{I}/\mathcal{E}} = \mathbf{g}_{\mathcal{I}/\mathcal{E}}^{-1} \left(\dot{\mathbf{g}}_{\mathcal{I}/\mathcal{B}} \mathbf{g}_{\mathcal{B}/\mathcal{E}} + \cdots + \mathbf{g}_{\mathcal{I}/\mathcal{R}_{R-1}} \dot{\mathbf{g}}_{\mathcal{R}_{R-1}/\mathcal{R}_R} \mathbf{g}_{\mathcal{R}_R/\mathcal{E}} + \mathbf{g}_{\mathcal{I}/\mathcal{F}} \dot{\mathbf{g}}_{\mathcal{F}/\mathcal{E}} \right) \quad (7)$$

$$= \mathbf{g}_{\mathcal{B}/\mathcal{E}}^{-1} \hat{\eta}_{\mathcal{I}/\mathcal{B}} \mathbf{g}_{\mathcal{B}/\mathcal{E}} + \cdots + \mathbf{g}_{\mathcal{R}_R/\mathcal{E}}^{-1} \hat{\eta}_{\mathcal{R}_{R-1}/\mathcal{R}_R} \mathbf{g}_{\mathcal{R}_R/\mathcal{E}} + \hat{\eta}_{\mathcal{F}/\mathcal{E}} \quad (8)$$

$$= \text{Ad}_{\mathbf{g}_{\mathcal{B}/\mathcal{E}}}^{-1} (\hat{\eta}_{\mathcal{I}/\mathcal{B}}) + \cdots + \text{Ad}_{\mathbf{g}_{\mathcal{R}_R/\mathcal{E}}}^{-1} (\hat{\eta}_{\mathcal{R}_{R-1}/\mathcal{R}_R}) + \hat{\eta}_{\mathcal{F}/\mathcal{E}} \quad (9)$$

where we have used Eq. (4) to rewrite the derivatives $\dot{\mathbf{g}}$ in Eq. (7). Equation (9) shows that the body velocity of the end-effector is obtained as the summation of the body velocities along the kinematic chain, where each contribution is acted upon by the Adjoint $\text{Ad}_{\mathbf{g}}$, resulting in a change of coordinates to the end-effector frame (see the Appendix for a detailed definition).

To conclude the discussion, we derive a more fundamental relationship that involves the joint rates for the rigid links and the strain field rate for the soft appendage. To this end, a few more considerations are in order. First, notice that equating the mixed partial derivatives of $\mathbf{g}$, $\partial/\partial t(\mathbf{g}') = \partial/\partial s(\dot{\mathbf{g}})$, leads to the compatibility equation at the strain and velocity level:

$$\hat{\eta}' = \dot{\hat{\xi}} - \text{ad}_{\hat{\xi}}(\hat{\eta}) \quad (10)$$

where the Lie algebra adjoint $\text{ad}_{\hat{\xi}}$ results directly from differentiation of $\text{Ad}_{\mathbf{g}}$ with respect to $\mathbf{g}$ (see the Appendix). Then, an analytical expression for the velocity field $\hat{\eta}(s)$ is obtained by integrating Eq. (10), as shown in the following lemma.

Lemma 1: Consider the compatibility relation expressed by Eq. (10). Then,

$$\hat{\eta}(s) = \text{Ad}_{g(s)}^{-1} \left(\int_0^s \text{Ad}_{g(\sigma)}(\dot{\hat{\xi}}(\sigma)) d\sigma \right) \quad (11)$$

Proof: We apply the linear map $\text{Ad}_g(\cdot)$ to both sides of Eq. (10), which yields

$$\text{Ad}_g(\hat{\eta}') + \text{Ad}_g(\text{ad}_{\hat{\xi}}(\hat{\eta})) = \text{Ad}_g(\dot{\hat{\xi}}) \quad (12)$$

and we recall that, by Proposition 9.3.8 in [34],

$$\frac{\partial}{\partial s}(\text{Ad}_g(\hat{\eta})) = \text{Ad}_g(\text{ad}_{\hat{\xi}}(\hat{\eta}) + \hat{\eta}') \quad (13)$$

The result in Eq. (13) can be used to rewrite Eq. (12), $(\partial/\partial s)(\text{Ad}_g(\hat{\eta})) = \text{Ad}_g(\dot{\hat{\xi}})$, whose integral is readily obtained as

$$\text{Ad}_{g(s)}(\hat{\eta}(\sigma)) \Big|_0^s = \int_0^s \text{Ad}_{g(\sigma)}(\dot{\hat{\xi}}(\sigma)) d\sigma \quad (14)$$

and, since $\hat{\eta}(0) = \mathbf{0}$,

$$\hat{\eta}(s) = \text{Ad}_{g(s)}^{-1} \left(\int_0^s \text{Ad}_{g(\sigma)}(\dot{\hat{\xi}}(\sigma)) d\sigma \right) \quad (15)$$

thus confirming the claim. $\square$

No assumptions were made in deriving Eq. (15), which is therefore equally valid for rigid and soft links. When dealing with rigid bodies, the preceding expressions simplify considerably and lead to the conventional velocity-level recursion for a rigid kinematic chain [33]. In fact, the transformation induced on a rigid link by the motion of its parent joint can be regarded as the composition of a sequence of infinitesimally small intermediate transformations encoded by the joint twist. The overall pose transformation is then obtained by integrating these infinitesimal motions from the initial configuration (denoted by $s = 0$, with a slight abuse of notation) to the final configuration at $s = 1$. Here, the joint twist, which remains constant between $s = 0$ and $s = 1$, serves as the infinitesimal generator of such transformation. Conversely, for a continuum link, the strain represents the infinitesimal part of the transformation from the base of the rod to its tip. This conceptual equivalence enables the treatment of both rigid and soft elements within the same framework, with the key difference being that, for rigid links, the strain $\hat{\xi}$ (i.e., the joint twist) is constant, whereas for a soft appendage it is a continuous function of the coordinate s along the soft link.

2. Rigid End-Effector Jacobian

Consider now a rigid body and the linear differential equation stemming from Eq. (3), $\mathbf{g}'(s) = \mathbf{g}(s)\hat{\xi}(s)$ with initial condition $\mathbf{g}(0) = \mathbf{I}_4$. Initial value problems of this type were investigated by Magnus [35], who proposed the solution

$$\mathbf{g}(s) = \exp(\mathbf{\Omega}(s)) \quad (16)$$

where $\mathbf{\Omega}(s)$ is an infinite series known as Magnus expansion. We refer the reader to [36,37] for a more thorough treatment of the underlying mathematical details. Here we only report the truncated Magnus expansion of the strain field:

$$\mathbf{\Omega}(s) = \int_0^s \hat{\xi}(\sigma) d\sigma - \frac{1}{2} \int_0^s \left[\int_0^\sigma \hat{\xi}(\tau) d\tau, \hat{\xi}(\sigma) \right] d\sigma + \dots \quad (17)$$

Similarly to the second integral in Eq. (17), higher-order terms in the expansion are computed through nested commutators of the integral of the strain field with the strain itself. For a rigid link, $\hat{\xi}$ is constant, and therefore the commutator is null, and thus the Magnus expansion reduces to $\mathbf{\Omega}(s) = s\hat{\xi}$. Recalling Eq. (16), we obtain the familiar expression of a pose transformation expressed as the exponential of the correspondent joint twist [38]:

$$\mathbf{g}(s) = \exp(s\hat{\xi}) \quad (18)$$

Substituting Eq. (18) into Eq. (11) and using that $\text{Ad}_{\exp(s\hat{\xi})} = \exp(s\text{ad}_{\hat{\xi}})$ yields

$$\hat{\eta}(1) = \text{Ad}_{\exp(\hat{\xi})}^{-1} \left(\int_0^1 \exp(s\text{ad}_{\hat{\xi}})(\dot{\hat{\xi}}(s)) ds \right) \quad (19)$$

The meaning of Eq. (19) can be further clarified when considering the case of a one-dimensional (e.g., revolute) joint undergoing a displacement q . In this case, $\mathbf{g}(s) = \exp(s\hat{\xi}q)$, where $s \in [0,1]$ acts as a parameter denoting the infinitesimal states between the initial and final configuration. Then, Eq. (19) becomes

$$\hat{\eta}(1) = \text{Ad}_{\exp(\hat{\xi}q)}^{-1} \left(\int_0^1 \exp(sq\text{ad}_{\hat{\xi}})(\hat{\xi}) ds \dot{q} \right) \quad (20)$$

Since $\hat{\xi}$ commutes with itself, it follows that $\text{ad}_{\hat{\xi}}(\hat{\xi}) = \mathbf{0}$. Consequently, the exponential series $\exp(sq\text{ad}_{\hat{\xi}}) = \mathbf{I} + sq\text{ad}_{\hat{\xi}} + [(sq)^2/2!]\text{ad}_{\hat{\xi}}^2 + \dots$ simplifies such that the argument of the integral reduces to $\hat{\xi}$. This, in turn, leads to

$$\hat{\eta}(1) = \text{Ad}_{\exp(\hat{\xi}q)}^{-1}(\hat{\xi}\dot{q}) \quad (21)$$

Exploiting again that $\hat{\xi}$ commutes with itself, the body twist further simplifies to

$$\hat{\eta}(1) = \hat{\xi}\dot{q} \quad (22)$$

Applying the result in Eq. (22) to the rigid-body velocities in Eq. (9), we obtain

$$\boldsymbol{\eta}_{I/E} = \text{Ad}_{g_{B/E}}^{-1}\boldsymbol{\xi}_B\dot{\mathbf{q}}_B + \dots + \text{Ad}_{g_{R/E}}^{-1}\boldsymbol{\xi}_R\dot{\mathbf{q}}_R + \boldsymbol{\eta}_{F/E} \quad (23)$$

$$= \left[\text{Ad}_{g_{B/E}}^{-1}\boldsymbol{\xi}_B, \dots, \text{Ad}_{g_{R/E}}^{-1}\boldsymbol{\xi}_R \right] \dot{\mathbf{q}}_R + \boldsymbol{\eta}_{F/E} \quad (24)$$

$$= \mathbf{J}_R^E(\mathbf{q}_R)\dot{\mathbf{q}}_R + \boldsymbol{\eta}_{F/E} \quad (25)$$

where $\boldsymbol{\eta}_{I/B} = \boldsymbol{\xi}_B\dot{\mathbf{q}}_B$ is a trivial equality since the free-floating base is modeled as a free joint (hence, $\boldsymbol{\xi}_B$ is the identity), and $\mathbf{J}_R^E$ is the rigid end-effector Jacobian defined by

$$\mathbf{J}_R^E(\mathbf{q}_R) = \left[\text{Ad}_{g_{B/E}}^{-1}\boldsymbol{\xi}_B, \dots, \text{Ad}_{g_{R/E}}^{-1}\boldsymbol{\xi}_R \right] \quad (26)$$

3. Soft Link

The formulation presented above cannot be extended to the soft appendage of the HSMS, where the strain field varies with s . In this case, one must truncate the Magnus expansion and numerically approximate its integral terms over a discrete set of nodes along the backbone. Other kinematic quantities, such as velocities and accelerations, can be evaluated at the nodes based on the approximated Magnus expansion. For a comprehensive overview of the solution strategy for soft elements, we refer the reader to [33].

4. Geometric Variable Strain Approach

As discussed previously, the strain field defines the configuration of the soft link in a manner analogous to how joint coordinates determine the configuration of rigid joints. However, while the rigid subsystem is described by a finite-dimensional configuration space, the strain field $\hat{\xi}$ is infinite-dimensional. In this section, and in order to make the problem tractable both analytically and numerically, we approximate the strain field by parameterizing it using a set of basis functions $\boldsymbol{\Psi}_{\hat{\xi}}$ [33] to obtain

$$\xi(s, q_S, t) = \Psi_\xi(s, q_S, t) + \xi^*(s) \quad (27)$$

where $\Psi_\xi: \mathbb{R} \times \mathbb{R}^S \times \mathbb{R} \mapsto \mathbb{R}^6$ is a function of the arc-length coordinate, of some generalized discretizing coordinates $q_S \in \mathbb{R}^S$ and of time, and ξ^* is a reference strain state. The approach, called geometric variable strain (GVS), was initially proposed in [39] and further developed in [40,41]. Its main advantage over other methods (e.g., finite-element approximations) is that it requires fewer degrees of freedom, rendering it particularly promising for onboard implementation. Moreover, the user maintains full control over the type and order of the GVS basis functions, allowing for an optimal balance between modeling accuracy and computational cost.

To continue with our analysis, hereafter, and for the sake of simplicity, the coordinates q_S will only appear linearly, and Eq. (27) can be reformulated as

$$\xi(s, q_S) = \Phi_\xi(s) q_S + \xi^*(s) \quad (28)$$

with $\Phi_\xi: \mathbb{R} \mapsto \mathbb{R}^{6 \times S}$. Remarkably, the proposed strain parameterization affords designers the flexibility to select an appropriate functional space to approximate the strain state, based on considerations such as the presence/absence of distributed and/or concentrated external loads, the configuration of the soft actuators, the tradeoff between accuracy and computational effort, etc. Furthermore, we will demonstrate that when the GVS approach is used to model a hybrid soft-rigid multibody system, it yields a minimal set of differential equations whose structure closely mirrors that of conventional rigid robotic manipulator models.

We now conclude the description of the hybrid kinematics of the HSMS. Inserting Eq. (28) into Eq. (11) and choosing the null strain state as the reference, $\xi^*(s) = \mathbf{0}_{6 \times 1}$, we obtain

$$\eta(s) = \text{Ad}_{g(s)}^{-1} \int_0^s \text{Ad}_{g(\sigma)} \Phi_\xi(\sigma) d\sigma \dot{q}_S \quad (29)$$

in which the right-hand side can be approximated, for example, with a quadrature method, and evaluated at a discrete set of nodes. Finally, Eq. (29) is substituted into the soft link velocity $\eta_{\mathcal{F}/\mathcal{E}}$ in Eq. (25), leading to

$$\eta_{\mathcal{I}/\mathcal{E}} = J_R^\mathcal{E}(q_R) \dot{q}_R + \text{Ad}_{g_{\mathcal{F}/\mathcal{E}}}^{-1} \int_0^L \text{Ad}_{g_{\mathcal{F}/\mathcal{X}}} \Phi_\xi(s) ds \dot{q}_S \quad (30)$$

$$= J_R^\mathcal{E}(q_R) \dot{q}_R + J_S^\mathcal{E}(q_S) \dot{q}_S \quad (31)$$

$$= J^\mathcal{E}(q) \dot{q} \quad (32)$$

where $q = [q_R^\top, q_S^\top]^\top \in \mathbb{R}^{n=6+R+S}$ is constructed by stacking the rigid degrees of freedom and the generalized strain coordinates. In Eqs. (31) and (32) we have defined the soft end-effector Jacobian

$$J_S^\mathcal{E} = \text{Ad}_{g_{\mathcal{F}/\mathcal{E}}}^{-1} \int_0^L \text{Ad}_{g_{\mathcal{F}/\mathcal{X}}} \Phi_\xi(s) ds \quad (33)$$

and the hybrid end-effector Jacobian that maps both rigid- and soft-body velocities into the end-effector task-space velocity:

$$J^\mathcal{E} = [J_R^\mathcal{E}, J_S^\mathcal{E}] \quad (34)$$

Note that a further acceleration-level relationship can be constructed by differentiating Eq. (32) [33] to yield

$$\dot{\eta}_{\mathcal{I}/\mathcal{E}} = \dot{J}^\mathcal{E}(q, \dot{q}) \dot{q} + J^\mathcal{E}(q) \ddot{q} \quad (35)$$

B. Hybrid Soft-Rigid Dynamics

The dynamic model of the HSMS can be derived in a systematic way by applying the D'Alembert's form of the principle of virtual work (PVW) for a multibody system in the presence of internal

elastic energy. Throughout the derivation, we assume that the relationship between the strain components ξ and the internal wrench μ along the soft rod is described by the linear law $\mu = \Sigma \xi$, where Σ is the stiffness matrix.

1. Principle of Virtual Work

For any rigid body j in the HSMS, the following holds:

$$\frac{\partial}{\partial t} \mathcal{E}_j^\mathcal{I} = \mathbf{w}_j^\mathcal{I} \quad (36)$$

where $\mathcal{E}_j^\mathcal{I} = \mathbf{M}_j \eta_{\mathcal{I}/j}^\mathcal{I}$ is the body momentum, $\mathbf{M}_j$ is the constant body inertia matrix, $\eta_{\mathcal{I}/j}^\mathcal{I}$ is the body velocity, and $\mathbf{w}_j^\mathcal{I}$ is the net wrench acting on j . The net wrench $\mathbf{w}_j^\mathcal{I}$ can be expanded as $\mathbf{w}_j^\mathcal{I} = \mathbf{w}_{j,e}^\mathcal{I} + \mathbf{w}_{j,a}^\mathcal{I} + \mathbf{w}_{j,c}^\mathcal{I}$, where $\mathbf{w}_{j,e}^\mathcal{I}$, $\mathbf{w}_{j,a}^\mathcal{I}$, and $\mathbf{w}_{j,c}^\mathcal{I}$ denote the wrenches due to external factors (e.g., gravity, disturbances), to the actuation and to the constraints, respectively. Equation (36) can be expressed in body coordinates recalling that $\mathcal{E}_j^\mathcal{I} = \text{Ad}_{g_{\mathcal{I}/j}}^* \mathcal{E}_j$ and $\mathbf{w}_j^\mathcal{I} = \text{Ad}_{g_{\mathcal{I}/j}}^* \mathbf{w}_j$,

$$\frac{\partial}{\partial t} (\text{Ad}_{g_{\mathcal{I}/j}}^* \mathcal{E}_j) = \text{Ad}_{g_{\mathcal{I}/j}}^* (\mathbf{w}_{j,e} + \mathbf{w}_{j,a} + \mathbf{w}_{j,c}) \quad (37)$$

Then, exploiting the identity $(\partial/\partial t) \text{Ad}_{g_{\mathcal{I}/j}}^* = \text{Ad}_{g_{\mathcal{I}/j}}^* \text{ad}_{\eta_{\mathcal{I}/j}}^*$, Eq. (37) becomes

$$\text{Ad}_{g_{\mathcal{I}/j}}^* \frac{\partial}{\partial t} \mathcal{E}_j + \text{Ad}_{g_{\mathcal{I}/j}}^* \text{ad}_{\eta_{\mathcal{I}/j}}^* \mathcal{E}_j = \text{Ad}_{g_{\mathcal{I}/j}}^* (\mathbf{w}_{j,e} + \mathbf{w}_{j,a} + \mathbf{w}_{j,c}) \quad (38)$$

$$\mathbf{M}_j \dot{\eta}_{\mathcal{I}/j} + \text{ad}_{\eta_{\mathcal{I}/j}}^* \mathbf{M}_j \eta_{\mathcal{I}/j} = \mathbf{w}_{j,e} + \mathbf{w}_{j,a} + \mathbf{w}_{j,c} \quad (39)$$

The dynamic equilibrium for frame $\mathcal{X}$, which is affixed to the center of a cross-section of the soft rod at coordinate s , has a similar formulation [33]:

$$\bar{\mathbf{M}}_{\mathcal{X}} \dot{\eta}_{\mathcal{I}/\mathcal{X}} + \text{ad}_{\eta_{\mathcal{I}/\mathcal{X}}}^* \bar{\mathbf{M}}_{\mathcal{X}} \eta_{\mathcal{I}/\mathcal{X}} = (\mu - \mathbf{w}_{\mathcal{X},a})' + \text{ad}_{\xi}^* (\mu - \mathbf{w}_{\mathcal{X},a}) + \bar{\mathbf{w}}_{\mathcal{X},e} \quad (40)$$

where $\bar{\mathbf{M}}_{\mathcal{X}}$ is the cross-section inertia matrix, $\mathbf{w}_{\mathcal{X},a}$ is the actuation wrench provided by the internal actuators, and $\bar{\mathbf{w}}_{\mathcal{X},e}$ is the net external load per unit of material length.

A principled method for deriving the multibody dynamic model of the HSMS is to employ D'Alembert's formulation of the PVW [42], stated as

$$\delta W = \delta W_{\text{int}} - \delta W_{\text{ext}} = 0 \quad (41)$$

The internal virtual work δW_{int} is performed by the internal wrench μ and by the tendon actuation $\mathbf{w}_{\mathcal{X},a}$ along a compatible variation of the strain $\delta \xi$. That is,

$$\delta W_{\text{int}} = \int_0^L \delta \xi \cdot (\mu - \mathbf{w}_{\mathcal{X},a}) ds \quad (42)$$

The total external virtual work is the sum of the work accomplished by the body loads and by the inertial forces acting on each body, whose expressions are given in Eqs. (38) and (40):

$$\delta W_{\text{ext}} = \delta \mathbf{r}_B \cdot (\mathbf{w}_{B,e} + \mathbf{w}_{B,a} - (\mathbf{M}_B \dot{\eta}_{\mathcal{I}/B} + \text{ad}_{\eta_{\mathcal{I}/B}}^* \mathbf{M}_B \eta_{\mathcal{I}/B})) + \quad (43)$$

$$+ \sum_{i=1}^R \delta \mathbf{r}_{\mathcal{R}_i} \cdot (\mathbf{w}_{\mathcal{R}_i,e} + \mathbf{w}_{\mathcal{R}_i,a} - (\mathbf{M}_{\mathcal{R}_i} \dot{\eta}_{\mathcal{I}/\mathcal{R}_i} + \text{ad}_{\eta_{\mathcal{I}/\mathcal{R}_i}}^* \mathbf{M}_{\mathcal{R}_i} \eta_{\mathcal{I}/\mathcal{R}_i})) + \quad (44)$$

$$+ \int_0^L \delta \mathbf{r}_{\mathcal{X}} \cdot (\bar{\mathbf{w}}_{\mathcal{X},e} - (\bar{\mathbf{M}}_{\mathcal{X}} \dot{\eta}_{\mathcal{I}/\mathcal{X}} + \text{ad}_{\eta_{\mathcal{I}/\mathcal{X}}}^* \bar{\mathbf{M}}_{\mathcal{X}} \eta_{\mathcal{I}/\mathcal{X}})) ds \quad (45)$$

Note that we have neglected the constraint reaction forces, as they do not contribute to the work performed during any admissible virtual displacement of the system.

2. Actuation Models

The terms $\mathbf{w}_{B,a}$ and $\mathbf{w}_{R_i,a}$ in Eqs. (43) and (44), along with $\mathbf{w}_{\mathcal{X},a}$ in Eq. (42), represent the body wrenches applied to the various components of the system as a result of the control input $\mathbf{u} \in \mathbb{R}^m$. The latter includes, for instance, the input tension assigned to the tendon actuators and the commanded gimbal velocity for the control moment gyros cluster, among other actuator commands. In order to explicitly incorporate $\mathbf{u}$ in the dynamic model of the HSMS, we derive the mapping between the actuation wrenches and their corresponding control inputs for each actuator type.

For generality, we assume that the spacecraft base is actuated by a set of n_{thr} thrusters and by n_{mom} momentum exchange devices. In particular, we consider a cluster of n_{mom} single-gimbal VSCMGs arranged in a pyramid configuration, as shown in Fig. 2 for the case where $n_{\text{mom}} = 4$. A possible configuration for the reaction control system is shown in Fig. 3 for $n_{\text{thr}} = 12$. Therefore, $\mathbf{w}_{B,a} = \mathbf{w}_{\text{thr}} + \mathbf{w}_{\text{mom}}$, where, in turn, $\mathbf{w}_{\text{thr}} \triangleq [\mathbf{m}_{\text{thr}}^T, \mathbf{f}_{\text{thr}}^T]^T$ and $\mathbf{w}_{\text{mom}} \triangleq [\mathbf{m}_{\text{mom}}^T, \mathbf{0}_{1 \times 3}^T]^T$. Here, $\mathbf{m}$ and $\mathbf{f}$ denote the torques and forces, respectively.

The overall torque exerted by the thrusters on the spacecraft body is obtained as follows:

$$\mathbf{m}_{\text{thr}} = \sum_{i=1}^{n_{\text{thr}}} \mathbf{r}_{B/\text{thr},i}^B \times \mathbf{f}_{\text{thr},i} = \sum_{i=1}^{n_{\text{thr}}} \mathbf{r}_{B/\text{thr},i}^B \times \mathbf{e}_{\text{thr},i} \|\mathbf{f}_{\text{thr},i}\| \quad (46)$$

where $\mathbf{r}_{B/\text{thr},i}^B$ is the position of the i th thruster with respect to frame B written in body frame coordinates, and $\mathbf{f}_{\text{thr},i}$ is the i th thruster force, directed as $\mathbf{e}_{\text{thr},i} = \mathbf{f}_{\text{thr},i} / \|\mathbf{f}_{\text{thr},i}\|$. Similarly, the net thruster force applied to the base is

$$\mathbf{f}_{\text{thr}} = \sum_{i=1}^{n_{\text{thr}}} \mathbf{f}_{\text{thr},i} = \sum_{i=1}^{n_{\text{thr}}} \mathbf{e}_{\text{thr},i} \|\mathbf{f}_{\text{thr},i}\| \quad (47)$$

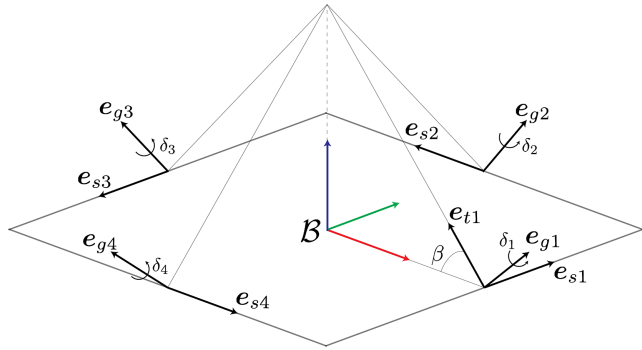


Fig. 2 Pyramid VSCMG configuration.

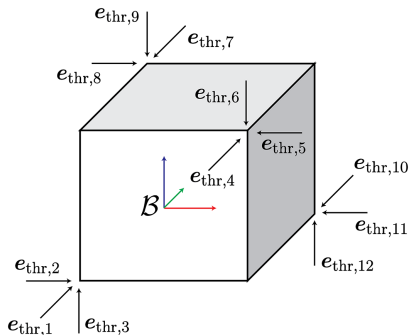


Fig. 3 Reaction control system configuration. Each $\mathbf{e}_{\text{thr},i}$ represents the direction of the force exerted by the i th thruster on the HSMS base.

The following thruster mapping matrices $\mathbf{H}_m \in \mathbb{R}^{3 \times n_{\text{thr}}}$ and $\mathbf{H}_f \in \mathbb{R}^{3 \times n_{\text{thr}}}$ are defined in [12]:

$$\mathbf{H}_m = [\mathbf{r}_{B/\text{thr},1}^B \times \mathbf{e}_{\text{thr},1}, \dots, \mathbf{r}_{B/\text{thr},n_{\text{thr}}}^B \times \mathbf{e}_{\text{thr},n_{\text{thr}}}] \quad (48)$$

$$\mathbf{H}_f = [\mathbf{e}_{\text{thr},1}, \dots, \mathbf{e}_{\text{thr},n_{\text{thr}}}] \quad (49)$$

which enables a compact expression of the net thruster wrench:

$$\mathbf{w}_{\text{thr}} = \begin{bmatrix} \mathbf{H}_m \\ \mathbf{H}_f \end{bmatrix} \mathbf{u}_{\text{thr}} \quad (50)$$

where $\mathbf{u}_{\text{thr}} \triangleq [u_{\text{thr},1}, \dots, u_{\text{thr},n_{\text{thr}}}]^T \in \mathbb{R}^{n_{\text{thr}}}$ collects the thrust values for each thruster, and $u_{\text{thr},i} = \|\mathbf{f}_{\text{thr},i}\|$.

The relationship between the torque provided by the VSCMGs, $\mathbf{m}_{\text{mom}}$, and the corresponding control input $\mathbf{u}_{\text{mom}} \triangleq [\dot{\boldsymbol{\delta}}^T, \dot{\boldsymbol{\Omega}}^T]^T$ can be obtained following the procedure outlined in [43]. In this formulation, the vector $\boldsymbol{\delta} \triangleq [\delta_1, \dots, \delta_{n_{\text{mom}}}]^T \in \mathbb{R}^{n_{\text{mom}}}$ contains the gimbal angles, while the vector $\boldsymbol{\Omega} \triangleq [\Omega_1, \dots, \Omega_{n_{\text{mom}}}]^T \in \mathbb{R}^{n_{\text{mom}}}$ represents the wheel angular velocities. We briefly recall the procedure for the sake of completeness.

Consider the following expression for the overall angular momentum of the spacecraft base—including the contribution from its VSCMG cluster—expressed in B -frame coordinates and denoted by $\mathbf{h}_{B,C}$:

$$\mathbf{h}_{B,C} = \mathbf{h}_B + \mathbf{h}_C = \mathbf{I}_B \boldsymbol{\omega}_{I/B} + \mathbf{A}_g \mathbf{I}_{cg} \dot{\boldsymbol{\delta}} + \mathbf{A}_s \mathbf{I}_{ws} \boldsymbol{\Omega} \quad (51)$$

where $\mathbf{h}_B = \mathbf{I}_B \boldsymbol{\omega}_{I/B}$ is the base momentum and $\mathbf{h}_C = \mathbf{A}_g \mathbf{I}_{cg} \dot{\boldsymbol{\delta}} + \mathbf{A}_s \mathbf{I}_{ws} \boldsymbol{\Omega}$ encompasses the contribution of the wheels cluster. The matrices $\mathbf{A}_\star \triangleq [\mathbf{e}_{\star 1}, \dots, \mathbf{e}_{\star n_{\text{mom}}}] \in \mathbb{R}^{3 \times n_{\text{mom}}}$ are constructed such that the i th column $\mathbf{e}_{\star i}$ is the unit vector parallel to the spin ($\star = s$), transverse ($\star = t$), or gimbal ($\star = g$) axis of the i th wheel mount. Matrices $\mathbf{I}_{c\star} = \mathbf{I}_{g\star} + \mathbf{I}_{w\star}$ represent the cluster's inertia, which is computed using the inertias of the gimbals and wheels about the axis identified by $\star$, and $\mathbf{I}_{g\star} \triangleq \text{diag}([I_{g\star 1}, \dots, I_{g\star n_{\text{mom}}}]) \in \mathbb{R}^{n_{\text{mom}} \times n_{\text{mom}}}$, and $\mathbf{I}_{w\star} \triangleq \text{diag}([I_{w\star 1}, \dots, I_{w\star n_{\text{mom}}}]) \in \mathbb{R}^{n_{\text{mom}} \times n_{\text{mom}}}$. With these definitions, the overall inertia $\mathbf{I}_B$ of the spacecraft base is expressed as the sum of the base platform inertia $\mathbf{I}_p$, which includes the wheels described as point masses, and the VSCMG inertia along axes s , t , and g , projected onto the base frame B :

$$\mathbf{I}_B = \mathbf{I}_p + \mathbf{A}_s \mathbf{I}_{cs} \mathbf{A}_s^T + \mathbf{A}_t \mathbf{I}_{ct} \mathbf{A}_t^T + \mathbf{A}_g \mathbf{I}_{cg} \mathbf{A}_g^T \quad (52)$$

Note that $\mathbf{I}_B$ is a function of the gimbal angles $\boldsymbol{\delta}$, since $\mathbf{A}_s = \mathbf{A}_s(\boldsymbol{\delta})$ and $\mathbf{A}_t = \mathbf{A}_t(\boldsymbol{\delta})$. In the absence of external torques, the overall angular momentum is conserved. Hence, differentiating Eq. (51) and moving the terms containing the control inputs $\dot{\boldsymbol{\delta}}$ and $\dot{\boldsymbol{\Omega}}$ to the right-hand side, the following is obtained:

$$\begin{aligned} \mathbf{I}_B \dot{\boldsymbol{\omega}}_{I/B} + \tilde{\boldsymbol{\omega}}_{I/B} \mathbf{I}_B \boldsymbol{\omega}_{I/B} \\ = -(\mathbf{A}_t [\dot{\boldsymbol{\delta}}]^d (\mathbf{I}_{cs} - \mathbf{I}_{ct}) \mathbf{A}_s^T + \mathbf{A}_s [\dot{\boldsymbol{\delta}}]^d (\mathbf{I}_{cs} - \mathbf{I}_{ct}) \mathbf{A}_t^T) \boldsymbol{\omega}_{I/B} - \end{aligned} \quad (53)$$

$$(\mathbf{A}_g \mathbf{I}_{cg} \ddot{\boldsymbol{\delta}} + \mathbf{A}_t \mathbf{I}_{ws} [\boldsymbol{\Omega}]^d \dot{\boldsymbol{\delta}} + \mathbf{A}_s \mathbf{I}_{ws} \dot{\boldsymbol{\Omega}}) - \tilde{\boldsymbol{\omega}}_{I/B} (\mathbf{A}_g \mathbf{I}_{cg} \dot{\boldsymbol{\delta}} + \mathbf{A}_s \mathbf{I}_{ws} \boldsymbol{\Omega}) \quad (54)$$

$$= \mathbf{m}_{\text{mom}} \quad (55)$$

From now on, the gimbal accelerations $\ddot{\boldsymbol{\delta}}$ will be neglected because their associated inertia is negligible compared to the other terms in the system [12]. Additionally, we define the diagonal matrix $[\mathbf{x}]^d \triangleq \text{diag}([x_1, \dots, x_N]) \in \mathbb{R}^{N \times N}$, where the elements of $\mathbf{x} \in \mathbb{R}^N$ populate the diagonal. Equations (53–55) are general and can be

easily adapted to the cases where the momentum exchange devices are used as reaction wheels (RWs) by setting $\dot{\delta} = \ddot{\delta} = \mathbf{0}$, or as control moment gyros (CMGs), where the condition $\dot{\Omega} = \mathbf{0}$ applies. The right-hand side of Eqs. (53) and (54) can be further manipulated to explicitly include the VSCMG control input $\mathbf{u}_{\text{mom}} = [\dot{\delta}^\top, \dot{\Omega}^\top]^\top$. Using the property that $[x]^d y = [y]^d x$, the right-hand side of Eq. (53) is rearranged as follows:

$$\left(A_t [\dot{\delta}]^d (I_{cs} - I_{ct}) A_s^\top + A_s [\dot{\delta}]^d (I_{cs} - I_{ct}) A_t^\top \right) \omega_{\mathcal{I}/B} \quad (56)$$

$$= \left(A_t (I_{cs} - I_{ct}) [A_s^\top \omega_{\mathcal{I}/B}]^d + A_s (I_{cs} - I_{ct}) [A_t^\top \omega_{\mathcal{I}/B}]^d \right) \dot{\delta} \quad (57)$$

$$= \mathbf{W} \dot{\delta} \quad (58)$$

For compactness, we define the following quantities:

$$\mathbf{C} \triangleq \mathbf{W} + A_t I_{ws} [\Omega]^d + \tilde{\omega}_{\mathcal{I}/B} A_g I_{cg} \quad (59)$$

$$\mathbf{D} \triangleq A_s I_{ws} \quad (60)$$

$$\mathbf{m}_{\text{gy}} \triangleq \tilde{\omega}_{\mathcal{I}/B} A_s I_{ws} \Omega \quad (61)$$

These definitions allow us to express the total torque $\mathbf{m}_{\text{mom}}$ induced on the spacecraft base by the VSCMGs as the sum of two distinct terms:

$$\mathbf{m}_{\text{mom}} = - \begin{bmatrix} \mathbf{C}(\omega_{\mathcal{I}/B}, \delta, \Omega) & \mathbf{D}(\omega_{\mathcal{I}/B}, \delta, \Omega) \end{bmatrix} \begin{bmatrix} \dot{\delta} \\ \dot{\Omega} \end{bmatrix} - \mathbf{m}_{\text{gy}}(\omega_{\mathcal{I}/B}, \delta, \Omega) \quad (62)$$

$$= -\mathbf{G} \mathbf{u}_{\text{mom}} - \mathbf{m}_{\text{gy}} \quad (63)$$

where $\mathbf{m}_{\text{gy}}$ represents the gyroscopic contribution. Given a desired torque $\mathbf{m}_d$ to be exerted on the spacecraft, the VSCMG control system allocates the gimbal rates and wheel accelerations in $\mathbf{u}_{\text{mom}}$ as follows:

$$\mathbf{u}_{\text{mom}} = -\mathbf{G}^\dagger (\mathbf{m}_d + \mathbf{m}_{\text{gy}}) \quad (64)$$

where $\mathbf{G}^\dagger$ is the weighted pseudoinverse,

$$\mathbf{G}^\dagger \triangleq \mathbf{P} \mathbf{G}^\top (\mathbf{P} \mathbf{G}^\top)^{-1} \quad (65)$$

and $\mathbf{P}$ is a diagonal weight matrix used to define the operational mode of the VSCMGs, prioritizing either the CMG mode or the RW one [12]. The overall actuation wrench exerted on the base is obtained by summing the torques produced by the VSCMGs and by the thrusters and accounting for the thruster force. In matrix form, the process yields

$$\mathbf{w}_{B,a} = \begin{bmatrix} -\mathbf{G} & \mathbf{H}_m \\ \mathbf{0}_{3 \times 2n_{\text{mom}}} & \mathbf{H}_f \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\text{mom}} \\ \mathbf{u}_{\text{thr}} \end{bmatrix} - \begin{bmatrix} \mathbf{m}_{\text{gy}} \\ \mathbf{0}_{3 \times 1} \end{bmatrix} = \mathbf{B}_B \mathbf{u}_B - \mathbf{w}_{B,\text{gy}} \quad (66)$$

The internal actuation $\mathbf{w}_{\mathcal{X},a}$ for the tendon-driven continuum appendage represents the resultant internal force and torque distributed along the rod's midline, generated by a pulling action exerted on the cable actuators. This is formalized as

$$\mathbf{w}_{\mathcal{X},a} = \sum_{k=1}^C \begin{bmatrix} \tilde{d}_k(s) \mathbf{t}_k(s) \\ \mathbf{t}_k(s) \end{bmatrix} u_{\text{cab},k} = \mathbf{B}_C \mathbf{u}_{\text{cab}} \quad (67)$$

where d_k denotes the distance between the midline and the k th tendon, $\mathbf{t}_k$ is the unit vector tangent to the k th actuator path (Fig. 4), $\mathbf{B}_C \in \mathbb{R}^{6 \times C}$ is the actuator configuration matrix [33], and $\mathbf{u}_{\text{cab}} \triangleq [u_{\text{cab},1}, \dots, u_{\text{cab},C}]^\top \in \mathbb{R}^C$ represents the vector of tendon tension inputs. A peculiar aspect of the cable actuation mechanism

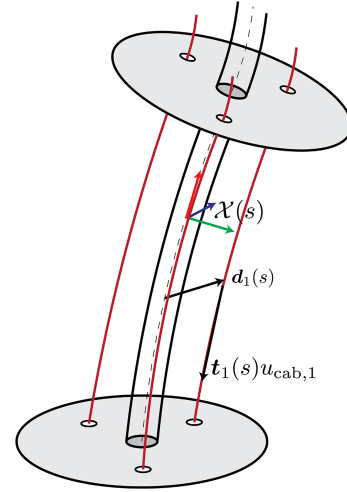


Fig. 4 Cable actuation for a continuum manipulator with three parallel tendons ($C = 3$).

described by Eq. (67) is that the cables can only be pulled, not pushed. In order to reflect this intrinsic physical limitation, we restrict the cable input values to the closed negative half-space of $\mathbb{R}$, i.e., $u_{\text{cab},i} \in (-\infty, 0]$, ($i = 1, \dots, R$). Henceforth, we refer to negative values of $u_{\text{cab},i}$ as “tensions.” While tendon–rod interaction occurs physically at the discrete routing disks, prior work [44] demonstrates that these discontinuous effects can be approximated via spatially continuous functions, as in Eq. (67). A representative design of the tendon-driven continuum appendage, illustrating the routing geometry and actuation principles, is provided in Fig. 4.

Let the vector of the control actions acting on the R revolute joints be $\mathbf{u}_{\text{rev}} \triangleq [u_{\text{rev},1}, \dots, u_{\text{rev},R}]^\top$. To establish a connection between the i th actuation wrench, $\mathbf{w}_{\mathcal{R}_i,a}$, and the corresponding joint control torque $u_{\text{rev},i}$, we consider $P_{\mathbf{w}_{\mathcal{R}_i,a}}$, the power delivered by $\mathbf{w}_{\mathcal{R}_i,a}$:

$$P_{\mathbf{w}_{\mathcal{R}_i,a}} = \mathbf{w}_{\mathcal{R}_i,a}^\top \mathbf{n}_{\mathcal{I}/\mathcal{R}_i} = \mathbf{w}_{\mathcal{R}_i,a}^\top \mathbf{J}^{\mathcal{R}_i} \dot{\mathbf{q}} \quad (68)$$

In Eq. (68), we have used the Jacobian, similarly to Eq. (25), to express the body velocity of frame $\mathcal{R}_i$. The power $P_{\mathbf{w}_{\mathcal{R}_i,a}}$ must equal $P_{u_{\text{rev},i}}$, the power of the corresponding control torque,

$$P_{u_{\text{rev},i}} = u_{\text{rev},i} \dot{q}_i = \mathbf{u}_{\text{rev},i}^\top \dot{\mathbf{q}} \quad (69)$$

where $\mathbf{u}_{\text{rev},i} = \mathbf{b}_i u_{\text{rev},i}$, and $\mathbf{b}_i \in \mathbb{R}^n$ is a vector of zeros with a unique “1” entry in the appropriate position. Setting $P_{\mathbf{w}_{\mathcal{R}_i,a}} = P_{u_{\text{rev},i}}$, we obtain

$$\dot{\mathbf{q}}^\top (\mathbf{J}^{\mathcal{R}_i})^\top \mathbf{w}_{\mathcal{R}_i,a} = \dot{q}_i u_{\text{rev},i} \quad (70)$$

and, since the equality must hold for any $\dot{\mathbf{q}}$, it follows that

$$(\mathbf{J}^{\mathcal{R}_i})^\top \mathbf{w}_{\mathcal{R}_i,a} = \mathbf{u}_{\text{rev},i} \quad (71)$$

We will leverage Eq. (71) in the next section to derive the dynamics of the HSMS. For convenience, we also define the overall HSMS control input as $\mathbf{u}$:

$$\mathbf{u} \triangleq [\mathbf{u}_B^\top, \mathbf{u}_{\text{rev}}^\top, \mathbf{u}_{\text{cab}}^\top]^\top \in \mathbb{R}^m \quad (72)$$

where $m = 2n_{\text{mom}} + n_{\text{thr}} + R + C$.

3. Equations of Motion

The virtual displacements introduced in Eqs. (42–44) can be expressed in terms of the generalized coordinates of the system:

$$\delta \xi = \Phi_{\xi} \delta q_S = \hat{\Phi}_{\xi} \delta q \quad (73)$$

$$\delta r_B = J^B \delta q \quad (74)$$

$$\delta r_{\mathcal{R}_i} = J^{\mathcal{R}_i} \delta q \quad (75)$$

$$\delta r_{\mathcal{X}} = J^{\mathcal{X}} \delta q \quad (76)$$

where the following quantities have been utilized in Eqs. (73–76):

$$\hat{\Phi}_{\xi} = [\mathbf{0}_{6 \times (6+R)}, \Phi_{\xi}] \quad (77)$$

$$J^B = [I_6, \mathbf{0}_{6 \times (R+S)}] \quad (78)$$

$$J^{\mathcal{R}_i} = [\text{Ad}_{g_{B/\mathcal{R}_i}}^{-1} \xi_B, \dots, \text{Ad}_{g_{\mathcal{R}_{i-1}/\mathcal{R}_i}}^{-1} \xi_{\mathcal{R}_{i-1}}, \xi_{\mathcal{R}_i}, \mathbf{0}_{6 \times (R-i+S)}], \quad i = 1, \dots, R \quad (79)$$

$$J^{\mathcal{X}} = [J_R^{\mathcal{X}}, J_S^{\mathcal{X}}] \quad (80)$$

and where $J_R^{\mathcal{X}} = [\text{Ad}_{g_{B/\mathcal{X}}}^{-1} \xi_B, \dots, \text{Ad}_{g_{\mathcal{R}_R/\mathcal{X}}}^{-1} \xi_{\mathcal{R}_R}] \in \mathbb{R}^{6 \times (6+R)}$ similarly to Eq. (24), and $J_S^{\mathcal{X}} = \text{Ad}_{g_{\mathcal{F}/\mathcal{X}}}^{-1} \int_0^s \text{Ad}_{g_{\mathcal{F}/\mathcal{X}(\sigma)}}^{-1} \Phi_{\xi}(\sigma) d\sigma \in \mathbb{R}^{6 \times S}$. Substituting Eqs. (67) and (73) into Eq. (42) and using the constitutive law, the internal work becomes

$$\delta W_{\text{int}} = \delta q^T \left(\int_0^L \hat{\Phi}_{\xi}^T \Sigma \hat{\Phi}_{\xi} ds q - \int_0^L \hat{\Phi}_{\xi}^T B_C ds u_{\text{cab}} \right) \quad (81)$$

Likewise, the external work in Eqs. (43–45) can be rewritten using Eqs. (74–76), expressing the body velocities using the Jacobians, and recalling the actuation models in Eqs. (66) and (71),

$$\delta W_{\text{ext}} = \delta q^T \left((J^B)^T (w_{B,e} + B_B u_B - w_{B,\text{gy}} - (M_B J^B \ddot{q} + M_B \dot{J}^B \dot{q} + \text{ad}_{J^B \dot{q}}^* M_B J^B \dot{q})) \right) + \quad (82)$$

$$\delta q^T \left(\sum_{i=1}^R b_i u_{\text{rev},i} + \sum_{i=1}^R (J^{\mathcal{R}_i})^T (w_{\mathcal{R}_i,e} - (M_{\mathcal{R}_i} J^{\mathcal{R}_i} \ddot{q} + M_{\mathcal{R}_i} \dot{J}^{\mathcal{R}_i} \dot{q} + \text{ad}_{J^{\mathcal{R}_i} \dot{q}}^* M_{\mathcal{R}_i} J^{\mathcal{R}_i} \dot{q})) \right) + \quad (83)$$

$$\delta q^T \left(\int_0^L (J^{\mathcal{X}})^T (\bar{w}_{\mathcal{X},e} - (\bar{M}_{\mathcal{X}} J^{\mathcal{X}} \ddot{q} + \bar{M}_{\mathcal{X}} \dot{J}^{\mathcal{X}} \dot{q} + \text{ad}_{J^{\mathcal{X}} \dot{q}}^* \bar{M}_{\mathcal{X}} J^{\mathcal{X}} \dot{q})) ds \right) \quad (84)$$

Note that, after enforcing the condition $\delta W_{\text{int}} = \delta W_{\text{ext}}$ via Eqs. (81–84), the variation δq can be omitted since it is arbitrary. In order to cast the equations of motion in a compact form, we proceed by defining the following system matrices:

$$M(q) = (J^B)^T M_B J^B + \sum_{i=1}^R (J^{\mathcal{R}_i})^T M_{\mathcal{R}_i} J^{\mathcal{R}_i} + \int_0^L (J^{\mathcal{X}})^T \bar{M}_{\mathcal{X}} J^{\mathcal{X}} ds \quad (85)$$

$$C(q, \dot{q}) = (J^B)^T (M_B \dot{J}^B + \text{ad}_{J^B \dot{q}}^* M_B J^B) + \sum_{i=1}^R (J^{\mathcal{R}_i})^T (M_{\mathcal{R}_i} \dot{J}^{\mathcal{R}_i} + \text{ad}_{J^{\mathcal{R}_i} \dot{q}}^* M_{\mathcal{R}_i} J^{\mathcal{R}_i}) \quad (86)$$

$$+ \int_0^L (J^{\mathcal{X}})^T (\bar{M}_{\mathcal{X}} \dot{J}^{\mathcal{X}} + \text{ad}_{J^{\mathcal{X}} \dot{q}}^* \bar{M}_{\mathcal{X}} J^{\mathcal{X}}) ds \quad (87)$$

$$K(q) = \int_0^L \hat{\Phi}_{\xi}^T \Sigma \hat{\Phi}_{\xi} ds \quad (88)$$

The contributions of the external forces and control action can be written in a compact fashion after introducing the following quantities:

$$W_{\text{ext}}(q) = (J^B)^T w_{B,e} + \sum_{i=1}^R (J^{\mathcal{R}_i})^T w_{\mathcal{R}_i,e} + \int_0^L (J^{\mathcal{X}})^T \bar{w}_{\mathcal{X},e} ds \quad (89)$$

$$B(q) = \left[(J^B)^T B_B, b_1, \dots, b_R, \int_0^L \hat{\Phi}_{\xi}^T B_C ds \right] \quad (90)$$

$$W_{\text{gy}}(q) = -(J^B)^T w_{B,\text{gy}} \quad (91)$$

Finally, the equations of motion for the HSMS are obtained as follows:

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + K(q) q = W_{\text{ext}}(q) + B(q) u + W_{\text{gy}}(q) \quad (92)$$

Notably, while these equations capture the coupled dynamics of both rigid and soft links, they retain the same structure as those governing purely rigid robotic manipulators [8,45]. This similarity represents a significant advantage, as it provides an advantageous foundation for control design by leveraging existing methodologies developed for conventional rigid manipulators. In fact, this structural consistency constitutes a key advantage of the proposed approach.

III. Control Design

The dynamic model presented in Eq. (92) can be exploited to design a controller that steers the system toward a desired steady state or, alternatively, to track a prescribed time-varying task-space reference. We refer to these two control objectives as regulation and tracking, respectively. In what follows, we provide a concise description of the synthesis of both configuration-space and task-space controllers: the former is designed to achieve regulation, while the latter is responsible for executing an end-effector position tracking maneuver. In the derivation of the two controllers, we assume that the external wrench acting on the system is negligible compared to the control action u , and therefore consider $W_{\text{ext}} = \mathbf{0}$. This assumption is representative of an in-orbit scenario in which the HSMS operates in the absence of contact forces and under only minor orbital perturbations. Therefore, the control system is of the form

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + K(q) q = B(q) u + W_{\text{gy}}(q) \quad (93)$$

Note also that, as discussed in Sec. II.B.2, the input values in u corresponding to the cable actuators must be negative (or zero), since the tendons can only undergo a pulling action. This constraint will be taken into account when computing u , as detailed in the next section.

A. Configuration-Space Regulation

In the following, we present the configuration-space control law and prove the global asymptotic stability of the resulting closed-loop system. Subsequently, we provide additional remarks on the numerical implementation of the proposed controller and explain how actuator limits are incorporated into the numerical solution. We begin by stating the control law.

Lemma 2: Let $q_d \in \mathbb{R}^n$ be a desired configuration for the system in Eq. (93), with q_d constant, that is, $\dot{q}_d = \mathbf{0}$ and $\ddot{q}_d = \mathbf{0}$. The state feedback control law

$$u = B^{\dagger}(q) [M(q) \phi + C(q, \dot{q}) \dot{q} + K(q) q - W_{\text{gy}}] \quad (94)$$

yields to the following decoupled closed-loop dynamics:

$$\ddot{\mathbf{q}} = \boldsymbol{\phi} \quad (95)$$

where $\boldsymbol{\phi}$ is an auxiliary control input. Moreover, if

$$\boldsymbol{\phi} = \ddot{\mathbf{q}}_d + \mathbf{K}_d(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) \quad (96)$$

with $\mathbf{K}_d, \mathbf{K}_p > 0$, then $\lim_{t \rightarrow \infty} \mathbf{q}(t) = \mathbf{q}_d$.

Proof: Equation (95) immediately results from substitution of the control law of Eq. (94) into Eq. (93). Then, plugging Eq. (96) into Eq. (95) yields the following error dynamics:

$$\ddot{\mathbf{e}} + \mathbf{K}_d \dot{\mathbf{e}} + \mathbf{K}_p \mathbf{e} = \mathbf{0} \quad (97)$$

where $\mathbf{e} \triangleq \mathbf{q}_d - \mathbf{q}$. Let $V: \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^+$ be a Lyapunov function candidate defined as follows:

$$V(\mathbf{e}, \dot{\mathbf{e}}) = \frac{1}{2} \dot{\mathbf{e}}^\top \dot{\mathbf{e}} + \frac{1}{2} \mathbf{e}^\top \mathbf{K}_p \mathbf{e} \quad (98)$$

where $V(\mathbf{e}, \dot{\mathbf{e}}) > 0$ since $\mathbf{K}_p > 0$ and $\dot{V} \leq 0$ since $\dot{V}(\mathbf{e}, \dot{\mathbf{e}}) = -\dot{\mathbf{e}}^\top \mathbf{K}_d \dot{\mathbf{e}}$. It follows from LaSalle's principle and from the radial unboundedness of V that the origin of Eq. (97) is globally asymptotically stable. Hence, $\lim_{t \rightarrow \infty} \mathbf{e}(t) = \mathbf{0}$, which implies $\lim_{t \rightarrow \infty} \mathbf{q}(t) = \mathbf{q}_d$. $\square$

From an implementation standpoint, the control action $\mathbf{u}$ in Eq. (94) can be computed by solving the following bound- and equality-constrained quadratic program:

$$\min_{\mathbf{u}} \quad \frac{1}{2} \mathbf{u}^\top \mathbf{H} \mathbf{u} \quad (99)$$

$$\text{subject to} \quad \mathbf{A}_{\text{eq}} \mathbf{u} = \mathbf{b}_{\text{eq}} \quad (100)$$

$$\boldsymbol{\ell}_b \leq \mathbf{u} \leq \mathbf{u}_b \quad (101)$$

In Eqs. (99) and (100), $\mathbf{H}$ is a constant diagonal weight matrix, while $\boldsymbol{\ell}_b$ and $\mathbf{u}_b$ are the lower and upper bounds on the components of the control vector, respectively. The linear equality in Eq. (100) is used to obtain $\mathbf{u}$ from Eq. (94) in a least-squares sense since $\mathbf{A}_{\text{eq}} \triangleq \mathbf{A}^\top \mathbf{A}$ and $\mathbf{b}_{\text{eq}} \triangleq \mathbf{A}^\top \mathbf{b}$, with

$$\mathbf{A} \triangleq \mathbf{B}(\mathbf{q}) \quad (102)$$

$$\mathbf{b} \triangleq \mathbf{M}(\mathbf{q})\boldsymbol{\phi} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{K}(\mathbf{q})\mathbf{q} - \mathbf{W}_{\text{gy}} \quad (103)$$

with $\boldsymbol{\phi}$ defined in Eq. (96). Notably, the bounds in Eq. (101) can be set in order to establish saturation limits and to enforce a pulling action on the cable actuators.

B. Task-Space Position Tracking

We wish to design a task-space controller guaranteeing tracking of a time-varying position $\mathbf{r}_{\mathcal{I}/\mathcal{D}}^T(t)$, where $\mathcal{D}$ denotes a desired frame. In order to take advantage of the kinematics framework established in Sec. II.A, we define the following reference trajectory in SE(3), $\mathbf{g}_{\mathcal{I}/\mathcal{D}}: \mathbb{R} \mapsto \text{SE}(3)$:

$$\mathbf{g}_{\mathcal{I}/\mathcal{D}}(t) = \begin{bmatrix} \mathbf{R}_{\mathcal{I}/\mathcal{D}} & \mathbf{r}_{\mathcal{I}/\mathcal{D}}^T(t) \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \quad (104)$$

where $\mathbf{R}_{\mathcal{I}/\mathcal{D}} \in \text{SO}(3)$ is the attitude of the desired frame, considered constant for the sake of simplicity. Given $\mathbf{g}_{\mathcal{I}/\mathcal{D}}$ as an explicit function of time, its time derivatives $\dot{\mathbf{g}}_{\mathcal{I}/\mathcal{D}}$ and $\ddot{\mathbf{g}}_{\mathcal{I}/\mathcal{D}}$ are easily computed. The end-effector pose error is expressed as the transformation from the current end-effector frame $\mathcal{E}$ to the target frame $\mathcal{D}$, $\mathbf{g}_{\mathcal{E}/\mathcal{D}} = \mathbf{g}_{\mathcal{I}/\mathcal{E}}^{-1} \mathbf{g}_{\mathcal{I}/\mathcal{D}}$. The desired end-effector velocity can be obtained according to Eq. (4):

$$\hat{\boldsymbol{\eta}}_{\mathcal{I}/\mathcal{D}} = \mathbf{g}_{\mathcal{I}/\mathcal{D}}^{-1} \dot{\mathbf{g}}_{\mathcal{I}/\mathcal{D}} \quad (105)$$

which can be expressed in the end-effector body frame $\mathcal{E}$ as

$$\hat{\boldsymbol{\eta}}_{\mathcal{I}/\mathcal{D}}^{\mathcal{E}} = \text{Ad}_{\mathbf{g}_{\mathcal{E}/\mathcal{D}}}(\hat{\boldsymbol{\eta}}_{\mathcal{I}/\mathcal{D}}) \quad (106)$$

From Eq. (106), we compute the end-effector velocity error, $\hat{\boldsymbol{\eta}}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}} = \hat{\boldsymbol{\eta}}_{\mathcal{I}/\mathcal{D}}^{\mathcal{E}} - \hat{\boldsymbol{\eta}}_{\mathcal{I}/\mathcal{E}}$, where $\boldsymbol{\eta}_{\mathcal{I}/\mathcal{E}}$ obeys the kinematics relationship in Eq. (32). The components of the linear velocity of the end-effector can be obtained by projecting the generalized velocity $\dot{\mathbf{q}}$ into the task space via $\mathbf{J}_I^{\mathcal{E}}$, the appropriate partition of the body Jacobian. Consequently, the linear velocity of the end-effector, expressed in its local $\mathcal{E}$ frame, is given by $\mathbf{v}_{\mathcal{I}/\mathcal{E}} = \mathbf{J}_I^{\mathcal{E}}(\mathbf{q})\dot{\mathbf{q}}$. Finally, the time derivative of the desired end-effector twist is obtained by differentiating Eq. (106):

$$\dot{\hat{\boldsymbol{\eta}}}_{\mathcal{I}/\mathcal{D}}^{\mathcal{E}} = \frac{d}{dt} \left(\text{Ad}_{\mathbf{g}_{\mathcal{E}/\mathcal{D}}}(\hat{\boldsymbol{\eta}}_{\mathcal{I}/\mathcal{D}}) \right) \quad (107)$$

$$= \text{ad}_{\hat{\boldsymbol{\eta}}_{\mathcal{E}/\mathcal{D}}} \left(\text{Ad}_{\mathbf{g}_{\mathcal{E}/\mathcal{D}}}(\hat{\boldsymbol{\eta}}_{\mathcal{I}/\mathcal{D}}) \right) + \text{Ad}_{\mathbf{g}_{\mathcal{E}/\mathcal{D}}}(\dot{\hat{\boldsymbol{\eta}}}_{\mathcal{I}/\mathcal{D}}) \quad (108)$$

where $\dot{\hat{\boldsymbol{\eta}}}_{\mathcal{I}/\mathcal{D}}$ can be computed by differentiating Eq. (105). Notice that the linear velocity error of the end effector, $\mathbf{v}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}} \in \mathbb{R}^3$, and its desired linear acceleration $\dot{\mathbf{v}}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}} \in \mathbb{R}^3$ are known since $\hat{\boldsymbol{\eta}}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}} = [(\boldsymbol{\omega}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}})^\top, (\mathbf{v}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}})^\top]^\top$ and $\dot{\hat{\boldsymbol{\eta}}}_{\mathcal{I}/\mathcal{D}}^{\mathcal{E}} = [(\dot{\boldsymbol{\omega}}_{\mathcal{I}/\mathcal{D}}^{\mathcal{E}})^\top, (\dot{\mathbf{v}}_{\mathcal{I}/\mathcal{D}}^{\mathcal{E}})^\top]^\top$.

Lemma 3: Consider the hybrid spacecraft manipulator system with dynamics described by Eq. (93), and let $\mathbf{r}_{\mathcal{I}/\mathcal{D}}^T(t)$ be a desired position for the end-effector frame $\mathcal{E}$. The feedback control law

$$\mathbf{u} = (\mathbf{J}_I^{\mathcal{E}} \mathbf{M}^{-1} \mathbf{B})^\dagger \left[\boldsymbol{\theta} + \mathbf{J}_I^{\mathcal{E}} \mathbf{M}^{-1} (\mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} - \mathbf{W}_{\text{gy}}) - \dot{\mathbf{J}}_I^{\mathcal{E}} \dot{\mathbf{q}} \right] \quad (109)$$

yields the following decoupled task-space dynamics:

$$\dot{\mathbf{v}}_{\mathcal{I}/\mathcal{E}}^{\mathcal{E}} = \boldsymbol{\theta} \quad (110)$$

where $\boldsymbol{\theta}$ is an auxiliary control input. Moreover, if

$$\boldsymbol{\theta} = \dot{\mathbf{v}}_{\mathcal{I}/\mathcal{D}}^{\mathcal{E}} + \mathbf{K}_d \mathbf{v}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}} + \mathbf{K}_p \mathbf{r}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}} \quad (111)$$

with $\mathbf{K}_d, \mathbf{K}_p > 0$, then $\lim_{t \rightarrow \infty} \mathbf{r}_{\mathcal{I}/\mathcal{E}}^T(t) = \mathbf{r}_{\mathcal{I}/\mathcal{D}}^T(t)$.

Proof: The linear acceleration of the end-effector obeys the following relationship:

$$\dot{\mathbf{v}}_{\mathcal{I}/\mathcal{E}}^{\mathcal{E}} = \dot{\mathbf{J}}_I^{\mathcal{E}} \dot{\mathbf{q}} + \mathbf{J}_I^{\mathcal{E}} \ddot{\mathbf{q}} \quad (112)$$

$$= \dot{\mathbf{J}}_I^{\mathcal{E}} \dot{\mathbf{q}} + \mathbf{J}_I^{\mathcal{E}} \left[\mathbf{M}^{-1} (-\mathbf{C}\dot{\mathbf{q}} - \mathbf{K}\mathbf{q} + \mathbf{B}\mathbf{u} + \mathbf{W}_{\text{gy}}) \right] \quad (113)$$

The decoupled dynamics in Eq. (110) results from the substitution of the control law in Eq. (109) into Eq. (113). Then, equating the terms in Eqs. (111) and (110) leads to a decoupled set of equations describing the task-space error dynamics:

$$\dot{\mathbf{v}}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}} + \mathbf{K}_d \mathbf{v}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}} + \mathbf{K}_p \mathbf{r}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}} = \mathbf{0} \quad (114)$$

After defining $\mathbf{e} \triangleq \mathbf{r}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}}$, $\dot{\mathbf{e}} \triangleq \mathbf{v}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}}$, and $\ddot{\mathbf{e}} \triangleq \dot{\mathbf{v}}_{\mathcal{E}/\mathcal{D}}^{\mathcal{E}}$, the proof is concluded using the same argument as in Lemma 2.

Note that the remarks reported at the end of Sec. III.A also apply to the task-space controller described in this section. Specifically, the quadratic program in Eqs. (99–103) can be adapted to the task-space formulation stemming from Eqs. (109) and (111), resulting in an equivalent numerical solution strategy. $\square$

IV. Numerical Results

We present the numerical results obtained from simulating the HSMS in both open-loop and closed-loop scenarios. The model derived in the preceding sections has been validated within a MATLAB-based numerical simulation environment. Throughout the implementation, a baseline model of the HSMS was developed leveraging the SoRoSim V6.3 library [46], which employs the GVS paradigm to construct a reduced-order model of a user-defined soft-rigid multibody system. Subsequently, we modified certain data structures and functionalities within SoRoSim to accurately capture the dynamic interplay between the actuators and the spacecraft base, thereby enabling the propagation of the dynamics described in Eq. (92).

Our case study considers the system depicted in Fig. 1, which comprises two rigid manipulator links mounted on the spacecraft base and actuated by two revolute joints, with a soft appendage attached to the tip of the second link via a fixed connection. The physical parameters of the HSMS are listed in Table 1. Table 2 reports the parameters of the VSCMGs for the configuration shown in Fig. 2, where $\beta = 54$ deg. The thruster layout is detailed in Table 3 and illustrated in Fig. 3. Finally, the tendon actuators for the continuum segment are described in Table 4 and shown in Fig. 4. The strain state of the continuum appendage is characterized solely by its bending components about the y - and z -axis, as these are the most significant contributors to the overall deformation of the structure.

Table 1 Physical parameters of the hybrid spacecraft-manipulator system (HSMS)

Body	Shape	Dimensions, m	Density, kg/m ³	Young modulus, N/m ²	Poisson's ratio
Base	R	Height: 1 Width: 1 Length: 1	100	— —	— —
Rigid links	C	Radius: 0.1 Height: 2	100	— —	— —
Soft link	C	Radius: 0.01 Height: 1	1000	10 ⁶	0.5

Table 2 Physical parameters of the VSCMG clusters for $i = 1, \dots, 4$

Wheels		Gimbals	
Mass, kg	3	Mass, kg	1
	$I_{ws,i}: 0.042$		$I_{gs,i}: 0.093$
Inertia, kg · m ²	$I_{wt,i}: 0.024$	Inertia, kg · m ²	$I_{gt,i}: 0.054$
	$I_{wg,i}: 0.024$		$I_{gg,i}: 0.054$

Table 3 Thruster configuration

Thruster ID	Position, m
$i = 1, \dots, 3$	$\mathbf{r}_{B/thr,i}^B = [-0.5, -0.5, -0.5]^T$
$i = 4, \dots, 6$	$\mathbf{r}_{B/thr,i}^B = [0.5, -0.5, 0.5]^T$
$i = 7, \dots, 9$	$\mathbf{r}_{B/thr,i}^B = [-0.5, 0.5, 0.5]^T$
$i = 10, \dots, 12$	$\mathbf{r}_{B/thr,i}^B = [0.5, 0.5, -0.5]^T$

Table 4 Soft actuators routing

Cable ID	Routing path, m
#1	$\mathbf{d}_1(s): [0, 0.05 \cos(\pi/6), 0.05 \sin(\pi/6)]$
#2	$\mathbf{d}_2(s): [0, -0.05 \cos(\pi/6), 0.05 \sin(\pi/6)]$
#3	$\mathbf{d}_3(s): [0, 0, -0.05]$

A. Model Simulation

Throughout the model validation phase, the bending strains were parameterized using second-order polynomials with a monomial basis defined over the curvilinear coordinate s . Consequently, the system has a total of 14 DoFs of freedom: 6 DoFs for the spacecraft base, 2 DoFs for the rigid joints, and 6 DoFs for the soft appendage. To validate our model, three distinct scenarios are considered in the absence of dissipative effects (i.e., no friction and material damping) and external forces. Specifically, the base thrusters remain inactive.

First, we consider the case in which the soft appendage is initialized with a nonzero strain, while the rigid components are at rest in their reference configuration. We aim to verify the conservation of the total mechanical energy E of the system. For this scenario, the HSMS model is constructed using the baseline SoRoSim library without incorporating the momentum exchange devices on the spacecraft base. We designate this case as V1, and its simulation parameters are provided in Table 5. The total energy E can be expressed as the sum of three contributions:

$$E = K_R + K_C + U \quad (115)$$

where K_R denotes the kinetic energy associated with the rigid components (i.e., the spacecraft base and the rigid manipulator links), K_C is the kinetic energy of the continuum appendage, and U is the elastic potential energy stored in the deformable element, with the following expressions:

$$K_R = \frac{1}{2} \left(\boldsymbol{\eta}_{I/B}^T \mathbf{M}_B \boldsymbol{\eta}_{I/B} + \sum_{i=1}^R \boldsymbol{\eta}_{I/R_i}^T \mathbf{M}_{R_i} \boldsymbol{\eta}_{I/R_i} \right) \quad (116)$$

$$K_C = \frac{1}{2} \int_0^L \boldsymbol{\eta}_{I/X}^T \mathbf{M}_X \boldsymbol{\eta}_{I/X} ds \quad (117)$$

$$U = \frac{1}{2} \int_0^L (\boldsymbol{\xi} - \boldsymbol{\xi}^*)^T \boldsymbol{\Sigma} (\boldsymbol{\xi} - \boldsymbol{\xi}^*) ds \quad (118)$$

Note that the body velocities in Eqs. (116) and (117) are computed by leveraging the forward kinematics relationships, such as the one in Eq. (25), which are specialized for each body along the kinematic chain, while the strain state $\boldsymbol{\xi}$ in Eq. (118) is found with the GVS discretization in Eq. (28). In scenario V1, under the prescribed initial conditions, the system is characterized by a non-zero initial elastic potential energy U , while the initial kinetic energy is zero. As the HSMS dynamics evolve, the total mechanical energy is conserved, with the elastic potential energy being intermittently converted into kinetic energy of both the soft appendage and the rigid links, and vice versa. This energy exchange is clearly illustrated in Fig. 5a, thereby supporting the validity of the baseline HSMS model.

We also aim to validate the augmented HSMS model, which accounts for the momentum exchange between the system and its VSCMG clusters. In this context, we assess the conservation of the overall angular momentum expressed in the inertial frame, $\mathbf{h}^I$. In doing so, two distinct contributions are highlighted:

$$\mathbf{h}^I = \mathbf{h}_b^I + \mathbf{h}_C^I \quad (119)$$

Table 5 Initial conditions for simulation scenarios V1, V2, and V3

Scenario	Initial configuration	Initial velocity
V1	$\mathbf{q}_R(0) = \mathbf{0}_{8 \times 1}$ $\mathbf{q}_S(0) = [2, 0, 0, 2, 0, 0]^T$	$\dot{\mathbf{q}}_R(0) = \mathbf{0}_{8 \times 1}$ $\dot{\mathbf{q}}_S(0) = \mathbf{0}_{6 \times 1}$
V2	$\mathbf{q}_R(0) = \mathbf{0}_{8 \times 1}$ $\mathbf{q}_S(0) = \mathbf{0}_{6 \times 1}$	$\dot{\mathbf{q}}_R(0) = \mathbf{0}_{8 \times 1}$ $\dot{\mathbf{q}}_S(0) = \mathbf{0}_{6 \times 1}$
V3	$\mathbf{q}_R(0) = [\mathbf{0}_{1 \times 6}, -\pi/4, 0]^T$ $\mathbf{q}_S(0) = [1, 0, 0, 1, 0, 0]^T$	$\dot{\mathbf{q}}_R(0) = [\pi/45, \pi/180, \pi/90, \mathbf{0}_{1 \times 5}]^T$ $\dot{\mathbf{q}}_S(0) = \mathbf{0}_{6 \times 1}$

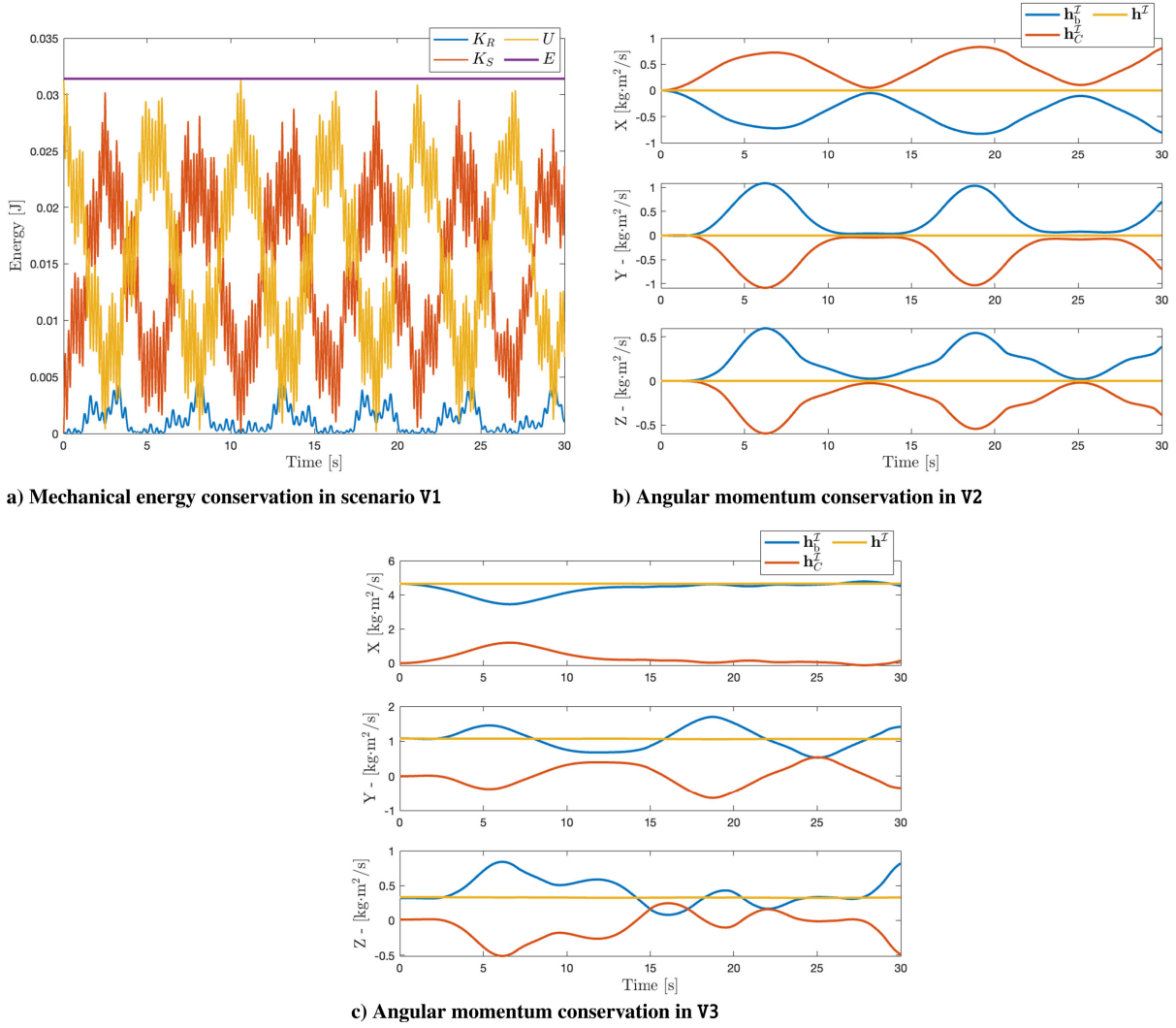


Fig. 5 Conservation of the mechanical constants for the validation scenarios V1, V2, and V3.

where $\mathbf{h}_b^T$ is the overall momentum of the HSMS structures (base and hybrid manipulator) and $\mathbf{h}_C^T$ represents the contributions of the VSCMG clusters. These two quantities are defined as follows:

$$\mathbf{h}_b^T = \mathbf{R}_{I/B} \mathbf{I}_B \boldsymbol{\omega}_{I/B} + \sum_{i=1}^R \mathbf{R}_{I/\mathcal{R}_i} \mathbf{I}_{\mathcal{R}_i} \boldsymbol{\omega}_{I/\mathcal{R}_i} + \int_0^L \mathbf{R}_{I/X} \bar{\mathbf{I}}_X \boldsymbol{\omega}_{I/X} ds \quad (120)$$

$$\mathbf{h}_C^T = \mathbf{R}_{I/B} \mathbf{h}_C \quad (121)$$

where $\mathbf{h}_C$ is defined in Eq. (51).

Momentum conservation was investigated in scenario V2, in which the gimbal angles δ are maintained constant and the VSCMGs are operated as reaction wheels, following the velocity profile $\boldsymbol{\Omega}$ in Fig. 6.

The total angular momentum $\mathbf{h}^T$ is conserved, as seen in Fig. 5b, while $\mathbf{h}_b^T$ and $\mathbf{h}_C^T$ display a consistent behavior with the reaction wheel rates.

The last validation scenario, designated as V3, is simulated in the presence of nonzero initial momentum (see Table 5), wheel spin rates as in Fig. 6, and constant gimbal velocities $\dot{\delta} = [1, 2.5, 5, 7.5]^T$ deg/s. The angular momentum profile for V3 is reported in Fig. 5c, confirming that the augmented HSMS model accurately captures the momentum exchange dynamics even for complex operating modes.

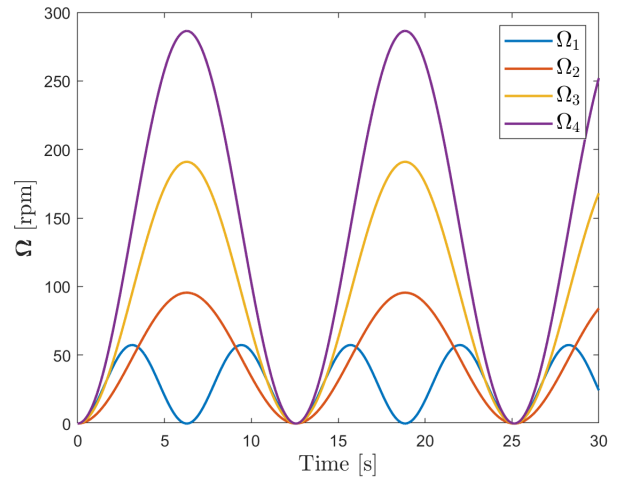


Fig. 6 Wheels velocity profile in scenario V2.

B. Closed-Loop

We examine two closed-loop simulation scenarios, C1 and C2, in which we employ the controllers derived in Sec. III.A and III.B, respectively. To simplify the analysis, we assume a fully actuated system and therefore replace the 6-DoF backbone model used in Sec. IV.A with a lower-dimensional one in which bending is linear around the y-axis (2 DoFs) and constant around the z-axis (1 DoF).

Table 6 Initial conditions for simulation scenarios C1 and C2

Scenario	Initial configuration	Initial velocity
C1	$\mathbf{e} = [0.792, 0.569, 0.220]^T$, $\theta = 29.645$, deg $\mathbf{r}_{I/B} = [-0.5, 0, 0.3]^T$, m $q_1 = 10$, $q_2 = 0$, deg $\mathbf{q}_S = \mathbf{0}_{3 \times 1}$	$\boldsymbol{\omega}_{I/B} = [-2, 2.5, 3]^T$, deg/s $\mathbf{v}_{I/B} = [0, -0.1, 0.07]^T$, m/s $\dot{q}_1 = 0$, $\dot{q}_2 = 0$, deg/s $\dot{\mathbf{q}}_S = \mathbf{0}_{3 \times 1}$
C2	$\mathbf{q}_R = \mathbf{0}_{8 \times 1}$ $\mathbf{q}_S = [0, 0, 1]^T$	$\dot{\mathbf{q}}_R = \mathbf{0}_{8 \times 1}$ $\dot{\mathbf{q}}_S = \mathbf{0}_{3 \times 1}$

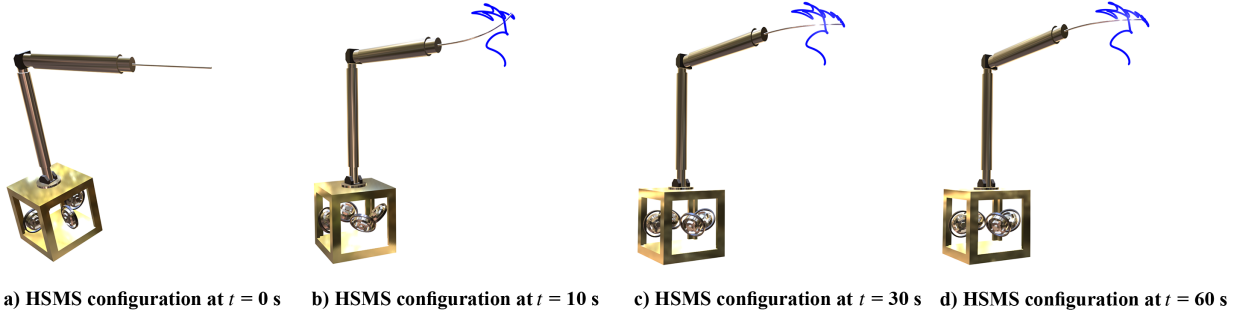
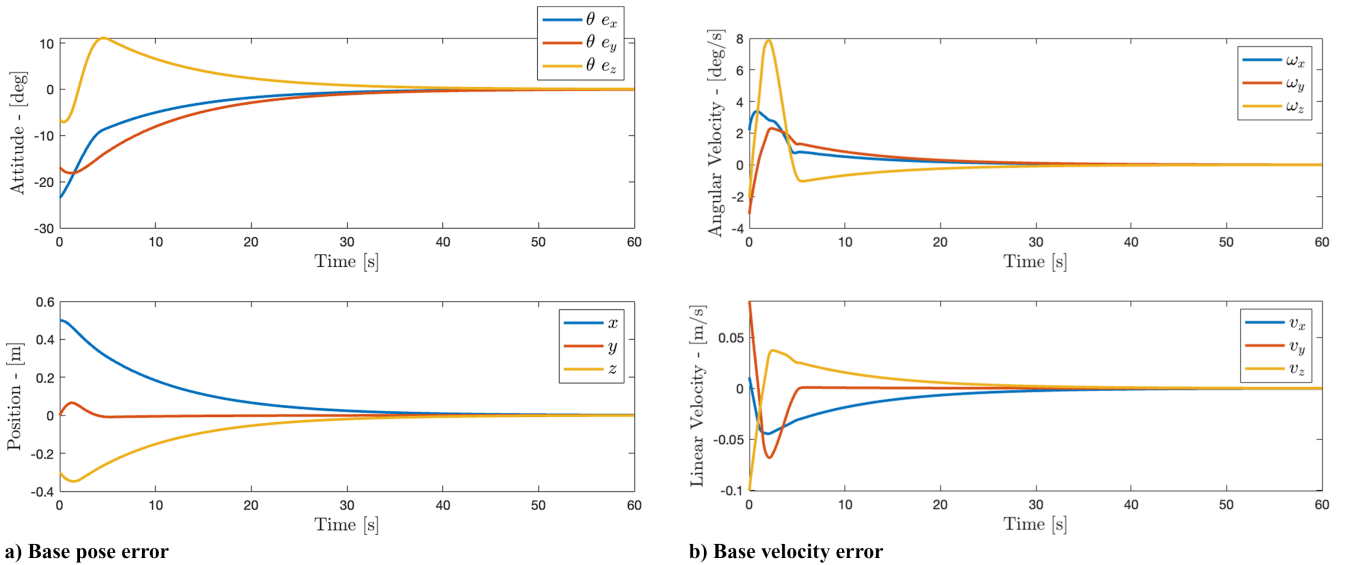
Consequently, the HSMS has 11 DoFs and an equal number of actuators. Because we only use 3 DoFs in the GVS model, this specific parameterization may not capture higher-order dynamic effects that would lead to more complex backbone deformation profiles. Indeed, by employing a single DoF for z -axis bending, we recover the usual constant-curvature case along the z -axis of the $\mathcal{X}(s)$ frame. Moreover, our controller's efficacy is heavily dependent on the accurate knowledge of the system's physical parameters, which could restrict its practical applicability. Nonetheless, this simplified case study offers valuable insights into the potential of an HSMS for in-orbit servicing tasks. Furthermore, the soft robotics control literature has shown successful examples of inverse dynamics controllers applied to real systems [47,48].

Scenario C1 is representative of a “detumbling” maneuver in which the HSMS is steered toward a desired rest configuration $\mathbf{q}_d = [\mathbf{q}_{R,d}^T, \mathbf{q}_{S,d}^T]^T$, with $\mathbf{q}_{R,d} = \mathbf{0}_{8 \times 1}$ and $\mathbf{q}_{S,d} = [0, 0, 1]^T$, starting from an initial configuration characterized by nonzero momentum. The selection of $\mathbf{q}_{S,d}$ is intended to preload the system with elastic energy that can be exploited at a later stage of the mission. The initial conditions for C1 (see Table 6) are chosen to create a highly

dynamic motion state for the HSMS platform, as visible from the snapshots of the end-effector displacement in Fig. 7. Animations of the whole maneuver can be found at https://youtu.be/xEbbdMqRz_0. The closed-loop behavior of the system is simulated for $T = 60$ s, during which the controller effectively transitions the system from its initial state to the desired configuration $\mathbf{q}_d$. The proportional and derivative gains in Eq. (96) are chosen as $\mathbf{K}_p = \text{diag}\{\mathbf{K}_{p,b}, \mathbf{K}_{p,r}, \mathbf{K}_{p,s}\}$ and $\mathbf{K}_d = \text{diag}\{\mathbf{K}_{d,b}, \mathbf{K}_{d,r}, \mathbf{K}_{d,s}\}$, where $\mathbf{K}_{p,b} = 0.5\mathbf{I}_6$, $\mathbf{K}_{p,r} = 0.5\mathbf{I}_2$, $\mathbf{K}_{p,s} = 0.1\mathbf{I}_3$, $\mathbf{K}_{d,b} = 5\mathbf{I}_6$, $\mathbf{K}_{d,r} = 10\mathbf{I}_2$, and $\mathbf{K}_{d,s} = 0.65\mathbf{I}_3$. The weight matrix in Eq. (99) is set to $\mathbf{H} = \mathbf{I}_{11}$. The actuator inputs are bound as follows:

- 1) $0 \leq u_{\text{thr},i} \leq u_{\text{thr}}^{\max}$, $i = 1, \dots, 12$,
- 2) $-\dot{\delta}^{\max} \leq \dot{\delta}_i \leq \dot{\delta}^{\max}$, $i = 1, \dots, 4$,
- 3) $-\dot{\Omega}^{\max} \leq \dot{\Omega}_i \leq \dot{\Omega}^{\max}$, $i = 1, \dots, 4$,
- 4) $-u_{\text{rev}}^{\max} \leq u_{\text{rev},i} \leq u_{\text{rev}}^{\max}$, $i = 1, 2$,
- 5) $-u_{\text{cab}}^{\max} \leq u_{\text{cab},i} \leq 0$, $i = 1, \dots, 3$,

with $u_{\text{thr}}^{\max} = 2.5$ N, $\dot{\delta}^{\max} = 25$ deg/s, $\dot{\Omega}^{\max} = 700$ rpm/s, $u_{\text{rev}}^{\max} = 10$ N m, and $u_{\text{cab}}^{\max} = 20$ N. These values are then used to construct the lower and upper bounds $\mathcal{L}_b$ and $\mathbf{u}_b$, respectively,

**Fig. 7** Snapshots of the detumbling scenario C1. The blue curve denotes the end-effector path throughout the maneuver.**Fig. 8** HSMS base pose and velocity errors for the detumbling maneuvers in C1. The attitude errors are displayed as θe using the axis-angle representation.

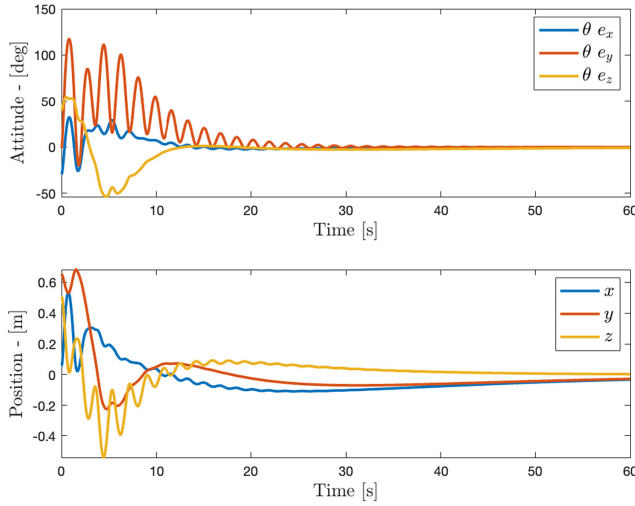
in Eq. (101). To facilitate an intuitive understanding of the closed-loop performance, we have transformed the configuration-space errors for both the base and the end-effector into the task space, as illustrated in Figs. 8 and 9. For the sake of compactness, we report the base and tip attitude errors in Figs. 8a and 9a in terms of their axis-angle representation (e, θ) , where $e = [e_x, e_y, e_z]^T$, with $\|e\| = 1$, and $\theta = [0, \pi]$ is the corresponding angle. The VSCMG states and thruster forces are reported in Figs. 10 and 11, whereas the control inputs for the hybrid manipulator are reported in Fig. 12. As expected from the controller implementation, the tendon input forces always have negative values (see Fig. 12b), which correspond to a physically sound pulling action exerted on the tendons.

In simulation scenario C2, we implemented the task-space tracking controller introduced in Sec. III.B over a simulation duration of $T = 300$ s. Our objective was to command the end-effector to follow the so-called Viviani curve, specified as $r_{T/D}^T(t) = [\rho(1 + \cos(\omega t)), \sin(\omega t), 2 \sin(0.5\omega t)]^T$, where $\rho = 0.3$ m and $\omega = 2\pi/T$, with $T = 150$ s. The controller gains in Eq. (114) are set to $K_p = 0.03I_3$ and $K_d = 0.3I_3$, while the actuator bounds are left equal to the ones in C1. For C2, we chose $H = \text{diag}\{H_b, H_r, H_s\}$, with $H_b = 10I_6$, $H_r = I_2$, and $H_s = 0.75I_3$. In the interest of brevity, we only present a few snapshots of the closed-loop simulations in C2 (Fig. 13), along with the corresponding tip position and linear velocity errors in Fig. 14. The error charts

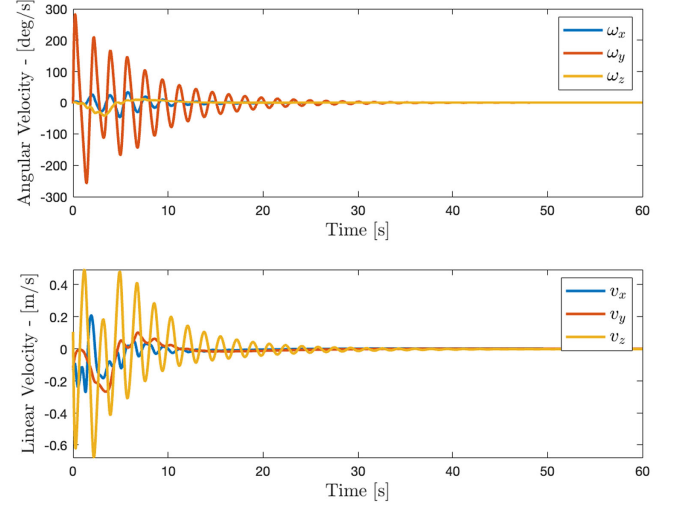
in Fig. 14 confirm that the proposed controller achieves highly satisfactory tracking performance. Moreover, the snapshots in Fig. 13 clearly demonstrate that both the rigid manipulator links and the soft appendage are essential for successfully executing the tracking maneuver. Animations can be found at <https://youtu.be/kU00tmeFRSU>. It is noteworthy that the state of the HSMS base undergoes significant changes during the simulation. This is expected because no specific setpoint was assigned to the base; as a result, the base drifts due to the large displacements undertaken by the manipulator. Such base-manipulator behavior might be desirable in mission phases where low power consumption is crucial, and one specific task (e.g., precise end-effector tracking) is prioritized over maintaining a fixed base configuration.

V. Conclusions

This paper formulates the dynamic model of a novel hybrid soft-rigid space manipulator system. Leveraging geometric mechanics and the GVS methodology—a soft robotics approach that parameterizes continuum body strain via user-defined basis functions—we derive a unified representation of the coupled rigid-soft dynamics. This framework yields a minimal set of ordinary differential equations structurally analogous to conventional rigid manipulator equations, enabling efficient controller synthesis. We further design two inverse dynamics controllers for system stabilization and

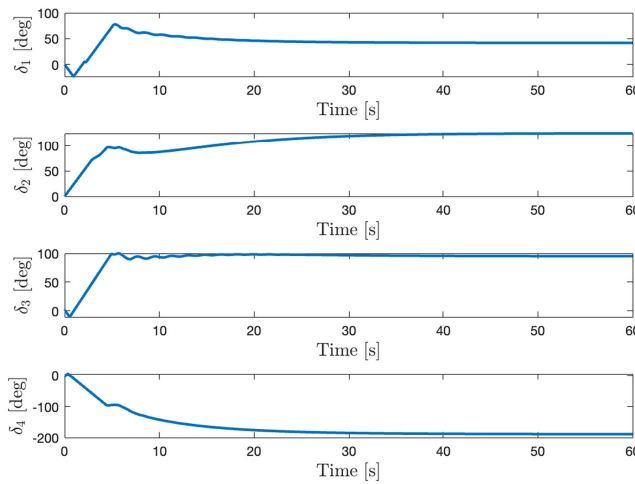


a) End-effector pose error

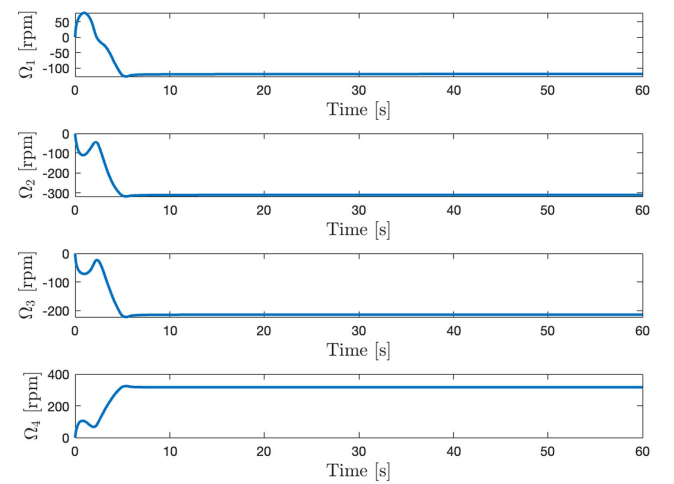


b) End-effector velocity error

Fig. 9 HSMS end-effector pose and velocity errors for the detumbling maneuvers in C1. The attitude errors are displayed as θe using the axis-angle representation.



a) Gimbal angles



b) Reaction wheel velocities

Fig. 10 VSCMG states (gimbal angles δ and wheel velocities Ω) for the detumbling maneuver in C1.

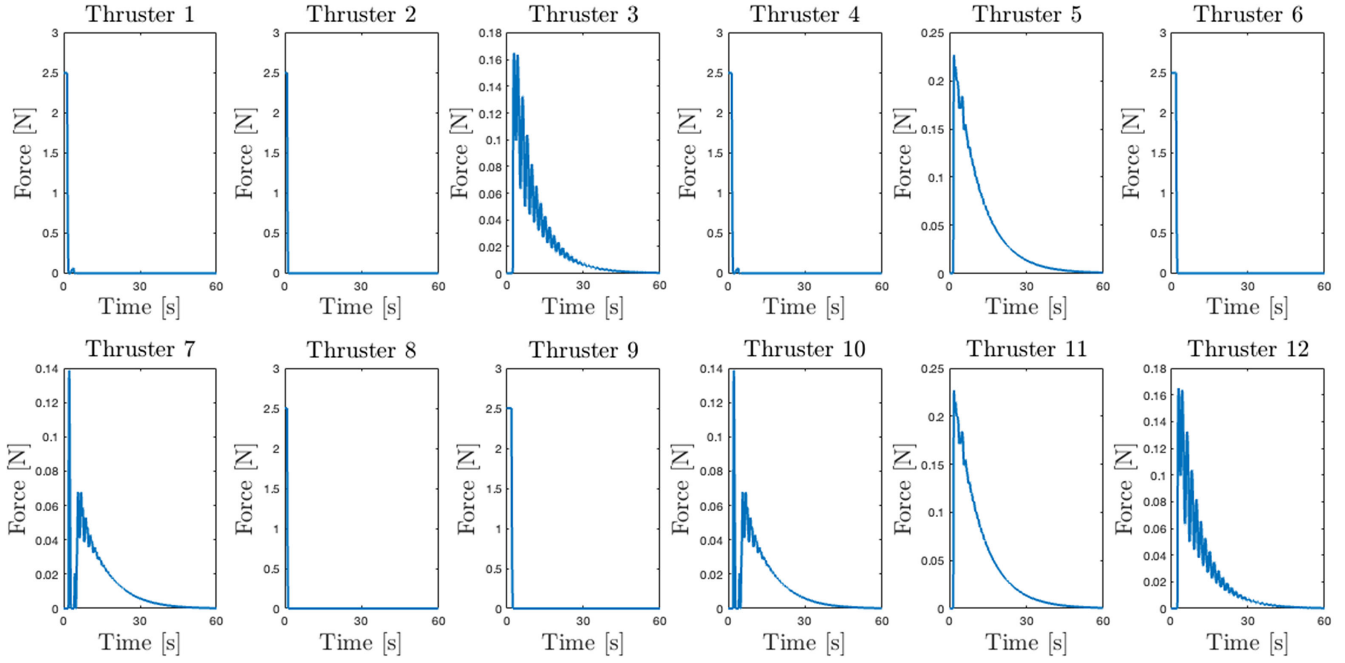


Fig. 11 Thruster force profiles in the detumbling maneuver in C1.

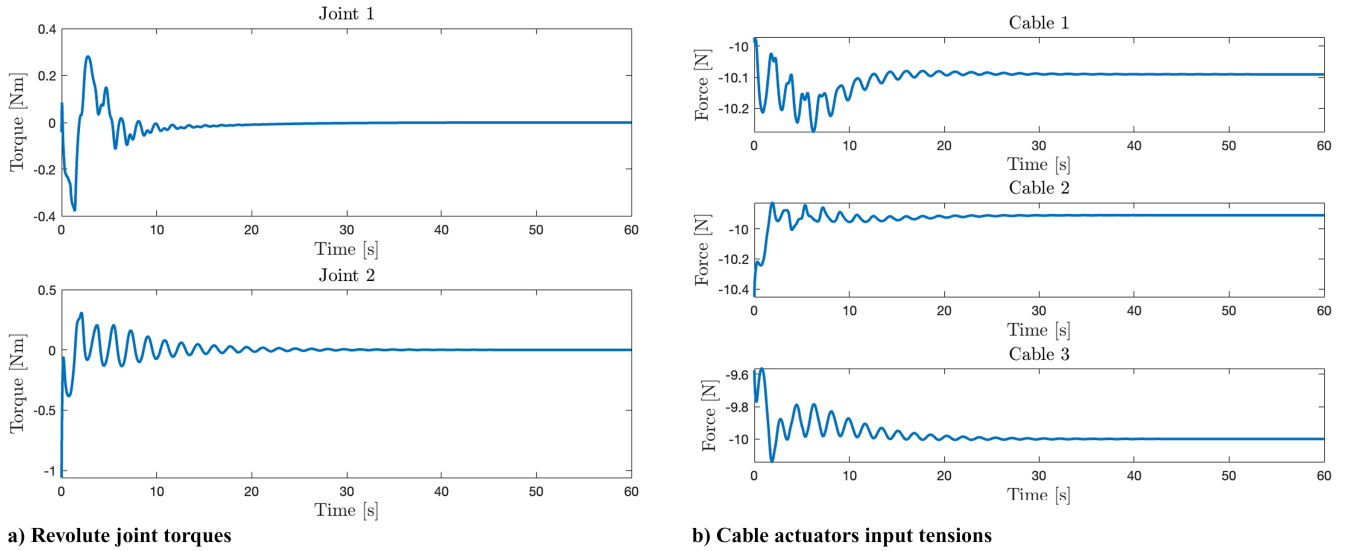


Fig. 12 Hybrid manipulator inputs (joint torques and cable tensions) in scenario C1.

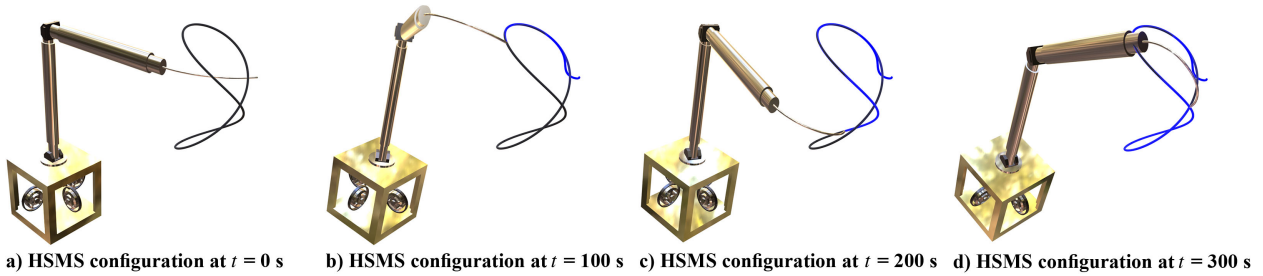


Fig. 13 Snapshots of the tracking scenario C2. The blue curve denotes the end-effector trajectory throughout the maneuver, while the black one represents the desired end-effector path.

end-effector tracking, incorporating realistic actuators such as thrusters and variable-speed control moment gyros to better represent a real spaceborne platform. The inherent advantage of the proposed modeling approach is demonstrated by the straightforward derivation and numerical implementation of two distinct

controllers: one in the configuration space and another one in the task space. The first controller brings the system to rest from an initial configuration characterized by nonzero momentum, while the task-space design successfully tracks a time-varying end-effector reference in the operational space. Overall, this work provides a first

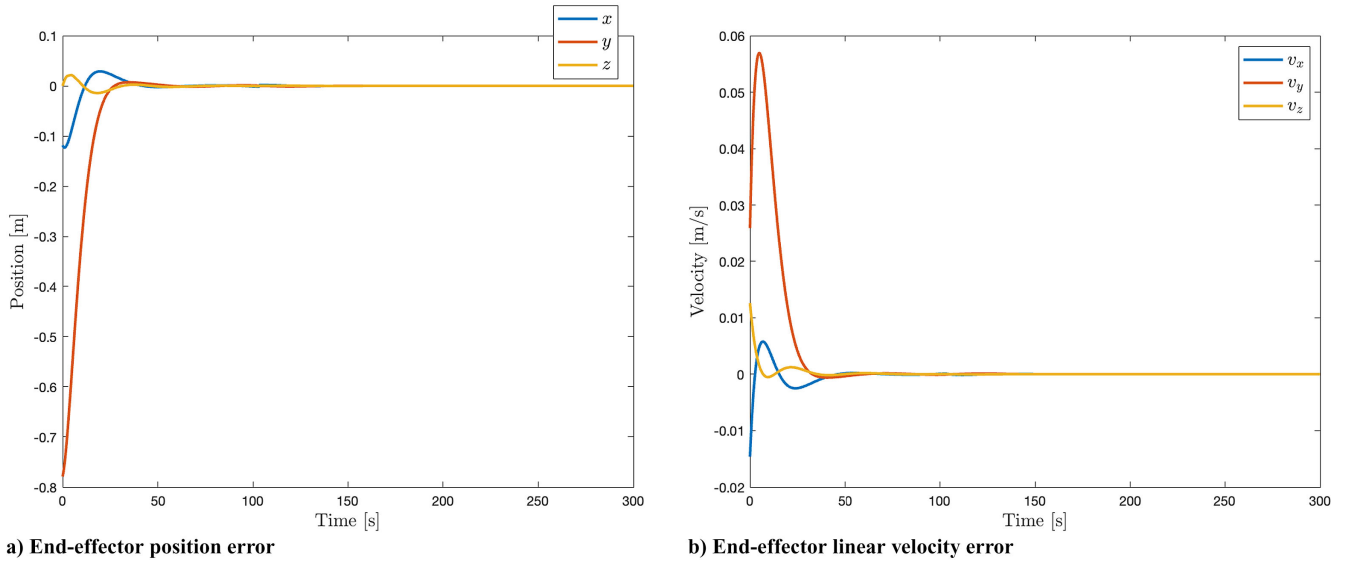


Fig. 14 End-effector position and velocity errors for the tracking scenario C2.

thorough investigation of the modeling and control of hybrid soft-rigid space robotic platforms, which offer higher dexterity and operational flexibility compared to purely rigid systems. Many promising research directions remain open in terms of further analysis, control synthesis, and experimental validation of the proposed approach. Among them, future studies should examine the impact of selecting a specific strain basis for the soft appendage. From the control design perspective, the development of an adaptive controller to address the uncertain nature of the system's parameters is also left as future work.

Appendix: Elements of Lie Theory

In this Appendix, we provide the reader with some elements of Lie theory that are used in the derivation of the equations of motion in this paper and that are necessary to follow the proposed methodology. While a thorough treatment of Lie theory is well beyond the scope of this section, we wish to expose the reader to some of the background information necessary for a more principled understanding of our general modeling framework used in the paper. We refer the reader to Chapter 9 in [34] for a complete coverage of the contents presented in the following. Another valuable resource is [49], perhaps more suitable to those who wish to approach Lie theory for the first time.

A.1. Lie Groups and Lie Algebras

For the sake of clarity, we briefly introduce the notation used in this paper, which is closely aligned to the one in [34,49].

Definition 1 (Tangent Bundle): Let $\mathcal{M}$ be a manifold, and denote the tangent space to $\mathcal{M}$ at point x as $T_x\mathcal{M}$. Then, the tangent bundle $T\mathcal{M}$ of $\mathcal{M}$ is defined as the disjoint union of the tangent spaces to $\mathcal{M}$ at all points $x \in \mathcal{M}$:

$$T\mathcal{M} = \bigcup_{x \in \mathcal{M}} T_x\mathcal{M} \quad (\text{A1})$$

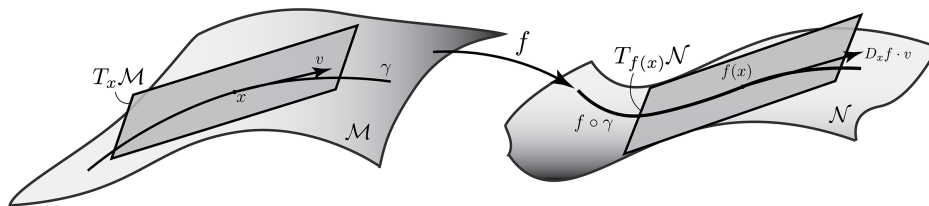


Fig. A1 Derivative of a smooth map.

Definition 2 (Vector Field): A vector field X on $\mathcal{M}$ is a map $X: \mathcal{M} \mapsto T\mathcal{M}$ such that $X(x) \in T_x\mathcal{M}$, for all $x \in \mathcal{M}$.

Definition 3 (Derivative): Consider a differentiable map $f: \mathcal{M} \mapsto \mathcal{N}$, where $\mathcal{M}$ and $\mathcal{N}$ are smooth manifolds. The derivative of f at a point $x \in \mathcal{M}$ is written as the linear map:

$$D_x f: T_x\mathcal{M} \mapsto T_{f(x)}\mathcal{N} \quad (\text{A2})$$

To construct the derivative, consider a curve $\gamma: (-\epsilon, \epsilon) \mapsto \mathcal{M}$, with $\gamma(0) = x$ and associated velocity vector $d\gamma/dt|_{t=0} = v$. Then, $D_x f \cdot v$ is the velocity vector at $t = 0$ of the curve $f \circ \gamma: \mathbb{R} \mapsto \mathcal{N}$ (see Fig. A1), as follows:

$$D_x f \cdot v = \left. \frac{d}{dt} f(\gamma(t)) \right|_{t=0}, \quad \gamma(0) = x \quad (\text{A3})$$

Given a manifold $\mathcal{M}$ and $x \in \mathcal{M}$, let $\gamma: I \mapsto \mathcal{M}$ be a curve defined on the interval $I \subseteq \mathbb{R}$, where $I \ni 0$ such that $\gamma(0) = x$. Two curves γ_1 and γ_2 are said to be equivalent at x if they have the same derivative at x , denoted as $\gamma_1 \sim_x \gamma_2$. A tangent vector at x is an equivalence class of curves under $\sim_x$, and the tangent space at x , denoted with $T_x\mathcal{M}$, is the collection of all such vectors.

The two main definitions used extensively in this work are now stated, namely Lie groups and Lie algebras. Then, we present a series of perhaps less familiar concepts, such as the one of left-invariant vector fields, which clarify the connection between Lie groups and Lie algebras. Beyond the mere definitions, we try to provide some intuition as to why these ideas are related to each other and how they fit into the description of complex mechanical systems.

Definition 4 (Lie Group): A Lie group $(\mathcal{G}, \blacksquare)$, also referred to as $\mathcal{G}$ for brevity, is a manifold $\mathcal{G}$ endowed with a group operation $\blacksquare: \mathcal{G} \times \mathcal{G} \mapsto \mathcal{G}$ and an element $e \in \mathcal{G}$ with the following axioms:

$$g \blacksquare e = e \blacksquare g = g \quad (\text{identity}), \quad \forall g \in \mathcal{G} \quad (\text{A4})$$

$$g^{-1} \blacksquare g = g \blacksquare g^{-1} = e \quad (\text{inverse}), \quad \forall g \in \mathcal{G} \quad (\text{A5})$$

$$g_1 \blacksquare (g_2 \blacksquare g_3) = (g_1 \blacksquare g_2) \blacksquare g_3 = g_1 \blacksquare g_2 \blacksquare g_3 \quad (\text{associativity}), \\ \forall g_1, g_2, g_3 \in \mathcal{G} \quad (\text{A6})$$

In general, the group operation is noncommutative, that is, $g \blacksquare h \neq h \blacksquare g$, for $h, g \in \mathcal{G}$. This is the case, for example, of matrix or quaternion groups. Groups for which the group operation is commutative are known as “abelian” [49].

Definition 5 (Lie Group Homomorphism): Given two Lie groups $(\mathcal{G}, \blacksquare)$ and $(\mathcal{H}, \star)$, a Lie group homomorphism is defined as a smooth map $\pi: \mathcal{G} \mapsto \mathcal{H}$ such that $\pi(g_1 \blacksquare g_2) = \pi(g_1) \star \pi(g_2)$.

If π is a bijection, then it is an isomorphism, and $\mathcal{G}, \mathcal{H}$ are said to be isomorphic, denoted as $\mathcal{G} \simeq \mathcal{H}$. If π is a bijection and its inverse π^{-1} is smooth, then π is called a diffeomorphism.

Definition 6 (Left Translation Map): Consider the Lie group $(\mathcal{G}, \blacksquare)$. For any $g, h \in \mathcal{G}$, the left translation map is the map $L_g: \mathcal{G} \mapsto \mathcal{G}$ such that

$$L_g(h) = g \blacksquare h \quad (\text{A7})$$

Note that L_g and its inverse $L_{g^{-1}}$ are smooth, and L_g is therefore a diffeomorphism of $\mathcal{G}$. Moreover, the tangent map $T_e L_g: T_e \mathcal{G} \mapsto T_g \mathcal{G}$ is an isomorphism between $T_e \mathcal{G}$ and $T_g \mathcal{G}$ [49]. The right translation map is defined in a similar fashion as $R_g: \mathcal{G} \mapsto \mathcal{G}$, that is, $R_g(h) = h \blacksquare g$.

Definition 7 (Lie Algebra): A Lie algebra $(\mathfrak{g}, [\cdot, \cdot]_{\mathfrak{g}})$, also referred to as $\mathfrak{g}$ for brevity, is a vector space endowed with a bilinear, anti-commutative bracket operation $[\cdot, \cdot]: \mathfrak{g} \times \mathfrak{g} \mapsto \mathfrak{g}$ satisfying Jacobi's identity:

$$[[\xi, \eta], \zeta] + [[\zeta, \xi], \eta] + [[\eta, \zeta], \xi] = 0 \quad (\text{A8})$$

for all $\xi, \eta, \zeta \in \mathfrak{g}$.

Definition 8 (Left-Invariant Vector Field): A vector field X on a group $\mathcal{G}$ is called left-invariant if, for all $g, h \in \mathcal{G}$, the following holds:

$$X(g \blacksquare h) = X(L_g(h)) = (T_h L_g)X(h) \quad (\text{A9})$$

where $T_h L_g: T_h \mathcal{G} \mapsto T_{g \blacksquare h} \mathcal{G}$ is the differential of L_g at h , also known as “pushforward” (Fig. A2).

As a consequence of Eq. (A9), we observe that $X(g) = (T_e L_g)X(e)$, which means that left-invariant vector fields are defined by their value at the identity. This is a key observation that highlights the particular significance of $T_e \mathcal{G}$, the tangent space at the identity. By denoting a generic vector belonging to $T_e \mathcal{G}$ with $\xi = X(e)$, the following vector field on $\mathcal{G}$ can be defined:

$$X_{\xi}(g) = (T_e L_g)\xi \quad (\text{A10})$$

Then,

$$X_{\xi}(g \blacksquare h) = (T_e L_{g \blacksquare h})\xi = T_e(L_g \circ L_h)\xi \quad (\text{A11})$$

$$= T_h L_g(T_e L_h \xi) = (T_h L_g)X_{\xi}(h) \quad (\text{A12})$$

from which we conclude that $X_{\xi}(g)$ is left invariant.

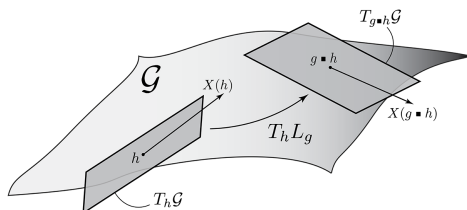


Fig. A2 Left invariant vector field.

Definition 9 (One-Parameter Subgroup): A one-parameter subgroup of the group $\mathcal{G}$ is defined as the image of a Lie group homomorphism $\gamma: \mathbb{R} \mapsto \mathcal{G}$.

It can be shown (see, e.g., [34]) that the (unique) integral curve $\gamma_{\xi}: \mathbb{R} \mapsto \mathcal{G}$ associated to the left-invariant vector field X_{ξ} , where $\gamma_{\xi}(0) = e$ and $\gamma'_{\xi}(t) = X_{\xi}(\gamma_{\xi}(t))$, is a smooth one-parameter subgroup. While this is an important consideration, in the next steps we provide more details regarding the form of smooth one-parameter subgroups. To this end, we define the exponential map.

Definition 10: The exponential map $\exp: \mathfrak{g} \mapsto \mathcal{G}$ is defined as

$$\exp(\xi) = \gamma_{\xi}(1) \quad (\text{A13})$$

It can be shown that $\exp(s\xi) = \gamma_{\xi}(s)$, for $s \in \mathbb{R}$. Let s be fixed, and consider the curve $t \mapsto \gamma_{\xi}(ts)$. Its associated differential equation reads

$$\frac{d}{dt} \gamma_{\xi}(ts) = s \gamma'_{\xi}(ts) = s X_{\xi}(\gamma_{\xi}(ts)) = X_{s\xi}(\gamma_{\xi}(ts)) \quad (\text{A14})$$

where we have used left invariance of X_{ξ} in the last equality. The integral curve $\gamma_{s\xi}(t)$ obeys the same equation since by definition $\gamma'_{s\xi}(t) = X_{s\xi}(\gamma_{s\xi}(t))$. Since both curves pass through the identity at $t = 0$, it follows that $\gamma_{s\xi}(t) = \gamma_{\xi}(ts)$. Choosing $t = 1$, and recalling the definition of the exponential in Eq. (A13), it follows that $\exp(s\xi) = \gamma_{\xi}(s)$. Therefore, the exponential map maps a line $s\xi \in T_e \mathcal{G}$ onto the one-parameter subgroup $\gamma_{\xi}(s)$ in $\mathcal{G}$. With a few more steps, skipped for brevity, it can be proved that any smooth one-parameter subgroup $\gamma(t)$ of $\mathcal{G}$ respects the equality $\gamma = \gamma_{\xi}$, with $\xi = \gamma'(0)$. An important consequence of this is that all smooth one-parameter subgroups of $\mathcal{G}$ can be written as $\exp(t\xi)$, for some $\xi \in T_e \mathcal{G}$.

Keeping in mind the definitions above, it is now possible to grasp the connection between a Lie group and its associated Lie algebra. We state the definition here below and provide some remarks in the following.

Definition 11 (Lie Algebra of a Lie group): The Lie algebra $\mathfrak{g}$ of a Lie group $\mathcal{G}$ is the tangent space at the identity, $T_e \mathcal{G}$, with the bracket defined as

$$[\xi, \eta] = [X_{\xi}, X_{\eta}](e) \quad (\text{A15})$$

Definition 11 reveals much of the power behind Lie group theory. In fact, we have shown how every straight path $t\xi \in \mathfrak{g}$ can be mapped to a corresponding curved path $\exp(t\xi)$ on the manifold via the exponential map. This connection is invaluable, as it allows us to reason about the nonlinear manifold $\mathcal{G}$ in terms of the much simpler vector space $\mathfrak{g}$. Notably, elements of $\mathfrak{g}$ can be expressed as a linear combination of basis elements along with their associated coordinates. Therefore, the elements of a Lie algebra of dimension m can be simply identified by a coordinate vector in $\mathbb{R}^m$.

A.2. Actions of Lie Groups

Another fundamental notion in the context of Lie theory, which has particular relevance in the study of mechanical systems, is the one of action of Lie groups on a manifold.

Definition 12 (Actions of Lie Groups): Given a Lie group $(\mathcal{G}, \blacksquare)$ and a manifold $\mathcal{M}$, the (left) action of $\mathcal{G}$ on $\mathcal{M}$ is defined as the smooth mapping $\Phi: \mathcal{G} \times \mathcal{M} \mapsto \mathcal{M}$ with the following properties:

$$\Phi(e, x) = x \quad (\text{A16})$$

$$\Phi(g_1, \Phi(g_2, x)) = \Phi(g_1 \blacksquare g_2, x) \quad (\text{A17})$$

for all $g_1, g_2 \in \mathcal{G}$ and $x \in \mathcal{M}$.

When $\mathcal{M}$ is a linear vector space, the action Φ of $\mathcal{G}$ on $\mathcal{M}$ is said to be a representation of $\mathcal{G}$ on $\mathcal{M}$ [34].

Definition 13 (Adjoint Representation): Given a Lie group $\mathcal{G}$ and its associated algebra $\mathfrak{g}$, let $g \in \mathcal{G}$ and define $I_g: \mathcal{G} \mapsto \mathcal{G}$, where

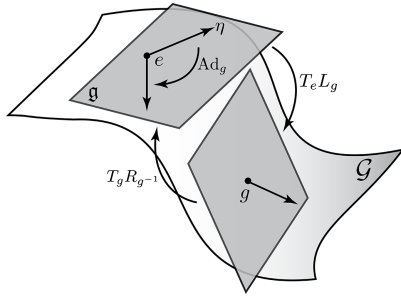


Fig. A3 Adjoint representation of $\mathcal{G}$ on $\mathfrak{g}$.

$I_g(h) = (R_{g^{-1}} \circ L_g)h$, for all $h \in \mathcal{G}$. The Adjoint representation of $\mathcal{G}$ on $\mathfrak{g}$, $\text{Ad}_g: \mathfrak{g} \mapsto \mathfrak{g}$, is defined as the differential of I_g at the identity (see Fig. A3):

$$\text{Ad}_g(\eta) = (T_e I_g)\eta = T_e(R_{g^{-1}} \circ L_g)\eta \quad (\text{A18})$$

for all $\eta \in \mathfrak{g}$.

Note that Ad_g satisfies Eqs. (A16) and (A17) since $\text{Ad}_e(\eta) = \eta$ and $\text{Ad}_g(\text{Ad}_h(\eta)) = \text{Ad}_{gh}(\eta)$. The Adjoint representation in rigid-body kinematics stands for a transformation between reference frames.

Definition 14 (Lie Algebra Adjoint). The Lie algebra adjoint $\text{ad}_\xi: \mathfrak{g} \mapsto \mathfrak{g}$ is defined as the differential of Ad_g with respect to g at e , in the direction ξ , that is,

$$\text{ad}_\xi(\eta) = T_e \text{Ad}_g(\eta) \cdot \xi \quad (\text{A19})$$

Given the curves $g(t), h(s)$ belonging to a matrix Lie group $\mathcal{G}$, with $g(0) = e, h(0) = e$, and $g'(0) = \xi, h'(0) = \eta$, recalling the definitions in Eqs. (A18) and (A19), the following holds:

$$\text{ad}_\xi(\eta) = \left. \frac{d}{dt} \frac{d}{ds} g(t)h(s)g(t)^{-1} \right|_{s=0, t=0} = \xi\eta - \eta\xi = [\xi, \eta] \quad (\text{A20})$$

where the last equality follows from Definition 7. Throughout this work, we denote the Adjoint representation by $\text{Ad}_g(\cdot): \mathfrak{g} \mapsto \mathfrak{g}$. When parentheses are omitted, $\text{Ad}_g \in \mathbb{R}^{n \times n}$ refers to its matrix representation. This convention extends to the Lie algebra adjoint $\text{ad}_\xi(\cdot): \mathfrak{g} \mapsto \mathfrak{g}$, with $\text{ad}_\xi \in \mathbb{R}^{n \times n}$ similarly denoting its matrix form.

Acknowledgment

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References

- [1] NASA Goddard Space Flight Center, "On-Orbit Satellite Servicing Study," 2010, <https://www.nasa.gov/wp-content/uploads/2023/10/nasa-satellite-servicing-project-report-0511.pdf> [retrieved 19 Feb. 2025].
- [2] Flores-Abad, A., Ma, O., Pham, K., and Ulrich, S., "A Review of Space Robotics Technologies for On-Orbit Servicing," *Progress in Aerospace Sciences*, Vol. 68, July 2014, pp. 1–26. <https://doi.org/10.1016/j.paerosci.2014.03.002>
- [3] Papadopoulos, E., Aghili, F., Ma, O., and Lampariello, R., "Robotic Manipulation and Capture in Space: A Survey," *Frontiers in Robotics and AI*, Vol. 8, July 2021. <https://doi.org/10.3389/frobt.2021.686723>
- [4] Yoshida, K., "ETS-VII Flight Experiments for Space Robot Dynamics and Control," *Experimental Robotics VII*, edited by D. Rus, and S. Singh, Springer, Berlin, 2001, pp. 209–218. https://doi.org/10.1007/3-540-45118-8_22
- [5] Shoemaker, J., and Wright, M., "Orbital Express Space Operations Architecture Program," *Spacecraft Platforms and Infrastructure*, edited by P. Thoryk, Jr., and M. Wright, Vol. 5419, International Soc. for Optics and Photonics (SPIE), Bellingham, WA, 2004, pp. 57–65. <https://doi.org/10.1117/12.544067>
- [6] ESA ESTEC, "ClearSpace-1 In-Orbit Demonstration Mission," 2024, https://indico.esa.int/event/516/contributions/9983/attachments/6342/10932/ClearSpace-1%20IOD%20Mission_Clean%20Space%20Days.pdf [retrieved 19 Feb. 2025].

- [7] Tsotras, P., King-Smith, M., and Ticozzi, L., "Spacecraft-Mounted Robotics," *Annual Review of Control, Robotics, and Autonomous Systems*, Vol. 6, No. 1, 2023, pp. 335–362. <https://doi.org/10.1146/annurev-control-062122-082114>
- [8] Dubowsky, S., and Papadopoulos, E., "The Kinematics, Dynamics, and Control of Free-Flying and Free-Floating Space Robotic Systems," *IEEE Transactions on Robotics and Automation*, Vol. 9, No. 5, 1993, pp. 531–543. <https://doi.org/10.1109/70.258046>
- [9] Carignan, C., and Akin, D., "The Reaction Stabilization of On-Orbit Robots," *IEEE Control Systems Magazine*, Vol. 20, No. 6, 2000, pp. 19–33. <https://doi.org/10.1109/37.887446>
- [10] Valverde, A., and Tsotras, P., "Dual Quaternion Framework for Modeling of Spacecraft-Mounted Multibody Robotic Systems," *Frontiers in Robotics and AI*, Vol. 5, Nov. 2018. <https://doi.org/10.3389/frobt.2018.00128>
- [11] Seddaoui, A., Saaj, C. M., and Nair, M. H., "Modeling a Controlled-Floating Space Robot for In-Space Services: A Beginner's Tutorial," *Frontiers in Robotics and AI*, Vol. 8, Dec. 2021. <https://doi.org/10.3389/frobt.2021.725333>
- [12] Antonello, A., Valverde, A., and Tsotras, P., "Dynamics and Control of Spacecraft Manipulators with Thrusters and Momentum Exchange Devices," *Journal of Guidance, Control, and Dynamics*, Vol. 42, No. 1, 2019, pp. 15–29. <https://doi.org/10.2514/1.G003601>
- [13] Yamano, M., Konno, A., and Uchiyama, M., "Experiments On Capturing a Floating Object by Two Flexible Manipulators," *IEEE International Conference on Robotics and Automation (ICRA)*, Vol. 1, IEEE Publ., Piscataway, NJ, 2000, pp. 482–487. <https://doi.org/10.1109/ROBOT.2000.844101>
- [14] Carabis, D. S., Oakes, K. I., and Wen, J. T., "Manipulation of Massive Objects in Space Using Flexible Joint Manipulators," *Journal of Guidance, Control, and Dynamics*, Vol. 44, No. 5, 2021, pp. 923–937. <https://doi.org/10.2514/1.G005347>
- [15] Dubanchet, V., "Modeling and Control of a Flexible Space Robot to Capture a Tumbling Debris," Ph.D. Thesis, Ecole Polytechnique de Montreal, Canada, Jan. 2016.
- [16] Zarafshan, P., and Moosavian, S. A. A., "Manipulation Control of a Space Robot with Flexible Solar Panels," *2010 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, IEEE Publ., Piscataway, NJ, 2010, pp. 1099–1104. <https://doi.org/10.1109/AIM.2010.5695818>
- [17] Scott, M. A., Gilbert, M. G., and Demeo, M. E., "Active Vibration Damping of the Space Shuttle Remote Manipulator System," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 2, 1993, pp. 275–280. <https://doi.org/10.2514/3.56598>
- [18] Sabatini, M., Gasbarri, P., Monti, R., and Palmerini, G. B., "Vibration Control of a Flexible Space Manipulator During on Orbit Operations," *Acta Astronautica*, Vol. 73, April–May 2012, pp. 109–121. <https://doi.org/10.1016/j.actaastro.2011.11.012>
- [19] Jia, S., Jia, Y., Xu, S., and Hu, Q., "Maneuver and Active Vibration Suppression of Free-Flying Space Robot," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 54, No. 3, 2018, pp. 1115–1134. <https://doi.org/10.1109/TAES.2017.2775780>
- [20] Xinxin Yang, S. S. G., and He, W., "Dynamic Modelling and Adaptive Robust Tracking Control of a Space Robot with Two-Link Flexible Manipulators under Unknown Disturbances," *International Journal of Control*, Vol. 91, No. 4, 2018, pp. 969–988. <https://doi.org/10.1080/00207179.2017.1300837>
- [21] Doggett, W. R., Dorsey, J. T., Jones, T. C., and King, B., "Development of a Tendon-Actuated Lightweight In-Space MANipulator (TALIS-MAN)," *42nd Aerospace Mechanisms Symposium*, NASA Goddard Space Flight Center, MD, 2014, <https://ntrs.nasa.gov/api/citations/20150004076/downloads/20150004076.pdf>
- [22] Dorsey, J., Doggett, W. R., Jones, T. C., and King, B., "Application of a Novel Long-Reach Manipulator Concept to Asteroid Redirect Missions," *2nd AIAA Spacecraft Structures Conference*, AIAA Paper 2015-0225, 2015. <https://doi.org/10.2514/6.2015-0225>
- [23] Sallaberger, C., "Canadian Space Robotic Activities," *Acta Astronautica*, Vol. 41, No. 4, 1997, pp. 239–246. [https://doi.org/10.1016/S0094-5765\(98\)00082-4](https://doi.org/10.1016/S0094-5765(98)00082-4)
- [24] Li, K., Zhang, Y., and Hu, Q., "Dynamic Modelling and Control of a Tendon-Actuated Lightweight Space Manipulator," *Aerospace Science and Technology*, Vol. 84, Jan. 2019, pp. 1150–1163. <https://doi.org/10.1016/j.ast.2018.11.018>

- [25] Kernot, J. E., and Ulrich, S., "Adaptive Control of a Tendon-Driven Manipulator for Capturing Non-Cooperative Space Targets," *Journal of Spacecraft and Rockets*, Vol. 59, No. 1, 2022, pp. 111–128. <https://doi.org/10.2514/1.A34881>
- [26] Do, T. N., Tjahjowidodo, T., Lau, M. W. S., and Phee, S. J., "Enhanced Performances for Cable-Driven Flexible Robotic Systems with Asymmetric Backlash Profile," *2015 IEEE International Conference on Technologies for Practical Robot Applications (TePRA)*, IEEE Publ., Piscataway, NJ, 2015, pp. 1–6. <https://doi.org/10.1109/TePRA.2015.7219674>
- [27] Della Santina, C., Catalano, M. G., and Bicchi, A., *Soft Robots*, Springer, Berlin, 2020, pp. 1–15. https://doi.org/10.1007/978-3-642-41610-1_146-2
- [28] Yang, J., Peng, H., Zhang, J., and Wu, Z., "Dynamic Modeling and Beating Phenomenon Analysis of Space Robots with Continuum Manipulators," *Chinese Journal of Aeronautics*, Vol. 35, No. 9, 2022, pp. 226–241. <https://doi.org/10.1016/j.cja.2021.08.007>
- [29] Yang, J., Peng, H., Zhou, W., and Wu, Z., "Integrated Control of Continuum-Manipulator Space Robots with Actuator Saturation and Disturbances," *Journal of Guidance, Control, and Dynamics*, Vol. 45, No. 12, 2022, pp. 2379–2388. <https://doi.org/10.2514/1.G006692>
- [30] Facchinei, F., and Pang, J.-S., *Finite-Dimensional Variational Inequalities and Complementarity Problems*, Springer, New York, 2003, pp. 866–869. <https://doi.org/10.1007/b97544>
- [31] Yang, J., Peng, H., Wu, S., Zhang, J., Wu, Z., and Wu, J., "Differential-Algebraic Equation-Based Model Predictive Control for Trajectory Tracking of Spacecraft-Mounted Continuum Manipulators," *Aerospace Science and Technology*, Vol. 155, Dec. 2024, Paper 109615. <https://doi.org/10.1016/j.ast.2024.109615>
- [32] Arnold, M., and Brüls, O., "Convergence of the Generalized-Alpha Scheme for Constrained Mechanical Systems," *Multibody System Dynamics*, Vol. 18, No. 2, 2007, pp. 185–202. <https://doi.org/10.1007/s11044-007-9084-0>
- [33] Mathew, A. T., Feliu-Talegon, D., Alkayas, A. Y., Boyer, F., and Renda, F., "Reduced Order Modeling of Hybrid Soft-Rigid Robots Using Global, Local, and State-Dependent Strain Parameterization," *International Journal of Robotics Research*, Vol. 44, No. 1, 2025, pp. 129–154. <https://doi.org/10.1177/02783649241262333>
- [34] Marsden, J. E., and Ratiu, T. S., "Introduction to Mechanics and Symmetry," *Texts in Applied Mathematics*, Vol. 17, Springer, New York, 1999. <https://doi.org/10.1007/978-0-387-21792-5>
- [35] Magnus, W., "On the Exponential Solution of Differential Equations for a Linear Operator," *Communications on Pure and Applied Mathematics*, Vol. 7, No. 4, 1954, pp. 649–673. <https://doi.org/10.1002/cpa.3160070404>
- [36] Blanes, S., Casas, F., Oteo, J. A., and Ros, J., "Magnus and Fer Expansions for Matrix Differential Equations: The Convergence Problem," *Journal of Physics A: Mathematical and General*, Vol. 31, No. 1, 1998, pp. 259–268. <https://doi.org/10.1088/0305-4470/31/1/023>
- [37] Hairer, E., Lubich, C., and Wanner, G., *Geometric Numerical Integration: Structure-Preserving Algorithms for Ordinary Differential Equations*, Springer, Berlin, 2006, pp. 121–123. <https://doi.org/10.1007/3-540-30666-8>
- [38] Murray, R. M., Li, Z., and Sastry, S. S., *A Mathematical Introduction to Robotics Manipulation*, CRC Press, Boca Raton, FL, 1994, pp. 39–41. <https://doi.org/10.1201/9781315136370>
- [39] Renda, F., Armanini, C., Lebastard, V., Candelier, F., and Boyer, F., "A Geometric Variable-Strain Approach for Static Modeling of Soft Manipulators with Tendon and Fluidic Actuation," *IEEE Robotics and Automation Letters*, Vol. 5, No. 3, 2020, pp. 4006–4013. <https://doi.org/10.1109/LRA.2020.2985620>
- [40] Boyer, F., Lebastard, V., Candelier, F., and Renda, F., "Dynamics of Continuum and Soft Robots: A Strain Parameterization Based Approach," *IEEE Transactions on Robotics*, Vol. 37, No. 3, 2021, pp. 847–863. <https://doi.org/10.1109/TRO.2020.3036618>
- [41] Renda, F., Mathew, A., and Talegon, D. F., "Dynamics and Control of Soft Robots With Implicit Strain Parametrization," *IEEE Robotics and Automation Letters*, Vol. 9, No. 3, 2024, pp. 2782–2789. <https://doi.org/10.1109/LRA.2024.3360813>
- [42] Reddy, J. N., *An Introduction to Continuum Mechanics*, 2nd ed., Cambridge Univ. Press, Cambridge, England, U.K., 2013, p. 338. <https://doi.org/10.1017/CBO9781139178952>
- [43] Yoon, H., and Tsotras, P., "Spacecraft Adaptive Attitude and Power Tracking with Variable Speed Control Moment Gyroscopes," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 6, 2002, pp. 1081–1090. <https://doi.org/10.2514/2.4987>
- [44] Rao, P., Peyron, Q., Lilge, S., and Burgner-Kahrs, J., "How to Model Tendon-Driven Continuum Robots and Benchmark Modelling Performance," *Frontiers in Robotics and AI*, Vol. 7, Feb. 2021. <https://doi.org/10.3389/frobt.2020.630245>
- [45] Khalil, W., and Siciliano, B., *Modelling and Identification*, Springer, London, 1996, pp. 3–58. https://doi.org/10.1007/978-1-4471-1501-4_1
- [46] Mathew, A. T., Hmida, I. B., Armanini, C., Boyer, F., and Renda, F., "SoRoSim: A MATLAB Toolbox for Hybrid Rigid–Soft Robots Based on the Geometric Variable-Strain Approach," *IEEE Robotics & Automation Magazine*, Vol. 30, No. 3, 2023, pp. 106–122. <https://doi.org/10.1109/MRA.2022.3202488>
- [47] Falkenhahn, V., Hildebrandt, A., Neumann, R., and Sawodny, O., "Model-Based Feedforward Position Control of Constant Curvature Continuum Robots Using Feedback Linearization," *IEEE International Conference on Robotics and Automation (ICRA)*, IEEE Publ., Piscataway, NJ, 2015, pp. 762–767. <https://doi.org/10.1109/ICRA.2015.7139264>
- [48] Kazemipour, A., Fischer, O., Toshimitsu, Y., Wong, K. W., and Katzschmann, R. K., "Adaptive Dynamic Sliding Mode Control of Soft Continuum Manipulators," *IEEE International Conference on Robotics and Automation (ICRA)*, IEEE Publ., Piscataway, NJ, 2022, pp. 3259–3265. <https://doi.org/10.1109/ICRA46639.2022.9811715>
- [49] Bullo, F., and Lewis, A. D., "Geometric Control of Mechanical Systems," *Texts in Applied Mathematics*, Vol. 49, Springer Verlag, Berlin, 2004. <https://doi.org/10.1007/978-1-4899-7276-7>

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