Paper Number

Trail-Braking Driver Input Parameterization for General Corner Geometry

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ABSTRACT

Trail-Braking (TB) is a common cornering technique used in rally racing to negotiate tight corners at high speeds. It has been shown that TB can be generated as the solution to the minimum time cornering problem, subject to fixed final positioning of the vehicle after the corner, using non-linear programming (NLP). In this work we formulate the optimization problem relaxing the final positioning of the vehicle with respect to the width of the road in order to study the optimality of late-apex trajectories typically followed by rally drivers. Non-linear programming optimization is applied on a variety of corners. The optimal control inputs are approximated by simple piecewise linear input profiles defined by a small number of parameters. It is shown that the proposed input parameterization can generate close to optimal TB along the various corner geometries.

INTRODUCTION

The problem of trajectory planning for high-speed ground vehicles presents an enormous technical challenge. Several numerical optimization approaches have been presented mainly for lap time simulation applications [1], [2], [3], [4]. These trajectory optimization schemes incorporate the full transient behavior of accurate, high order dynamical models, thus producing realistic results.

Rally racing involves additional challenges compared to closed circuit racing, as it takes place in open, changing and uncontrolled environments. Unlike closed circuit racing of high performance vehicles (e.g. F1), to date there has been limited amount of work correlating driving techniques used by expert rally drivers with mathematical models. In [5] a numerical optimization approach, involving a single-track vehicle model, was proposed to study the optimality properties of Trail-Braking and Pendulum-Turn cornering techniques used in rally racing, by solving several minimum time cornering scenarios along a 90deg corner. In [6] the steering, throttle and brake inputs of an expert driver were recorded during the execution of aggressive cornering maneuvers. Simple parameterizations of the recorded inputs were optimized to reproduce the same maneuvers along a 90deg corner using a high fidelity vehicle model.

In this work we concentrate on a high speed cornering technique commonly used in rally racing, the Trail-Braking (TB). Trail-Braking is a technique used by rally drivers to negotiate single tight corners at high speeds [7], [8]. Typical characteristics of the TB maneuver include high vehicle slip angles and yaw rates. In addition, expert rally drivers follow the so-called “late-apex” line through the corner during execution of TB, that is, they exit the corner close to the inner edge of the road. Rally drivers use such aggressive techniques in order to bring the vehicle back to a stable straight line driving condition in a short distance after the corner, allowing themselves to react to unexpected changes in road conditions ahead, which are typical during open-road racing. The above characteristics of the TB maneuver were verified using data collected during the execution of the TB technique along a 90deg corner by Mr. Tim O’Neil, a five times US and North American Rally Champion and rally driver instructor [6]. It has been shown [5] that TB along a 90deg corner can be generated as the solution to a special case of the minimum time cornering problem, subject to fixed final positioning of the vehicle, using non-linear programming (NLP).

In this work we apply the methodology of [5] on a variety of corner geometries to validate the optimality properties of the Trail-Braking maneuver. In addition, we relax the boundary conditions corresponding to the final positioning of the vehicle with respect to the width of the road and study the optimality of late apex trajectories typically followed by rally drivers. The derived optimal control inputs are then approximated by piecewise linear profiles, defined by a small number of parameters. Using an alternative optimization scheme we demonstrate that the parameterized input profiles can be adjusted to generate close to optimal TB along the various corner geometries. The parameterized inputs optimization scheme is also implemented using a high fidelity vehicle model to generalize the results of [6] with respect to the corner geometry.

TRAIL-BRAKING OPTIMALITY VALIDATION FOR GENERAL CORNER GEOMETRY

In this section we apply the optimization scheme introduced in [5] to reproduce Trail-Braking maneuvers along a variety of corner geometries. This time we allow free final positioning of the vehicle with respect to the width of the road in order to study the optimality of late-apex trajectories typically followed by rally drivers.

VEHICLE MODEL

Rally drivers take advantage of the normal load transfer from the front to the rear axle and vice versa during acceleration and braking in order to control the yaw motion of the vehicle [5], [6], [7], [8]. The single track vehicle model used in [5] (Fig.1), is of low dimensionality and can be efficiently incorporated in a numerical optimization scheme. At the same time the above vehicle model takes into consideration the essential load transfer effects.

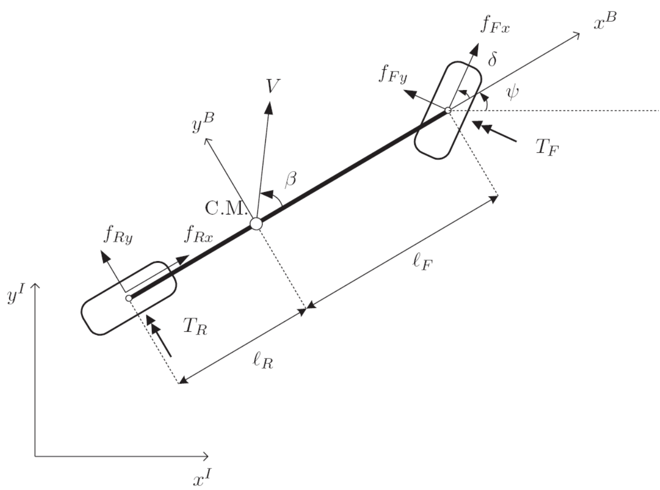

Figure 1: The single track model.

The equations of motion of the single track model are given as follows:

$$m\ddot{x} = f_{Fx}\cos(\psi+\delta) - f_{Fy}\sin(\psi+\delta) + f_{Rx}\cos\psi - f_{Ry}\sin\psi \qquad (1)$$

$$m\ddot{y} = f_{Fx}\sin(\psi+\delta) + f_{Fy}\cos(\psi+\delta) + f_{Rx}\sin\psi + f_{Ry}\cos\psi \qquad (2)$$

$$I_z\ddot{\psi} = (f_{Fy}\cos\delta + f_{Fx}\sin\delta)\ell_F - f_{Ry}\ell_R \qquad (3)$$

$$I_i\dot{\omega}_i = T_i - f_{ix}r_i, \qquad i = F,R \qquad (4)$$

In the above equations m is the vehicle's mass, I_z is the polar moment of inertia of the vehicle, I_i *(i = F,R)* are the moments of inertia of the front and rear wheels about the axis of rotation, r_i *(i = F,R)* is the radius of each wheel, x and y are the Cartesian coordinates of the center of mass in the inertial frame of reference, ψ is the yaw angle of the vehicle and ω_i *(i = F,R)* is the angular rate of the front and rear wheel respectively. By f_{ij} *(i = F,R and j = x,y)* we denote the longitudinal and lateral friction forces at the front and rear wheels respectively. The inputs are the driving/braking torques T_F and T_R at the front and rear wheels, and δ is the steering angle of the front wheel.

Assuming linear dependence of the friction forces on the normal load at each wheel, one obtains

$$f_{ij} = f_{iz}\mu_{ij}, \qquad i = F,R, \qquad j = x,y$$

where f_{iz} is the normal load at each of the front and rear axles, and μ_{ij} is the longitudinal and lateral friction coefficients of the front and rear tires. The friction coefficients μ_{ij} are calculated Pacejka's Magic Formula [9], normalized by the corresponding axle normal load.

Neglecting the suspension dynamics, the normal load transfer effect is incorporated in the vehicle model using a static map of the acceleration of the vehicle in the longitudinal direction:

$$f_{Fz} = \frac{mg(\ell_R - h\mu_{Rx})}{L + h(\mu_{Fx}\cos\delta - \mu_{Fy}\sin\delta - \mu_{Rx})}$$
$$f_{Rz} = mg - f_{Fz},$$

where L is the distance between front and rear axles and h is the vertical distance of the center of mass of the vehicle from the ground.

The following maps are used to calculate the control inputs, T_F, T_R and δ from the non-dimensional throttle/brake u_T and steering u_δ commands.

$$\delta = C_\delta u_\delta, \quad u_\delta \in [-1,+1]$$
$$T_i = \begin{cases} -\operatorname{sgn}(\omega_i)C_{ibrk}u_T, & u_T \in [0,+1]\ (braking) \\ -C_{iacc}u_T, & u_T \in [-1,0]\ (acceleration) \end{cases},$$

where $i = F,R$. The constants C_δ, C_{Facc}, C_{Racc}, C_{Fbrk} and C_{Rbrk} determine the vehicle's performance. In this work we assume a Front Wheel Drive (FWD) vehicle, hence $C_{Racc} = 0$.

OPTIMAL CONTROL FORMULATION

In the following we formulate the minimum-time cornering problem for the single-track model (1)-(4) along the 60, 90, 135 and 180deg corners of Figs. 3, 5, 7 and 9 respectively, on a low μ surface (μ = 0.5 for gravel).

All corners are of inner radius of 10m and outer radius 20m. The vehicle is required to remain within the road limits, which translates to a state constraint in the optimal control formulation. We define:

$$C_S(x,y)=\begin{cases}\sqrt{x^2+y^2} & for \quad y\geq 0 \\ 15 & otherwise\end{cases}.$$

The following state inequality constraint has to be satisfied at all times in order for the vehicle's C.G. to remain within the limits of the road:

$$10m \leq C_S(x,y) \leq 20m \tag{5}$$

The state constraint (5) is the same for all the different corners considered in this work.

The boundary conditions consist of fixed initial position, orientation and velocity of the vehicle, partially fixed final position and orientation and free final speed:

$$\begin{aligned} &x_0=18m, \quad y_0=-30m, \quad \dot{x}_0=0, \quad \dot{y}_0=60kph, \quad \dot{y}_f=0 \\ &\psi_0=\pi/2, \quad \dot{\psi}_0=0, \quad \psi_f=\psi_0+\alpha_c, \quad \dot{\psi}_f=0. \end{aligned} \tag{6}$$

with the corner angle α_c = 60, 90, 135, 180deg.

As in [5] we require that the vehicle returns to the straight line driving condition immediately after it reaches the geometric end of the corner. In the current formulation, however, we allow free positioning of the vehicle with respect to the width of the road. Specifically we use the following boundary conditions at the end point of the optimization:

$$\begin{aligned} &y_f/x_f=\tan\alpha_c \qquad \text{for} \qquad \alpha_c=60,\ 90^o \\ &y_f/x_f=\tan\alpha_c,\ \ x_f<0 \qquad \text{for} \qquad \alpha_c=135,\ 180^o \end{aligned} \tag{7}$$

The selection of the above boundary conditions is motivated by the challenges encountered during high-speed rally driving. Unlike closed circuit racing, rally racing involves an unpredictably changing environment and lack of detailed information about the condition of the road. Rally drivers bring their vehicles in a controllable straight line driving state in a short distance after the corner, a strategy that allows them to react to emergencies and unexpected changes in the environment after each corner.

Considering the dynamics of the system (1)-(4) we wish to find the optimal control inputs $u_T(t)$ and $u_\delta(t)$ that minimize the following cost function:

$$J=t_f,$$

subject to the state constraint (5) and the boundary conditions (6) and (7).

We use collocation to transcribe the above optimal control problem to a nonlinear programming problem by discretizing the continuous system dynamics (1)-(4). Consequently, the control inputs $u_T(t)$ and $u_\delta(t)$ are approximated with constant functions during each time interval. The numerical calculations are performed using EZOPT [10], by Analytical Mechanics Associates Inc, which provides a gateway to NPSOL, a well-known nonlinear optimization algorithm.

DRIVER INPUT PARAMETERIZATION

In the following we present an alternative optimization scheme to reproduce TB maneuvers, where we approximate the driver steering and throttle/brake inputs with piecewise linear profiles (Fig.2) defined by a small number of parameters t_{si}, c_{si}, t_{bi}, c_{bi}, [5]. Thus we achieve a considerable reduction of the optimization search space compared to the previous scheme where the control inputs are optimized at each time step. In addition, we replace the previous optimization algorithm (NPSOL) with the simplex method of [11] (Nelder-Mead), a direct search method that does not use numerical or analytic gradients.

The simplified optimization scheme is used to reproduce TB maneuvers along the 60, 90, 135 and 180deg corners of the previous formulation using the control input parameterization of Fig.2. The dynamics of the vehicle and the initial conditions (position, velocity and orientation of the vehicle) remain the same as in the previous formulation. The position of the vehicle at the end of the optimization is partially fixed. As in the previous formulation, while the final positioning of the vehicle with respect to the width of the road is free, the end point of the optimization satisfies (7). The speed of the vehicle at the end of the optimization is free.

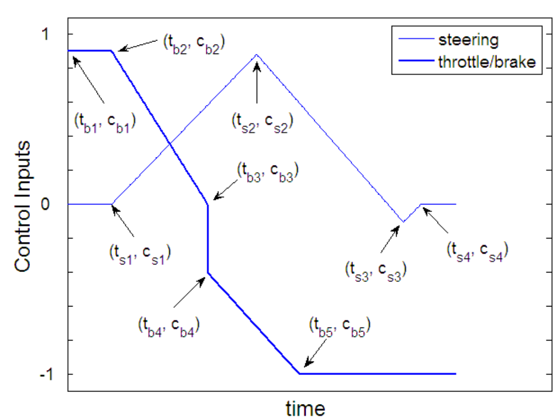

Figure 2: Parameterized steering and throttle/brake inputs for Trail-Braking.

Assuming that the trajectory is known at discrete instants t_0 = t_1 <…< t_N = t_f, we wish to find the optimal parameters t_{si}, c_{si}, t_{bi}, c_{bi}, that minimize the following cost function:

$$J=W_t t_f+W_r\sum_{k=1}^{N}e_r(t_k)+W_\psi e_\psi(t_f)+W_v e_v(t_f)+W_y e_y(t_f),$$

where t_f is the final time, e_r is the absolute value of the position error from the road limits, $e_\psi(t_f)$ is the final absolute orientation error, $e_v(t_f)$ is the final absolute lateral velocity of the vehicle and $e_y(t_f)$ is the final absolute yaw rate of the vehicle. The weights W_i are used for non-dimensionalization and to adjust the relative significance between the terms in the cost function. The optimization was performed in Matlab using an unconstrained nonlinear minimization algorithm (Nelder-Mead).

OPTIMIZATION RESULTS

In the following we compare the results produced by the two optimization schemes described above along the different corner geometries.

Figures 3, 5, 7 and 9 show the Trail-Braking trajectory generated using the input parameterization along the 60, 90, 135 and 180deg corners respectively. While the optimization ends at the geometric end of the corners, as shown in the Figs. 3, 5, 7, 9, the simulation continues with the car accelerating hard (u_T = -1) on a straight line (u_δ = 0) to demonstrate that the final boundary conditions have been satisfied. The optimal control inputs, vehicle speed and slip angle along the 60, 90, 135 and 180deg corners, from both optimization schemes, are shown in Figs 4, 6, 8 and 10 respectively.

In all of the optimization scenarios the control inputs (Figs 4, 6, 8, 10) are in agreement with the empirical guidelines provided by an expert rally driver [8] and the data collected during execution of the TB technique along a 90deg corner [6]. That is, hard braking is followed by progressive increase of the steering command towards the direction of the corner and simultaneous progressive release of the brakes. The driver counter-steers and applies throttle transferring load to the rear wheels to control the oversteer at the exit of the corner. We notice that aggressive slip angles are developing during the execution of the TB technique (Figs 4, 6, 8, 10), in agreement with the driving style of rally drivers. The slip angles increase noticeably with increasing corner angle. As expected, the vehicle maintains higher speed along the smaller corner angles. In addition, in all of the optimization scenarios the vehicle follows a "late-apex" line, that is, it finishes cornering at the geometric end of the corner and exits close to the inner limit of the road (Figs 3, 5, 7, 9).

The parameterization of the control inputs of Fig.2 was used in [5] to approximate the TB solution along a 90deg corner. We observe that the same input parameterization successfully reproduces TB along all the different corner geometries with the modified boundary conditions in relation to [5]. In addition, the solutions generated using the simplified optimization scheme with the parameterized inputs, are very close to the ones generated using the NPSOL algorithm, where the driver command are optimized at each time step (Table 1). We notice that in the case of the 180deg corner the vehicle exits the corner with a slight understeer, as the reversal of the slip angle sign at t = 6 sec reveals. We also notice that in the steering command generated by the NPSOL algorithm counter-steering is followed by a steering command towards the direction of the corner (5 ≤ t ≤ 7sec). The maneuver is successfully recreated using the input parameterization by assigning a non-zero value to the c_{s4} parameter.

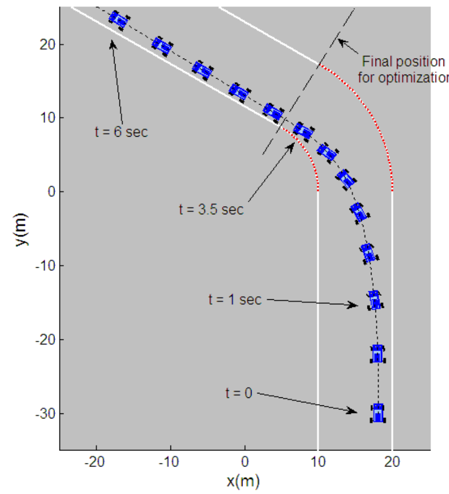

Figure 3: Trail-Braking trajectory along a 60deg corner.

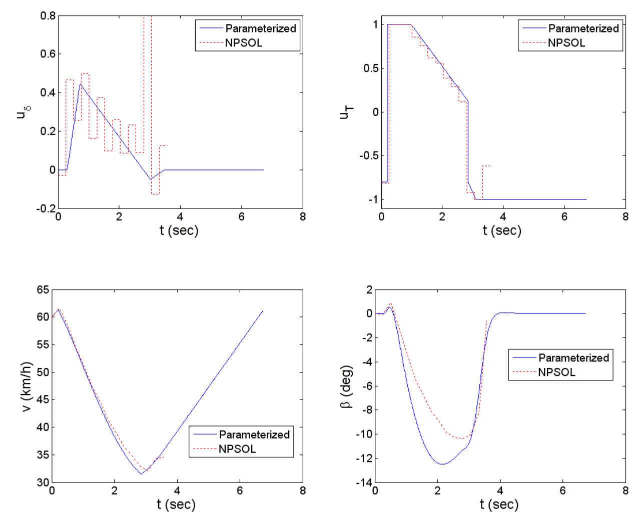

Figure 4: Driver inputs, vehicle speed and slip angle during a 60deg Trail-Braking.

OPTIMALITY OF THE "LATE-APEX" LINE

In this section we discuss the results of an additional optimization scenario in order to underline the optimality of the "late-apex" line followed during the execution of the TB technique.

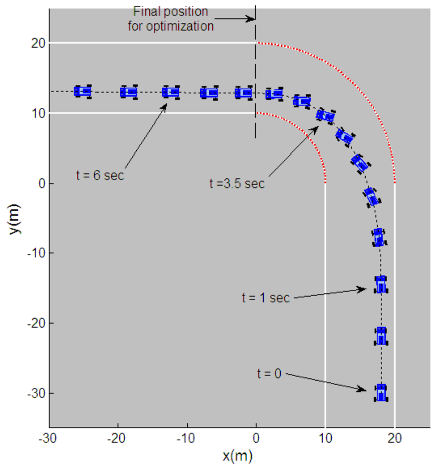

Figure 5: Trail-Braking trajectory along a 90deg corner.

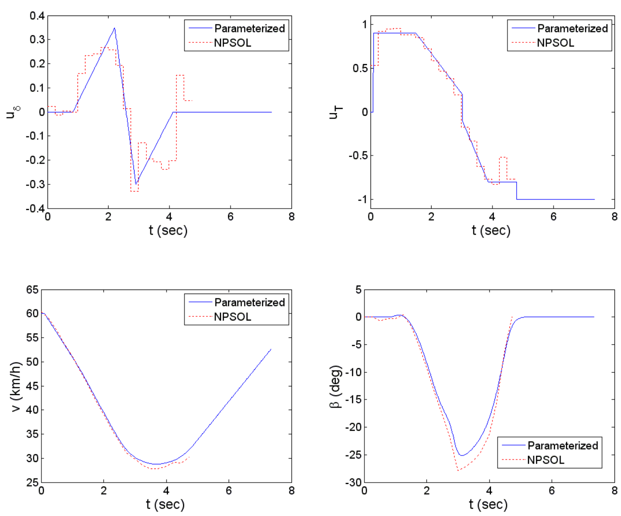

Figure 6: Driver inputs, vehicle speed and slip angle during a 90deg Trail-Braking.

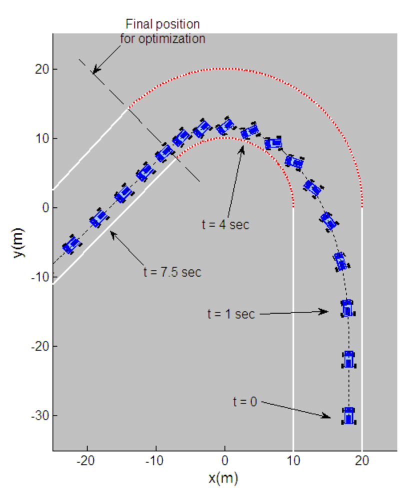

Figure 7: Trail-Braking trajectory along a 135deg corner.

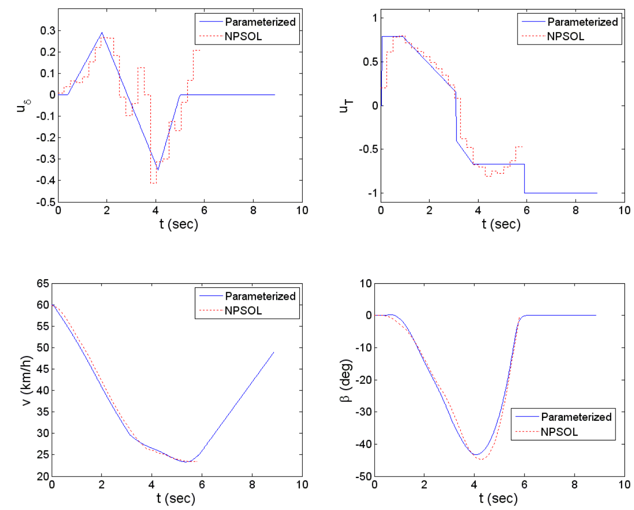

Figure 8: Driver inputs, vehicle speed and slip angle during a 135deg Trail-Braking.

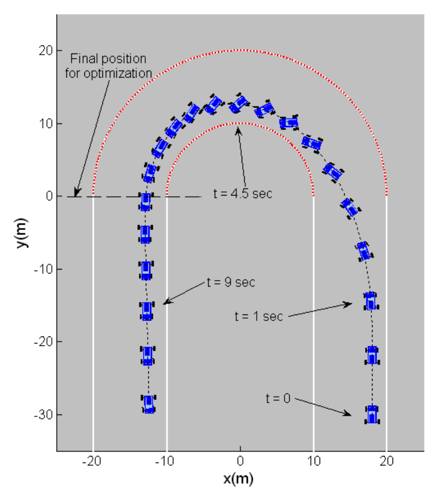

Figure 9: Trail-Braking trajectory along a 180deg corner.

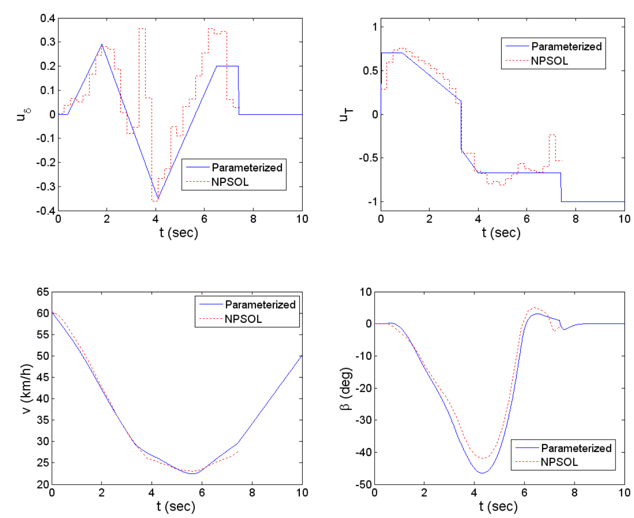

Figure 10: Driver inputs, vehicle speed and slip angle during a 180deg Trail-Braking.

Table 1: Optimization results.

Corner Angle	t_f (NPSOL)	t_f (parameterization)	% Δt_f
60deg	3.57sec	3.64sec	1.9
90deg	4.72sec	4.80sec	1.7
135deg	5.80sec	5.90sec	1.7
180deg	7.40sec	7.42sec	0.2

We consider the minimum time cornering problem along a 90deg corner with modified boundary conditions. In particular we replace the boundary condition (7) with x_f = -30m, allowing significant space for the vehicle to return to the stable, straight line driving condition after the corner. We will refer to the solution to the new optimization scenario as the baseline solution.

The baseline (x_f = -30m) trajectory is shown in Fig.11, while the optimal control inputs, velocity profiles and vehicle slip angles of both baseline and TB (90deg) cases are shown in Fig. 12. We notice that in the baseline trajectory the optimal path takes advantage of the whole width of the road, similar to the racing line followed by closed circuit race drivers. This is in contrast to the TB case, where the vehicle remains close to the inner edge of the road after the corner. The TB maneuver lacks in speed compared to the baseline solution. In fact the baseline velocity profile is piecewise greater than the TB profile as shown in Fig. 12, while the slip angles developing in the baseline solution are considerably smaller than the ones of the TB case. The TB maneuver, however, minimizes time when the vehicle is required to return to a straight driving condition in a short distance after the corner, in which case the optimal path is the “late-apex” line.

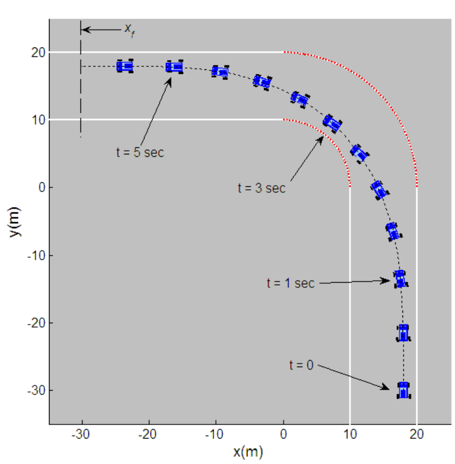

Figure 11: Baseline trajectory along a 90deg corner.

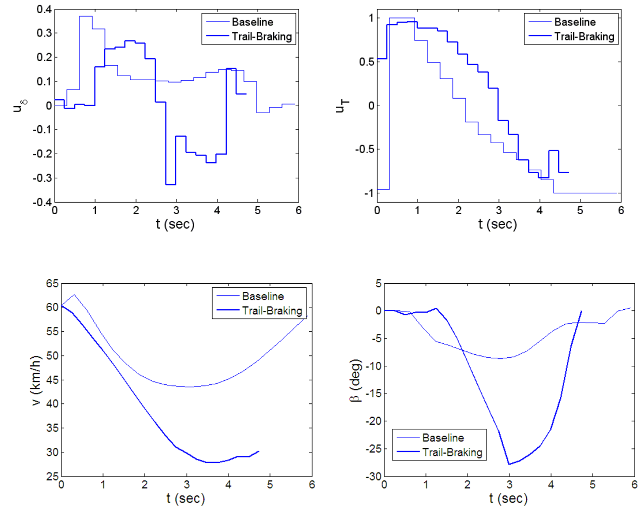

Figure 12: Baseline vs Trail-Braking optimal solutions: Steering, throttle/brake commands, vehicle speed and slip angle.

IMPLEMENTATION USING A HIGH-FIDELITY VEHICLE MODEL

The direct search method used in the input parameterization optimization scheme does not require numerical or analytic gradients. Hence, this optimization scheme does not require analytic expressions for the vehicle model, and external vehicle dynamics simulation software, such as CarSim [12], may be incorporated.

In [6] a parameterization of the steering, throttle and braking commands was proposed to generate a TB maneuver along a 90deg corner (Fig.13) using a high fidelity vehicle model. The CarSim vehicle model incorporates realistic powertrain and brake system models. To this end, we choose to decouple the braking and throttle commands as shown in Fig.13, rather than use the compound throttle/brake command u_T of the previous formulations. In the following we validate this parameterization of the control inputs by applying the optimization scheme of the previous section, in conjunction with CarSim, along 60, 90, 135 and 180deg corners. A light weight (1000kg), All-Wheel-Drive (50/50 torque distribution), 2.5L-115kw engine sedan was chosen for these calculations.

In Fig. 13 the optimal steering, braking and throttle commands are shown. Notice that certain parameters (p_{si}, p_{bi} and, p_{ai}) are common for all optimization cases. We have deliberately fixed the values of these parameters in order to further reduce the optimization search space. This simplified optimization scheme, however, is still successful in reproducing TB maneuvers along all corner geometries. The velocity profile and vehicle slip angle along each optimal trajectory are shown in Fig.14. In the same figure the front and rear normal loads are shown demonstrating the longitudinal load transfer effects that take place during acceleration and deceleration and play a key role in the TB maneuver. The braking command results in load transfer from the rear to the front axle, assisting in the initial rotation of the vehicle. Conversely, the throttle command results in load transfer from the front to the rear axle, aiming to control the yaw motion at the exit of the corner. Figures 15, 16, 17 and 18 show the optimal trajectory of the vehicle along the 60, 90, 135 and 180deg corners respectively.

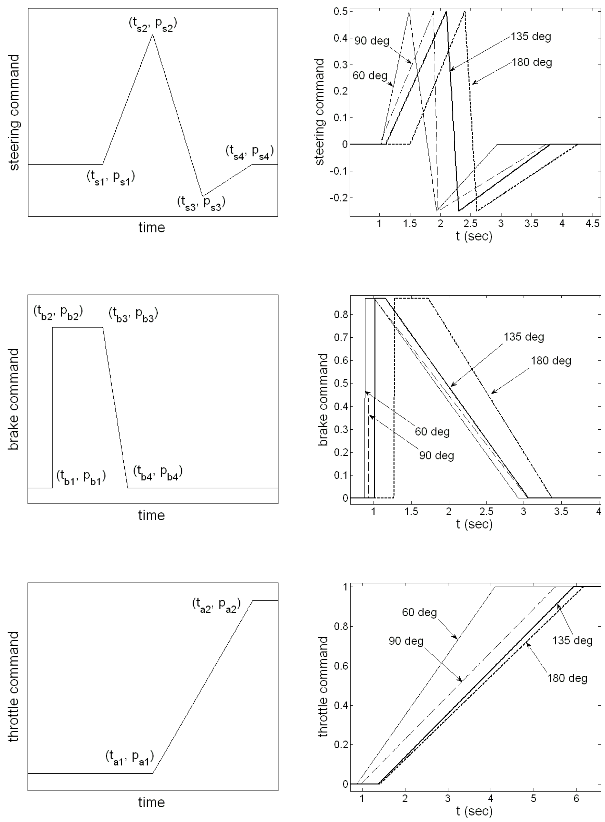

Figure 13: Parameterized steering, and decoupled brake/throttle inputs for the Trail-Braking maneuver; Optimized input profiles for different corner geometries.

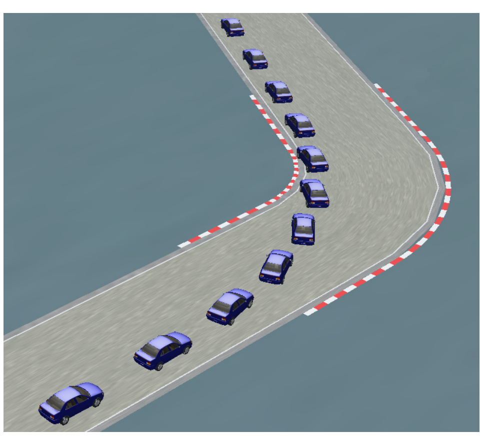

Figure 15: Trail-Braking through the 60deg corner.

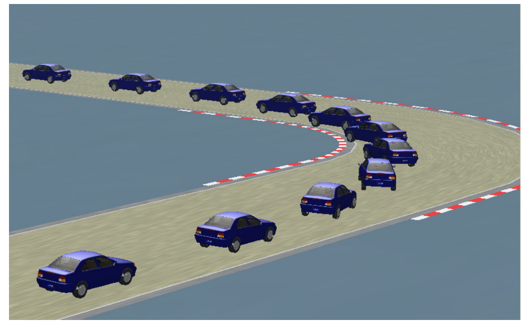

Figure 16: Trail-Braking through the 90deg corner.

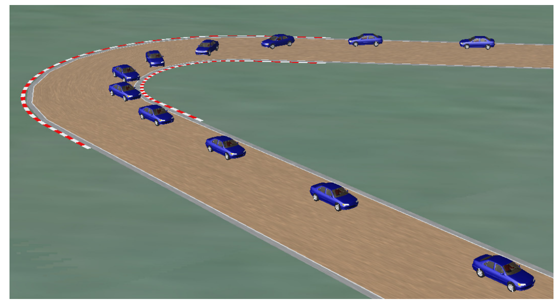

Figure 17: Trail-Braking through the 135deg corner.

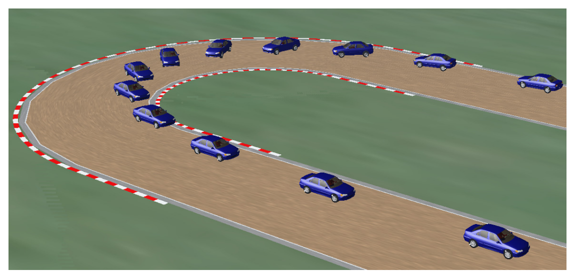

Figure 18: Trail-Braking through the 180deg corner.

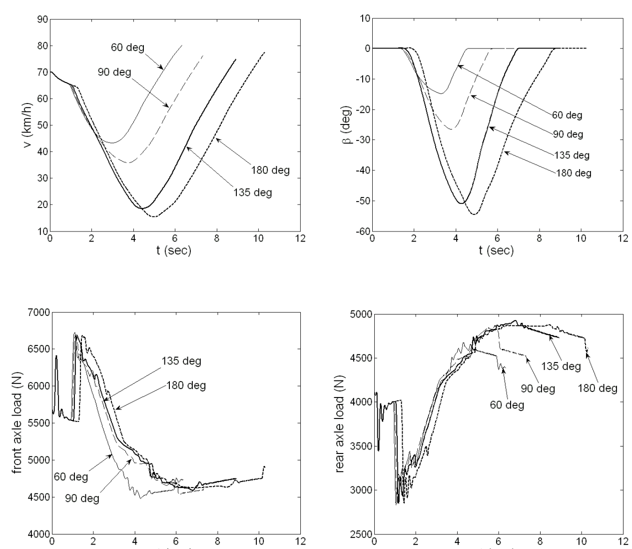

Figure 14: Optimal vehicle speed, vehicle slip angle, front and rear axle normal loads through the 60, 90, 135 and 180deg corners.

CONCLUSION

In this work we presented several numerical optimization schemes to reproduce a high-speed cornering maneuver used in rally racing. A previously introduced nonlinear programming (NLP) optimization approach was used in order to verify the optimality properties of Trail-Braking along a variety of corner geometries. In addition, the optimality of "late-apex" lines, typically followed by rally drivers, was demonstrated by appropriately modifying the boundary conditions of the optimization formulation. A simplified optimization scheme, based on a parameterization of the control inputs, was presented. In this work we demonstrated that the same input parameterization can be used to reproduce TB trajectories along different corner geometries, while the trajectories generated are very close to the optimal ones. The new optimization scheme was also efficiently implemented in various corner scenarios using a highly accurate vehicle model, providing further validation of the proposed input parameterization for Trail-Braking.

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REFERENCES

1. J. Hendrikx, T. Meijlink, and R. Kriens, "Application of optimal control theory to inverse simulation of car handling," *Vehicle System Dynamics*, vol. 26, pp. 449-461, 1996.
2. D. Casanova, R. S. Sharp, and P. Symonds, "Minimum time maneuvering: The significance of yaw inertia," *Vehicle System Dynamics*, vol. 34, pp. 77-115, 2000.
3. B. Siegler, and D. Crolla, "Racing Car Simulation and the Virtual Race Track," *2001 ASME International Mechanical Engineering Congress and Exposition*, New Tork, NY, pp. 231-238, 2001.
4. E. Velenis and P. Tsiotras, "Minimum time vs maximum exit velocity path optimization during cornering," *2005 IEEE International Symposium on Industrial Electronics*, Dubrovnic, Croatia, June 2005, pp. 355-360.
5. E. Velenis, P. Tsiotras, and J. Lu, "Modeling aggressive maneuvers on loose surfaces: The cases of trail-braking and pendulum-turn," *Proceedings of the 2007 European Control Conference*, Kos, Greece, July 2-5 2007.
6. E. Velenis, P. Tsiotras, and J. Lu, "Aggressive maneuvers on loose surfaces: Data Analysis and Input Parameterization," *15th Mediterranean Conference on Control and Automation*, Athens, Greece, June 27-29 2007.
7. T. O'Neil, private communication, 2006.
8. T. O'Neil, "Rally Driving Manual," Team O'Neil Rally School and Car Control Center, 2006.
9. E. Bakker, L. Nyborg, and H.B. Pacejka, "Tyre Modelling for Use in Vehicle Dynamics Studies," *SAE Paper No.870421*, 1987.
10. P.E. Gill, W. Murray, M.A. Saunders, and M.H. Wright, "User's Guide for NPSOL (version 4.0)," Dept. of Operations Research, Stanford University, CA, 1986.
11. J.C. Lagarias, J.A. Reeds, M.H. Wright and P.E. Wright, "Convergence Properties of the Nelder-Mead Simplex Methods in Low Dimensions," *SIAM Journal of Optimization*, vol. 9, n1, pp. 112-147, 1998.
12. www.carsim.com, Mechanical Simulation Corp., Ann Arbor, MI.