CONTROL OF SPACECRAFT SUBJECT TO ACTUATOR FAILURES: STATE-OF-THE-ART AND OPEN PROBLEMS

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In this paper we review the main results in the area of active control of spacecraft with one actuator failure. We emphasize the qualitative characteristics that make this a challenging control problem. We present a series of new results that solve the problem of detumbling with simultaneous attitude stabilization about the unactuated axis, the complete attitude stabilization problem, and the feasible trajectory generation problem for a spacecraft with one actuator failure. We present several numerical examples that demonstrate the efficacy of the proposed control algorithms. We conclude with a brief review of some open problems in the general area of spacecraft control subject to actuator and/or sensor failures.

INTRODUCTION

Recent advances in spacecraft and satellite control systems have succeeded in solving several challenging problems dealing with the attitude tracking, robust control of rigid and flexible spacecraft, optimal slew maneuvers, precision pointing, formation flying, etc. Techniques from nonlinear [1, 2, 3, 4, 5, 6], adaptive [7, 8, 9, 10, 11, 12], optimal [13, 14, 15, 16, 17, 18, 19, 20, 21] and robust control [9, 22, 23, 24, 25] have been used to this end with a lot of success. Most (if not all) of these results assume that the spacecraft is actively controlled with a sufficient number of actuators equal to, or larger than, the degrees of freedom of the system. Although this is certainly the case

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with most current spacecraft, these control laws – by and large – do not account for unexpected actuator and/or sensor failures. It appears that the issue of spacecraft control in case of actuator and sensor failures has not received its due attention in the literature.

The scope of this paper is to emphasize the challenges associated with the attitude stabilization problem in case of actuator failures. As it turns out, this problem – apart from its obvious practical importance – requires answering several interesting control-theoretic questions as well. We first present a brief survey of the main stabilization results for the case of spacecraft actuator failures. Subsequently, we introduce a series of new results including detumbling maneuvers, complete attitude stabilization, and feasible trajectory generation.

The first theoretical investigation of the equations of the rotational motion of a rigid body is probably due to Crouch [26], where he provided necessary and sufficient conditions for the controllability of a rigid body in the case of one, two and three independent control torques. This paper also showed that in the case of momentum exchange devices, controllability is impossible with fewer than three devices. This article sparked a renewed interest in the area of control of rigid spacecraft with less than three control torques. Additional results on the small-time controllability of the rigid body equations were given by Keraï [27], where it was shown that, with one control, the system is never small-time locally controllable. Stabilization of the angular velocity equations has been addressed, for example, in Refs. [28, 29, 30, 31, 32, 33]. In Ref. [28] it was shown that the angular velocity equations can be made asymptotically stable about the origin by means of two torques, each applied along a principal axis. In Ref. [29] the author addresses the problem of feedback stabilization of the zero solution of Euler's angular velocity equations using one torque aligned with a principal axis. It is shown that there exists a smooth stabilizing feedback control law if the moment of inertia of the rigid body along that principal axis is either the larger or smaller than the remaining two. Moreover, this control law is robust relative to changes in the parameters defining the control law. In Ref. [30] the authors provide a methodology to *asymptotically* stabilize Euler's equations with a single (in fact, linear) control law. It is also shown that a single control aligned with a principal axis cannot asymptotically stabilize the system. The control law construction in Ref. [30] requires that the body has no symmetry axes. Sontag and Sussman [31] extended the results of Ref. [30] by showing that the angular velocity equations can be smoothly stabilized with a single (nonlinear) torque for a rigid body with an axis of symmetry. This result was rederived in Ref. [32] as an application of the Jurdjevic-Quinn method [34]. Andriano [33] showed that the angular velocity equations of an axi-symmetric rigid body can be globally asymptotically stabilized by means of a liner feedback when two control torques act on the body. References [35] and [36] derive a control law that stabilizes a uniform rotation of a rigid body about its intermediate axis using a single torque about its major or minor axis.

The complete attitude equations were addressed in Ref. [37] and [38]. In particular, Byrnes and Isidori [38] proved that there is no smooth state variable feedback law that locally asymptotically stabilizes a rigid spacecraft about a desired reference attitude with two control torques. Stabilization about an equilibrium manifold is, however, possible [37, 38]. In Refs. [39, 40] the authors investigated the attitude stabilization of an axially symmetric spacecraft. The control torques were assumed to be supplied by two pairs of gas jet actuators about axes spanning the two-dimensional plane orthogonal to the axis of symmetry. According to the results of Refs. [39, 40], the complete dynamics of the spacecraft system fails to be controllable or even accessible in this case. It is, however, strongly accessible and small-time locally controllable in a restricted sense, namely when the unactuated axis angular velocity is zero (for the case of gas jets) or when the total angular momentum is zero (for the case of momentum/reaction wheels). Even in this case, the restricted spacecraft dynamics cannot be asymptotically stabilized using smooth feedback, due to failure of Brockett's necessary condition [28]. In Refs. [39, 40] the authors provide a nonsmooth feedback control strategy for the restricted case which achieves arbitrary reorientation of the spacecraft. The stabilization problem of the axi-symmetric spacecraft was revisited by Tsiotras et al. [41] and by Tsiotras and Luo [42, 43]. Bounded stabilizing and tracking control laws were also derived by the same authors in Ref. [44]. Thus, the stabilization problem for the axi-symmetric case can be considered solved, although there is still work to be done in addressing robustness questions (see, for example, Ref. [45] for a discussion on robustness for the angular velocity stabilization problem of an underactuated rigid body.) The stabilization problem for a non-symmetric spacecraft in case of one actuator failure turned out to be much more challenging, but finally several approaches were proposed with much success [46, 47, 48, 49]. The authors in Ref. [47] and [48] use time-varying control laws to circumvent the topological obstruction to smooth stabilizability due to Brockett's condition. Especially innovative is the control law proposed by Coron and Keraï [49], where they construct an almost continuous, periodic control law by switching between two different control laws.

In this paper we provide some new results for the problem of the attitude stabilization of a non-symmetric rigid body spacecraft using two pairs of gas jet actuators. We propose both time-invariant and time-varying discontinuous stabilizing control laws. We emphasize the problem of feasible trajectory generation for an underactuated spacecraft and we provide a solution to this problem using ideas from the theory of differentially flat systems. Numerical simulations accompany the theoretical developments. We also provide some discussion on open problems in this framework. It turns out that these control problems are significant not only because of their use in applications but also because of their challenging theoretical aspects.

STABILIZATION RESULTS

In this section we present some new results for the attitude stabilization problem in case of a single actuator failure. For simplicity, it will be assumed that the available control torques are provided by pairs of thrusters. We will also restrict the discussion at the kinematic level. Thus, we assume that the actuators can implement angular velocity commands directly. This is not a major restriction if the actuators have sufficiently high bandwidth and saturation limits. In this case these kinematic (i.e., angular velocity) control laws can be extended to torque control laws using known results from the theory of cascaded systems [50, 51, 52, 53].

Euler's Equations

Assuming a principal axes reference frame at the center of mass, Euler's equations of motion take the form

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3 + M_1 \tag{1a}$$

$$I_2\dot{\omega}_2 = (I_3 - I_1)\omega_3\omega_1 + M_2 \tag{1b}$$

$$I_3\dot{\omega}_3 = (I_1 - I_2)\omega_1\omega_2 + M_3 \tag{1c}$$

where ω_1, ω_2, ω_3 are the components of the angular velocity vector along the principal axes of this body-fixed reference frame, M_1, M_2, M_3 denote the external torques and I_1, I_2, I_3 denote the principal moments of inertia. Here we assume that there is no control along the 3-axis of the vehicle due to, say, an actuator failure. In contrast to the results of Refs. [41] and [39], we do not impose any symmetry assumptions along the 3-axis (i.e. $I_1 \neq I_2$). Also, it is not necessary to impose the (somewhat artificial) condition $\omega_3(0) = 0$, in order to ensure controllability [39].

Introducing the variables

$$a = \frac{I_2 - I_3}{I_1}, \qquad \varepsilon = \frac{I_1 - I_2}{I_3}$$

we obtain I_1 and I_2 in terms of a, ε and I_3

$$I_1 = \frac{\varepsilon + 1}{1 - a} I_3, \qquad I_2 = \frac{1 + a\varepsilon}{1 - a} I_3$$

Letting $M_3 = 0$, and $\tilde{u}_1 = M_1/I_1$, $\tilde{u}_2 = M_2/I_2$, and substituting into equations (1) yields

$$\dot{\omega}_1 = a\omega_2\omega_3 + \tilde{u}_1 \tag{2a}$$

$$\dot{\omega}_2 = -\frac{a + \varepsilon}{1 + a\varepsilon}\omega_2\omega_3 + \tilde{u}_2 \tag{2b}$$

$$\dot{\omega}_3 = \varepsilon\omega_1\omega_2 \tag{2c}$$

A trivial redefinition of the control inputs finally yields the dynamic equations

$$\dot{\omega}_1 = u_1 \tag{3a}$$

$$\dot{\omega}_2 = u_2 \tag{3b}$$

$$\dot{\omega}_3 = \varepsilon\,\omega_1\omega_2 \tag{3c}$$

The variable ε gives a measure of the body asymmetry about its unactuated axis. In case the spacecraft is nearly symmetric about the unactuated axis, $|\varepsilon| \ll 1$. The axi-symmetric case corresponds to $\varepsilon = 0$. In this case Eq. (3c) reduces to $\dot{\omega}_3 = 0$ and, without additional assumptions, the system is not controllable at the origin. The symmetric case has been addressed, for example, in Refs. [41, 43, 40]. In this work will assume that $\varepsilon \neq 0$.

Attitude Equations

We use the variables introduced in Refs. [54, 41] to describe the attitude of the spacecraft. According to the results of Ref. [41], the relative orientation between two reference frames can be represented by the following system of differential equations

$$\dot{w}_1 = \omega_3 w_2 + \frac{1}{2}(1 + w_1^2 - w_2^2)\,\omega_1 + w_1 w_2 \omega_2 \tag{4a}$$

$$\dot{w}_2 = -\omega_3 w_1 + \frac{1}{2}(1 - w_1^2 + w_2^2)\,\omega_2 + w_1 w_2 \omega_1 \tag{4b}$$

$$\dot{z} = \omega_3 + w_1 \omega_2 - w_2 \omega_1 \tag{4c}$$

The state variables w_1, w_2 and z are true kinematic coordinates for the attitude, in the sense that they provide a unique parameterization of the rotation matrix [54]

$$R(w,z) = \frac{1}{1 + w_1^2 + w_2^2} \begin{bmatrix} (1 + w_1^2 - w_2^2)\,\mathrm{c}z - 2w_1 w_2\,\mathrm{s}z & (1 + w_1^2 - w_2^2)\,\mathrm{s}z + 2w_1 w_2\,\mathrm{c}z & -2w_2 \\ 2w_1 w_2\,\mathrm{c}z - (1 - w_1^2 + w_2^2)\,\mathrm{s}z & 2w_1 w_2\,\mathrm{s}z + (1 - w_1^2 + w_2^2)\,\mathrm{c}z & 2w_1 \\ 2w_2\,\mathrm{c}z + 2w_1\,\mathrm{s}z & 2w_2\,\mathrm{s}z - 2w_1\,\mathrm{c}z & 1 - w_1^2 - w_2^2 \end{bmatrix}$$

where $w = [w_1\ w_2]^T$ and cz and sz denote $\cos z$ and $\sin z$, respectively. Conversely, for any rotation matrix R, the parameters w and z can be computed from [54]

$$w_1 = \frac{R_{23}}{1 + R_{33}}, \qquad w_2 = -\frac{R_{13}}{1 + R_{33}} \tag{5}$$

and

$$\cos z = \frac{1}{2}\left((1 + w_1^2 + w_2^2)\,\mathrm{trace}(R) + |w|^2 - 1\right) \tag{6a}$$

$$\sin z = \frac{1}{1 + w_1^2 + w_2^2}\left[(1 + w_1^2 - w_2^2)R_{12} + 2\,w_1 w_2 R_{22} + 4\,w_1 w_2 R_{32}\right] \tag{6b}$$

Time-Invariant Control Laws

From Eqs. (3) the control inputs u_1 and u_2 affect directly only the angular velocity components ω_1 and ω_2, which then can be used to control ω_3 from (3c) and w_1, w_2 and z from Eqs. (4). The angular velocities ω_1 and ω_2 play the role of "virtual" control inputs for the system (3c)-(4) that must be designed to achieve the control objectives. Once an acceptable control action in terms of the angular velocities $(\omega_{d1}, \omega_{d2})$ has been selected, standard results can be used to design the control inputs at the dynamic level. For example, the control law

$$u_1 = -\gamma(\omega_1 - \omega_{d1}) + \dot{\omega}_{d1} \tag{7a}$$
$$u_2 = -\gamma(\omega_2 - \omega_{d2}) + \dot{\omega}_{d2} \tag{7b}$$

with $\gamma > 0$, will force ω_i track ω_{di} $(i = 1,2)$ exponentially with rate of convergence γ. For large enough γ, one can ensure that the trajectories of the complete system in Eqs. (3)-(4) with the control law in Eq. (7) will be arbitrarily close to those of (3c)-(4) with $\omega_i = \omega_{di}$ $(i = 1,2)$.

The previous discussion allows one to basically reduce the original control problem to one of controlling the differential system in Eqs. (3c)-(4), with ω_1 and ω_2 as the control inputs. This system is re-written below for future reference as follows

$$\dot{\omega}_3 = \varepsilon\,\omega_1\omega_2 \tag{8a}$$
$$\dot{w}_1 = \omega_3 w_2 + \frac{1}{2}(1 + w_1^2 - w_2^2)\,\omega_1 + w_1 w_2 \omega_2 \tag{8b}$$
$$\dot{w}_2 = -\omega_3 w_1 + \frac{1}{2}(1 - w_1^2 + w_2^2)\,\omega_2 + w_1 w_2 \omega_1 \tag{8c}$$
$$\dot{z} = \omega_3 + w_1\omega_2 - w_2\omega_1 \tag{8d}$$

For a symmetric spacecraft ($\varepsilon = 0$) the previous system is controllable only under the additional assumption that $\omega_3(0) = 0$. In this case, $\omega_3(t) = 0$ for all $t \geq 0$ and the system (8) reduces to

$$\dot{w}_1 = \frac{1}{2}(1 + w_1^2 - w_2^2)\,\omega_1 + w_1 w_2 \omega_2 \tag{9a}$$
$$\dot{w}_2 = \frac{1}{2}(1 - w_1^2 + w_2^2)\,\omega_2 + w_1 w_2 \omega_1 \tag{9b}$$
$$\dot{z} = w_1\omega_2 - w_2\omega_1 \tag{9c}$$

In Ref. [41] it was shown that the following control law

$$\omega_1 = -\kappa w_1 + \mu \frac{z}{w_1^2 + w_2^2} w_2 \tag{10a}$$
$$\omega_2 = -\kappa w_2 - \mu \frac{z}{w_1^2 + w_2^2} w_1 \tag{10b}$$

where $\mu > \kappa/2 > 0$ globally asymptotically stabilizes (9). To avoid the singularity when $w = 0$, a saturated version of the previous control law can be used [55]

$$\omega_1 = -\kappa \frac{w_1}{\sqrt{1 + |w|^2}} + \mu \,\mathrm{sat}\left(\frac{z}{|w|}\right) \frac{w_2}{|w|} \tag{11a}$$
$$\omega_2 = -\kappa \frac{w_2}{\sqrt{1 + |w|^2}} - \mu \,\mathrm{sat}\left(\frac{z}{|w|}\right) \frac{w_1}{|w|} \tag{11b}$$

where $|w|^2 = w_1^2 + w_2^2$ and where $\mu > \kappa/2 > 0$ if $|z|/|w| \leq 1$ and $\mu > -\kappa > 0$ if $|z|/|w| > 1$.

In order to extend the control laws (10) or (11) to the non-symmetric spacecraft case, one needs to consider the time history of ω_3 from (8a). Notice that by imposing $w \to 0$ and $z \to 0$ as $t \to \infty$ it follows from Eq. (4c) that necessarily $\omega_3 \to 0$, assuming that the control inputs ω_1 and ω_2 remain bounded. The following two control laws are thus proposed

$$\omega_1 = -\kappa w_1 + \mu \frac{z - \lambda\omega_3}{w_1^2 + w_2^2} w_2 \tag{12a}$$

$$\omega_2 = -\kappa w_2 - \mu \frac{z - \lambda\omega_3}{w_1^2 + w_2^2} w_1 \tag{12b}$$

and

$$\omega_1 = -\kappa \frac{w_1}{\sqrt{1 + |w|^2}} + \mu \,\mathrm{sat}\left(\frac{z - \lambda\omega_3}{w_1^2 + w_2^2}\right) \frac{w_2}{|w|} \tag{13a}$$

$$\omega_2 = -\kappa \frac{w_2}{\sqrt{1 + |w|^2}} - \mu \,\mathrm{sat}\left(\frac{z - \lambda\omega_3}{w_1^2 + w_2^2}\right) \frac{w_1}{|w|} \tag{13b}$$

with $\lambda > 0$ and $\mu > 0$, and $\kappa > 0$. The control laws in (10) and (11) represent special cases of Eqs. (12) and (13), respectively, with $\lambda = 0$.

It is conjectured that for $\omega_3(0) \approx 0$ and for all initial conditions $(w(0), z(0)) \in B_\varepsilon$, a compact set* of $\mathbb{R}^2 \times S^1$, there exist constant positive gains κ, μ and λ such that the system in (8) will be asymptotically stable, i.e., $w(t) \to 0$, $z(t) \to 0$ and $\omega_3(t) \to 0$ as $t \to \infty$. The rationale behind this conjecture is that if ω_3 is initially small, and since the asymmetry parameter ε is small, ω_3 will remain small for all times. That is, Eqs. (9) will be a good approximation to Eqs. (4). In addition, since the control law (10) forces $(w, z) \to 0$ then – assuming that the control inputs ω_1 and ω_2 remain bounded – necessarily $\omega_3 \to 0$ as well. The extra term $-\lambda\omega_3$ in (12) then is used to add robustness for stabilizing (4c). Notice that with (12) the equation for z is given by

$$\dot{z} = -\mu z + (1 - \lambda)\omega_3$$

Figure 1 shows a numerical simulation using the control law (12). The following initial conditions and control gains were used: $w_1(0) = -0.1$, $w_2(0) = -0.2$, $z = 0.7$ (rad), $\kappa = 0.05$, $\mu = 3$, $\lambda = 12$, $\omega_3(0) = 0$ (rad/sec). The asymmetry parameter was chosen $\varepsilon = 0.2$. The corresponding angular velocity (control input) history is shown in Fig. 2.

Although in these simulations we used $\omega_3(0) = 0$, our experience has shown that the control law (12) works even when $\omega_3(0) \neq 0$ as long as it is small.

If ω_3 is initially large, a detumbling maneuver may be necessary to make the value of ω_3 small enough. The control laws of Refs [28] or [33] can be used for this purpose. An alternative detumbling maneuver that also achieves partial attitude stabilization is presented next.

*The set B_ε will, in general, depend on the asymmetry parameter ε.

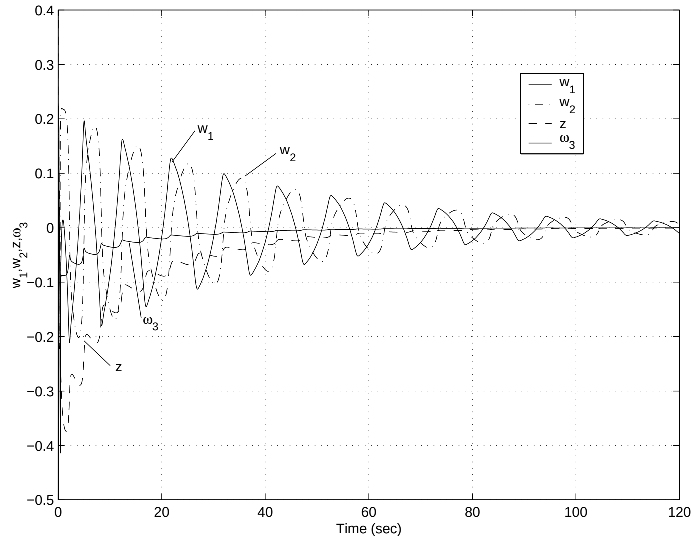
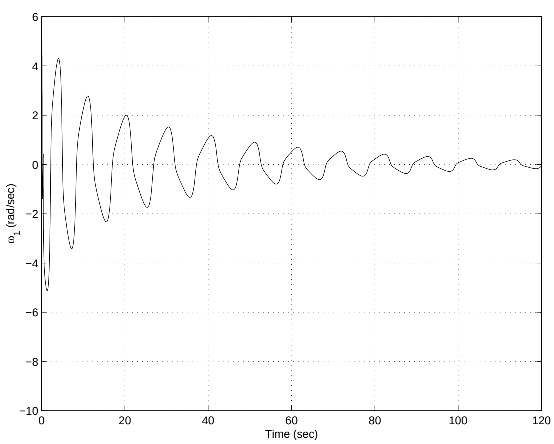
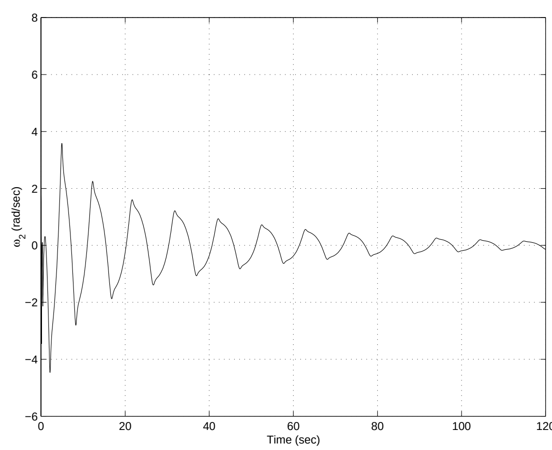


Figure 1: Time history of w, z and ω_3 $(\mu = 3, \lambda = 12, \kappa = 0.05, \varepsilon = 0.2)$.



(a) Time history of ω_1.



(b) Time history of ω_2.

Figure 2: Time history of angular velocities $(\mu = 3, \lambda = 12, \kappa = 0.05, \varepsilon = 0.2)$.

Detumbling with Partial Attitude Stabilization

In this section we present a control law that will achieve $\omega_3 = 0$, while at the same time drive w to zero. The last objective, although not necessary for applying the control laws (12) and (13), is beneficial for cases of partial attitude stabilization, namely, stabilization of the spacecraft about the unactuated axis. Before presenting the detumbling control law we need to introduce some mathematical preliminaries.

Let $\lambda > 0$ and any set of positive scalars $r_i > 0$, $i = 1,\dots,n$. Then the *dilation operator* δ_λ with weights r_i is defined by $\delta_\lambda(x_1,\dots,x_n) = (\lambda^{r_1}x_1,\dots,\lambda^{r_n}x_n)$. A function $h : \mathbb{R}^n \to \mathbb{R}$ is said to be *positively homogeneous* of degree k with respect to a given dilation δ_λ if $h(\delta_\lambda(x_1,\dots,x_n)) = \lambda^k h(x_1,\dots,x_n)$. A vector field $f : \mathbb{R}^n \to \mathbb{R}^n$ is said to be *homogeneous of degree k* with respect to a given dilation δ_λ if its i^{th} coordinate is a homogeneous function of degree $r_i + k$, i.e.,

$$f_i(\delta_\lambda(x_1,\dots,x_n)) = \lambda^{r_i+k} f_i(x_1,\dots,x_n)$$

where f_i denotes the i^{th} component of the vector field f. Having introduced the concept of homogeneity, it is now possible to state some important properties of homogeneous functions and vector fields. The following Theorem is taken from Ref. [56]; see also [57].

Theorem 1 *Let f be a homogeneous vector field of degree k with respect to a given dilation δ_λ and let g be a sum of continuous homogeneous vector fields of degree greater than k. Then if the trivial solution $x = 0$ of $\dot{x} = f(x)$ is locally asymptotically stable, the same is true for the trivial solution of the perturbed system $\dot{x} = f(x) + g(x)$*

Homogeneous systems, defined by homogeneous vector fields have certain appealing properties. The following fact, taken from Ref. [58], along with Theorem 1, justify our interest in homogeneous systems.

Theorem 2 *Let f be a homogeneous vector field of degree zero. Then local asymptotic stability of the origin of $\dot{x} = f(x)$ is equivalent to global exponential stability with respect to the homogeneous norm ρ, defined by $\rho(x) = |x_1^{c/r_1} + x_2^{c/r_2} + \cdots + x_n^{c/r_n}|^{1/c}$ where c is a positive integer evenly divisible by r_i.*

To apply the previous results to the underactuated spacecraft problem, re-write system (8a)-(8c) as

$$\begin{aligned} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{\omega}_3 \end{bmatrix} &= \begin{bmatrix} \frac{1}{2}\omega_1 \\ \frac{1}{2}\omega_2 \\ \varepsilon\omega_1\omega_2 \end{bmatrix} + \begin{bmatrix} w_1w_2\omega_2 + w_2\omega_3 + \frac{1}{2}\omega_1\left(w_1^2 - w_2^2\right) \\ w_1w_2\omega_1 - w_1\omega_3 + \frac{1}{2}\omega_2\left(w_2^2 - w_1^2\right) \\ 0 \end{bmatrix} \\ &= f(\omega_1,\omega_2) + g(\omega_3, w_1, w_2, \omega_1, \omega_2) \end{aligned} \tag{14}$$

and introduce the following dilation operator

$$\delta_\lambda(\omega_3, w_1, w_2, \omega_1, \omega_2) = \left(\lambda^2\omega_3, \lambda w_1, \lambda w_2, \lambda\omega_1, \lambda\omega_2\right) \tag{15}$$

It can be easily seen that f is homogeneous of degree zero with respect to the previous dilation operator, since

$$f_1(\delta_\lambda(\cdot)) = \lambda\left(\tfrac{1}{2}\omega_1\right) = \lambda f_1(\cdot), \qquad f_2(\delta_\lambda(\cdot)) = \lambda\left(\tfrac{1}{2}\omega_2\right) = \lambda f_2(\cdot)$$
$$f_3(\delta_\lambda(\cdot)) = \lambda^2(\varepsilon\omega_1\omega_2) = \lambda^2 f_3(\cdot)$$

where f_i is the i^{th} component of the vector field f. To make the appropriate use of Theorem 1, the homogeneity properties of the vector-valued function g need to be investigated as well. To this end, notice that

$$\lim_{\lambda\to 0}\frac{g_1(\delta_\lambda(\cdot))}{\lambda} = \lim_{\lambda\to 0}\frac{\lambda^3\left(w_1w_2\omega_2 + \omega_2\omega_3 + \frac{1}{2}\omega_1\left(w_1^2 - w_2^2\right)\right)}{\lambda} = 0$$

$$\lim_{\lambda\to 0}\frac{g_2(\delta_\lambda(\cdot))}{\lambda} = \lim_{\lambda\to 0}\frac{\lambda^3\left(w_1w_2\omega_1 - \omega_1\omega_3 + \frac{1}{2}\left(w_2^2 - w_1^2\right)\right)}{\lambda} = 0$$

$$\lim_{\lambda\to 0}\frac{g_3(\delta_\lambda(\cdot))}{\lambda^2} = \lim_{\lambda\to 0}\frac{0}{\lambda^2} = 0$$

where g_i is the i^{th} component of the vector field g. By Theorem 1 it is therefore sufficient to find a homogeneous control law of degree at least one, such that the trivial solution for the system

$$\dot{\omega}_3 = \varepsilon\,\omega_1\omega_2, \qquad \dot{w}_1 = \frac{1}{2}\omega_1, \qquad \dot{w}_2 = \frac{1}{2}\omega_2 \tag{16}$$

is asymptotically stable. Such a control law is presented next.

Without loss of generality assume that $\varepsilon > 0$. Then the transformation

$$\omega_1^n = \sqrt{\varepsilon}\,\omega_1, \quad \omega_2^n = \sqrt{\varepsilon}\,\omega_2, \quad w_1^n = \sqrt{4\varepsilon}\,w_1, \quad w_2^n = \sqrt{4\varepsilon}\,w_2, \quad \omega_3^n = \omega_3 \tag{17}$$

results in the system

$$\dot{\omega}_3^n = \omega_1^n\omega_2^n, \qquad \dot{w}_1^n = \omega_1^n, \qquad \dot{w}_2^n = \omega_2^n \tag{18}$$

Dropping the superscript n for notational simplicity, let the control for the system (18) defined by

$$\omega_1 = -w_1 + s\,w_2\,\psi, \qquad \omega_2 = -w_2 + s\,w_1\,\psi \tag{19}$$

where $s = \omega_3 + \frac{1}{2}w_1w_2$ and

$$\psi = \frac{\mu}{\sqrt{w_1^4 + w_2^4 + (\mu s)^2}}, \qquad \mu > 4 \tag{20}$$

Then this control law will locally asymptotically stabilize the system given by (18) for all initial conditions such that

$$\left|\frac{s(0)}{w_1(0)^2+w_2(0)^2}\right| \leq \frac{\sqrt{\mu^2-16}}{4\mu} \tag{21}$$

This, in turn, will guarantee asymptotic stability for the original system (14)[†]. The proof of this result can be found in Tsiotras and Schleicher [59].

In addition, notice that since $s(\delta_\lambda(\cdot)) = \lambda^2 s(\cdot)$ and

$$\begin{aligned}
\omega_1(\delta_\lambda(\cdot)) &= -\lambda w_1 + \frac{1}{2}\frac{\lambda^3}{\lambda^2}\frac{\mu w_2 s}{\sqrt{w_1^4+w_2^4+(\mu s)^2}} = \lambda\omega_1(\cdot) \\
\omega_2(\delta_\lambda(\cdot)) &= -\lambda w_2 + \frac{1}{2}\frac{\lambda^3}{\lambda^2}\frac{\mu w_1 s}{\sqrt{w_1^4+w_2^4+(\mu s)^2}} = \lambda\omega_2(\cdot)
\end{aligned}$$

the control inputs ω_1 and ω_2 are homogeneous of degree one with respect to the given dilation. This implies that the closed-loop system (18)-(19) is homogeneous of degree zero. According to Theorem 2 the closed-loop system is globally exponentially stable with respect to the homogeneous norm associated with the dilation operator (15).

Figure 3 shows numerical simulations of the system (14) with the control law in (19)-(20). The following initial conditions were used in these simulations: $w_1(0) = 5$, $w_2(0) = 1$ (rad), $\omega_3(0) = -0.5$ (rad/sec), $\mu = 7$ and $\varepsilon = 0.2$.

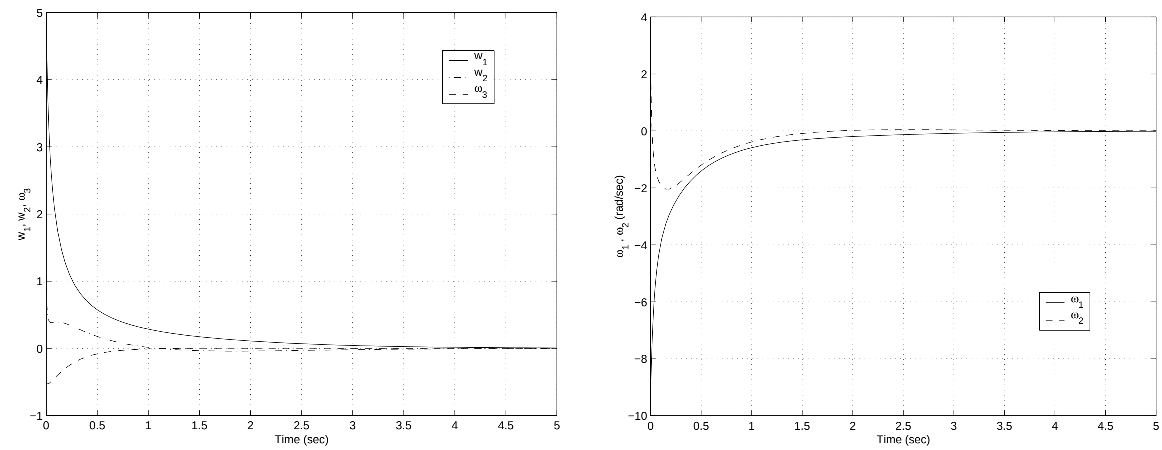
A Time-Varying Control Law

The control laws in Eqs. (12) and (13) are time-invariant. Any time-invariant control law that stabilizes (8) cannot be continuous at the origin due to the violation of Brockett's necessary condition for stabilization [28]. A time-varying control law can be used instead, to avoid this condition. Such time-varying control laws have been proposed, for example, by Coron and Keraï [49] and Morin and Samson [60]. The control law of Coron and Keraï is almost continuous. The control by Morin and Samson is continuous, but not smooth at the origin. Smooth, periodic control laws have been proposed in Ref. [48]. It is worthwhile mentioning that smooth control laws cannot achieve exponential convergence rates [60, 61]. This justifies our interest in non-smooth or even discontinuous control laws.

In this section we propose a time-varying control law that locally asymptotically stabilizes the complete attitude equations (8). To this end, consider the new control inputs

$$\omega_1 = v_1 + |v|^{\frac{1}{2}}\cos(t/e), \qquad \omega_2 = v_2 + \mathrm{sgn}(v)|v|^{\frac{1}{2}}\cos(t/e) \tag{22}$$

[†]Here one cannot apply Theorem 1 directly since the initial conditions satisfying (21) do not form an open neighborhood of the origin. A more careful argument is needed.



(a) Time history of w_1, w_2 and ω_3.

(b) Time history of ω_1 and ω_2.

Figure 3: Time history of states and angular velocities ($\mu = 7, \varepsilon = 0.2$).

where v_1, v_2 and v are auxiliary controls and $0 < e \ll 1$. Substituting (22) in Eq. (8), letting $\tau = t/e$ and considering the averaged system, one obtains[‡]

$$\bar{\omega}_3' = \varepsilon\,(v_1 v_2 + \frac{v}{2}) \tag{23a}$$

$$\bar{w}_1' = \bar{\omega}_3 \bar{w}_2 + \frac{1}{2}(1 + \bar{w}_1^2 - \bar{w}_2^2)\, v_1 + \bar{w}_1 \bar{w}_2 v_2 \tag{23b}$$

$$\bar{w}_2' = -\bar{\omega}_3 \bar{w}_1 + \frac{1}{2}(1 - \bar{w}_1^2 + \bar{w}_2^2)\, v_2 + \bar{w}_1 \bar{w}_2 v_1 \tag{23c}$$

$$\bar{z}' = \bar{\omega}_3 + \bar{w}_1 v_2 - \bar{w}_2 v_1 \tag{23d}$$

where $\bar{\omega}_3, \bar{w}_1, \bar{w}_2, \bar{z}$ are the states of the averaged system with v_1, v_2 and v the new inputs. Notice that the averaged system has an extra control input, v. The idea for a stabilizing control law for the averaged system is then simple: use the extra control input to drive $\bar{\omega}_3$ to zero in finite time, say t_c. For all $t \geq t_c$ the system (23) reduces to (9). The control law (10) or (11) can then be used to stabilize the complete system. Results from averaging theory[§] then ensure that the original system is also asymptotically stable about an arbitrarily small neighborhood of the origin.

Following these observations, a control law that stabilizes the averaged system (23) is given by

$$v = -2\, v_1 v_2 - 2\,\alpha\, \mathrm{sgn}(\varepsilon\,\bar{\omega}_3)|\bar{\omega}_3|^{\frac{1}{2}} \tag{24a}$$

$$v_1 = -\kappa\, \bar{w}_1 + \mu\, \frac{\bar{z}}{\bar{w}_1^2 + \bar{w}_2^2}\, \bar{w}_2 \tag{24b}$$

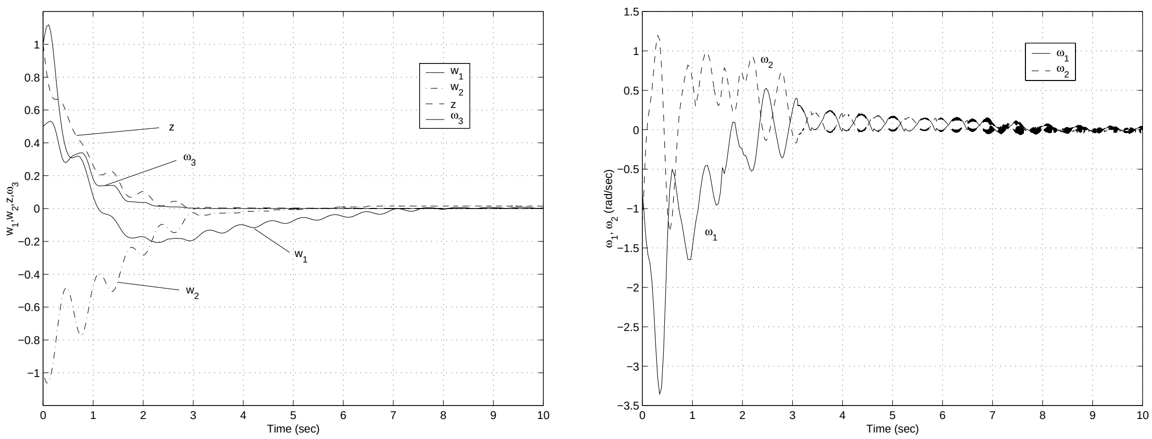
$$v_2 = -\kappa\, \bar{w}_2 - \mu\, \frac{\bar{z}}{\bar{w}_1^2 + \bar{w}_2^2}\, \bar{w}_1 \tag{24c}$$

[‡] The reader is referred to Refs. [62, 63] for a more in-depth discussion on averaging.

[§] A straightforward extension of the standard results of averaging theory may be necessary since the closed-loop system with the control laws (10) or (11) is not continuous at the origin.

where $\mu > \kappa/2 > 0$ and $\alpha > 0$. The corresponding stabilizing control for (8) in terms of ω_1 and ω_2 is then given by (22).

Figures 4 show numerical simulations of the closed-loop system (8) with the control law (22) and (24). The initial conditions and the control gains we chosen as $w_1(0) = 1,\ w_2(0) = -1,\ z(0) = 1$ (rad), $\omega_3(0) = 0.5$ (rad/sec), $\kappa = 1,\ \mu = 2,\ \alpha = 5,\ e = 0.1,\ \varepsilon = 0.2$.



(a) Time history of w_1, w_2, z and ω_3.

(b) Time history of ω_1 and ω_2.

Figure 4: Time history of states and angular velocities $(\mu = 2, \kappa = 1, \varepsilon = 0.2)$.

An alternative control law that is continuous at the origin can also be implemented instead of (24) using the results of Ref. [64]. This control law[¶] is given by

$$v_1 = -\kappa \bar{w}_1 + \mu \bar{z} \cos\theta \tag{25a}$$

$$v_2 = -\kappa \bar{w}_2 - \mu \bar{z} \sin\theta \tag{25b}$$

$$\dot{\theta} = \bar{z} \tag{25c}$$

where $\mu > 0, \kappa > 0$. Figures 5-6 show numerical simulations with this control law.

FEASIBLE TRAJECTORY GENERATION

In contrast to the stabilization problem of an underactuated spacecraft, which has been extensively treated in the literature, the attitude tracking problem has not been dealt with in great detail. An exception is Ref. [44] where the attitude tracking problem for an axially symmetric spacecraft unactuated about its symmetry axis was solved. Closely associated to tracking is the feasible (reference)

[¶]Notice that this control law is dynamic.

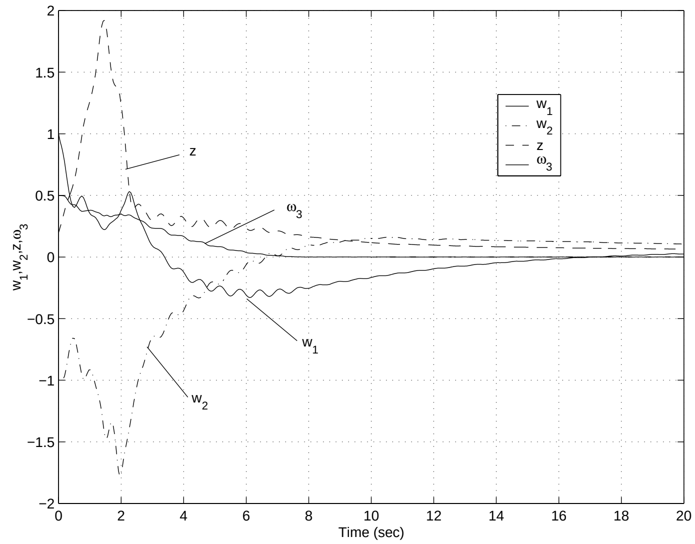
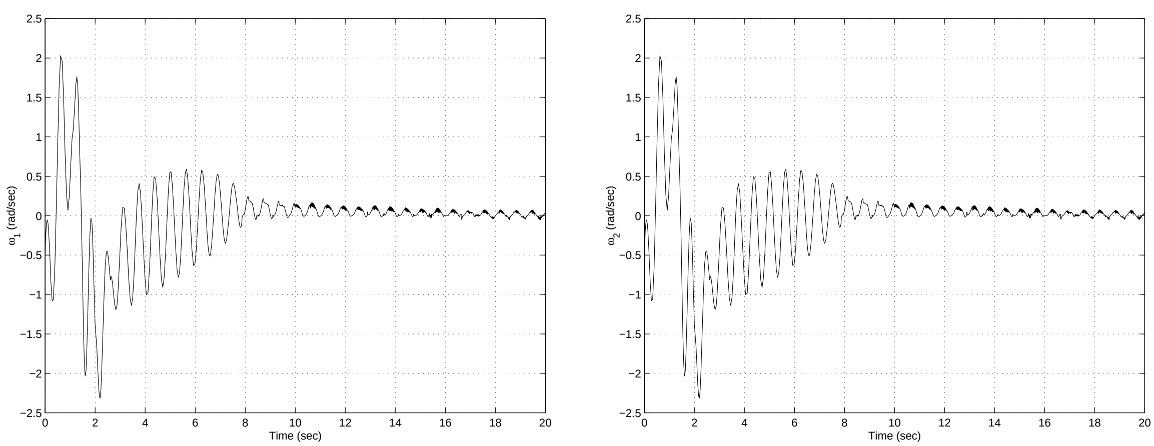


Figure 5: Time history of w_1, w_2, z and ω_3 ($\mu = 2, \kappa = 1, \varepsilon = 0.2, e = 0.1$).



(a) Time history of ω_1.

(b) Time history of ω_2.

Figure 6: Time history of angular velocities ($\mu = 2, \kappa = 1, \varepsilon = 0.2, e = 0.1$).

trajectory generation problem. For the axi-symmetric spacecraft the feasible generation problem was completely characterized and solved in Ref. [65] using ideas from differentially flat systems.

In this section we address the feasible generation problem for a general (not necessarily axi-symmetric) spacecraft with one actuator failure along the 3-axis. That is, given the dynamical system in Eq. (8) and a time history of the state variables $(\omega_3(t), w_1(t), w_2(t), z(t))$ defined over some interval $0 \leq t \leq t_f$, it is desired to find control input histories $\omega_1(t), \omega_2(t)$ such that the differential equations (8) are satisfied for all $0 \leq t \leq t_f$. We call such state trajectories *feasible* since there exists a control input that generates them. Feasible trajectories are important in tracking problems since they offer reference trajectories that (subject to the correct initial conditions) can be tracked *exactly* by a suitable control law.

To appreciate the difficulties associated with the feasible generation problem for the system at hand, notice that since (8) has less control inputs than states, not every trajectory is feasible. To see this, assume we are given some time histories $(\omega_3(t), w_1(t), w_2(t), z(t))$. Rewrite (8b)-(8d) as follows

$$\begin{aligned}
\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{z} \end{bmatrix} &= \begin{bmatrix} \frac{1}{2}(1 + w_1^2 - w_2^2) & w_1 w_2 \\ w_1 w_2 & \frac{1}{2}(1 - w_1^2 + w_2^2) \\ -w_2 & w_1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} w_2 \\ -w_1 \\ 1 \end{bmatrix} \omega_3 \\
&= F_1(w) \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + F_2(w)\, \omega_3
\end{aligned} \tag{26}$$

or

$$\left(\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{z} \end{bmatrix} - F_2(w)\, \omega_3 \right) = F_1(w) \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \tag{27}$$

For every $t \in [0, t_f]$ the previous equation is satisfied for some $\omega_1(t)$ and $\omega_2(t)$ if and only if the vector in the left hand side of (27) is in the range of the matrix $F_1(w)$. Clearly, for arbitrary functions $(\omega_3(t), w_1(t), w_2(t), z(t))$ this does not hold, in general. Even when one can solve (27) for every $t \in [0, t_f]$, from

$$\begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = F_1^{\dagger}(w) \left(\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{z} \end{bmatrix} - F_2(w)\, \omega_3 \right) \tag{28}$$

with $F_1^{\dagger}(w)$ the pseudo-inverse of $F_1(w)$, this solution must satisfy the additional dynamic constraint $\dot{\omega}_3 = \varepsilon\, \omega_1 \omega_2$ for all $t \in [0, t_f]$, an unlikely possibility. Not every trajectory in the (ω_3, w_1, w_2, z) space is thus feasible.

Recall that perfect tracking can be achieved only for feasible trajectories. In the literature the difficulty in generating feasible trajectories for systems that are not (stably) left-invertible is typically being avoided by using a copy of the plant[‖] for generating the reference trajectories. This does not solve the problem of feasible trajectory generation, but rather relegates it to the choice of the command input for the exosystem. This approach has been used in Ref. [44] where the attitude tracking problem (in terms of w and z) for the an axi-symmetric spacecraft with two control inputs has been addressed. In Ref. [44] the exosystem was called the "virtual" spacecraft. The advantage of this approach was that one could guarantee *a priori* that the trajectories of this exosystem are feasible and perfect tracking can be achieved. As mentioned previously, such an assumption is not realistic. In practice, a target attitude is typically provided and the control logic must use this information to generate a trajectory to be followed by the actual spacecraft. If an arbitrary target reference history is given, we wish to find (and subsequently track) a feasible trajectory that is as close to the given trajectory as possible.

Before presenting a feasible trajectory generation algorithm for the underactuated rigid spacecraft problem, we briefly review the main facts and definitions from differentially flat systems.

Differential Flatness and Flat Outputs

To solve the feasible trajectory generation problem, we will use the notion of differential flatness [66, 67]. Let the system

$$\dot{x} = f(x,u) \tag{29}$$

where $x \in \mathbb{R}^n$ is the state, and $u \in \mathbb{R}^m$ are the control variables. This system is differentially flat if one can find outputs $y \in \mathbb{R}^m$ (the same as the number of inputs) of the form

$$y = y(x,u,\dot{u},\ldots,u^{(p)}) \tag{30}$$

such that all states and inputs of the system can be written as algebraic functions of these flat outputs and their derivatives. In other words, equation (30) can be inverted, such that

$$x = x(y,\dot{y},\ldots,y^{(q)}) \tag{31}$$

$$u = u(y,\dot{y},\ldots,y^{(q)}) \tag{32}$$

From the previous equations it becomes evident why flat outputs play a significant role in trajectory generation problems. If the flat output history $y(t)$ is known, then (31) and (32) can be used to compute the corresponding state and input trajectories. Every path in the flat output space is mapped to a feasible trajectory and thus, the trajectory generation problem for flat systems is trivial.

Differentially flat systems are extremely nice since, they are equivalent[**] to an algebraic system, i.e., a system without dynamics. The downside of this approach is that most nonlinear systems are

[‖]The so-called "exosystem".

[**]This type of equivalence is called Lie-Bäcklund equivalence and it is quite well-known in physics. This transformation does not necessarily preserve state dimension. See also Ref. [68].

not flat. Also, except the case of configuration flat Lagrangian systems with n degrees of freedom and $n-1$ controls [69], to date, there does not exist a systematic way for finding the flat outputs (if they exist) although very often they have intrinsic physical significance [67].

Flat Outputs for the Attitude Problem

In this section we use vibrational control and averaging to construct a differentially flat system that approximates (8). We show that this new system is differentially flat by explicitly computing its flat outputs. In fact, the approximated system is exactly the averaged system introduced in (23). The use of vibrational control for approximating non-flat systems with flat ones has been used previously by Fliess et al. [67] to control several mechanical systems.

To this end, we rewrite the averaged system in (23), dropping the bars for convenience, as follows

$$\dot{\omega}_3 = \varepsilon(v_1 v_2 + \frac{v}{2}) \tag{33a}$$

$$\dot{w}_1 = \omega_3 w_2 + \frac{1}{2}(1 + w_1^2 - w_2^2)\, v_1 + w_1 w_2 v_2 \tag{33b}$$

$$\dot{w}_2 = -\omega_3 w_1 + \frac{1}{2}(1 - w_1^2 + w_2^2)\, v_2 + w_1 w_2 v_1 \tag{33c}$$

$$\dot{z} = \omega_3 + w_1 v_2 - w_2 v_1 \tag{33d}$$

It is claimed that system (33) with inputs v_1, v_2 and v is differentially flat. The flat outputs are given by

$$y_1 = 2 \arctan\left(\frac{w_2}{w_1}\right) + z \tag{34a}$$

$$y_2 = z \tag{34b}$$

$$y_3 = \omega_3 \tag{34c}$$

First note that, trivially, z and ω_3 can be written as functions of y_1 y_2 and y_3. Next we show that w_1 and w_2 can be written as functions of y_1, y_2, y_3 and, possibly, their derivatives.

Differentiating Eq. (34a) we get

$$\dot{y}_1 = \frac{1-|w|^2}{|w|^2}(\dot{y}_2 - y_3) - 2y_3 + \dot{y}_2 = \frac{\dot{y}_2}{|w|^2} - \frac{1+|w|^2}{|w|^2} y_3 \tag{35}$$

or

$$|w|^2 = \frac{\dot{y}_2 - y_3}{\dot{y}_1 + y_3} \tag{36}$$

Moreover, we have that

$$\arctan\left(\frac{w_2}{w_1}\right) = \frac{y_1 - y_2}{2} \tag{37}$$

The previous two equations together imply that

$$w_1 = \sqrt{\frac{\dot{y}_2 - y_3}{\dot{y}_1 + y_3}} \cos\left(\frac{y_1 - y_2}{2}\right), \qquad w_2 = \sqrt{\frac{\dot{y}_2 - y_3}{\dot{y}_1 + y_3}} \sin\left(\frac{y_1 - y_2}{2}\right) \tag{38}$$

which, together with equations,

$$z = y_2, \qquad \text{and} \qquad \omega_3 = y_3 \tag{39}$$

provide the desired result.

We have shown that w_1, w_2, z and ω_3 can be written as algebraic functions of y_1, y_2, y_3 and their time derivatives. Let us now rewrite Eqs. (33b)-(33d) in the compact form

$$\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{z} \end{bmatrix} = F_1(w) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + F_2(w)\, \omega_3 \tag{40}$$

Once y_1, y_2, y_3, w_1, w_2 and their time derivatives are known, one can solve Eq. (40) for v_1 and v_2 as follows

$$\begin{aligned} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} &= \left(F_1^T(w) F_1(w)\right)^{-1} F_1^T(w) \left(\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{y}_2 \end{bmatrix} - F_2(w)\, y_3 \right) \\ &= \frac{4}{(1 + w_1^2 + w_2^2)^2} \times \\ &\quad \begin{bmatrix} \frac{1}{2}(1 + w_1^2 - w_2^2) & w_1 w_2 & -w_2 \\ w_1 w_2 & \frac{1}{2}(1 - w_1^2 + w_2^2) & w_1 \end{bmatrix} \left(\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{y}_2 \end{bmatrix} - \begin{bmatrix} w_2 \\ -w_1 \\ 1 \end{bmatrix} y_3 \right) \end{aligned} \tag{41}$$

Once v_1 and v_2 have been computed from Eq. (41), the remaining input v can be computed from Eq. (33a)

$$v = \frac{2}{\varepsilon} \dot{y}_3 - 2 v_1 v_2 \tag{42}$$

It follows that v_1, v_2 and v can be written as functions of y_1, y_2, y_3 and their time derivatives. Therefore, y_1, y_2 and y_3 are flat outputs for the system in Eqs. (33), as claimed.

The procedure to follow in order to generate a feasible trajectory is relatively simple [70]. Given an initial point $(\omega_{30}, w_{10}, w_{20}, z_0)$, a final point $(\omega_{3f}, w_{1f}, w_{2f}, z_f)$, a final time t_f, and a series of ℓ intermediate points $(\omega_{3k}, w_{1k}, w_{2k}, z_k)$, $k = 1, \ldots, \ell$, we want to meet, we calculate the corresponding points in the flat output space $y(0), y(t_f), y(t_1), \ldots, y(t_\ell)$. We then introduce some smooth basis functions $\phi_j(t)$, $j = 1, \ldots, N$, such that

$$y_i(0) = \sum_{j=1}^{N} A_{ij}\, \phi_j(0), \quad y_i(t_f) = \sum_{j=1}^{N} A_{ij}\, \phi_j(t_f), \quad y_i(t_k) = \sum_{j=1}^{N} A_{ij}\, \phi_j(t_k), \quad k = 1, \ldots, \ell \tag{43}$$

where $1 \leq i \leq 3$. We therefore obtain the system of linear equations

$$\begin{bmatrix} y(0) \\ y(t_f) \\ y(t_1) \\ \vdots \\ y(t_\ell) \end{bmatrix} = \begin{bmatrix} \phi_1(0) & \dots & \phi_N(0) \\ \phi_1(t_f) & \dots & \phi_N(t_f) \\ \phi_1(t_1) & \dots & \phi_N(t_1) \\ \vdots & \ddots & \vdots \\ \phi_1(t_\ell) & \dots & \phi_N(t_\ell) \end{bmatrix} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ \vdots & \vdots & \vdots \\ A_{1N} & A_{2N} & A_{3N} \end{bmatrix} = \Phi A^T \tag{44}$$

which can be solved for the unknown coefficients A_{ij}, $1 \leq i \leq 3, 1 \leq j \leq N$ assuming that the $(\ell+2) \times N$ matrix Φ has full rank. Once the time history of the flat outputs $y_1(t), y_2(t), y_3(t)$ is known, the corresponding state histories can be computed from (38) and (39) and the corresponding inputs $v_1(t), v_2(t)$ and $v(t)$ and from (41) and (42). The angular velocities commands $\omega_{d1}(t)$ and $\omega_{d2}(t)$ that generate the solution $(\omega_3(t), w_1(t), w_2(t), z(t))$ for the given initial, final and intermediate conditions can be computed from (22).

Care must be taken when choosing the basis functions ϕ_j in order to make sure that the flat output histories satisfy the additional requirement that $(\dot{y}_2 - y_3)(\dot{y}_1 + y_3) \geq 0$; see (38). One way to circumvent this difficulty is by time re-parameterization in the flat output space [65].

Figure 7 shows a feasible trajectory for the system in (8) calculated using the previous procedure. The initial conditions are given by $(\omega_3(0), w_1(0), w_2(0), z(0)) = (0.5, 1, -1, 0)$ and the final conditions are given by $(\omega_3(t_f), w_1(t_f), w_2(t_f), z(t_f)) = (-1, 0, 2, 0.5)$. The final time was chose $t_f = 20$ sec. The solid lines in Fig. 7 indicate the trajectory computed from the flat outputs (shown in Fig. 8(a)) and the dashed lines show the actual trajectory followed when using the angular velocity commands ω_{1d} and ω_{2d}. These angular velocity commands are shown in Fig. 8(b). The angular velocity input, as expected, is oscillatory, but in practice the frequency also depends on the total maneuver time.

For simplicity, four Legendre polynomials were used for basis functions in (43). Also, in order to allow for any possible value for $|w(0)|$ and $|w(t_f)|$ we impose the extra constraints that $\dot{y}_2 - y_3 = \dot{y}_1 + y_3 = 0$ for $t = 0$ and $t = t_f$.

OPEN PROBLEMS

The angular velocity stabilization problem for the case of one or two actuator failures is well-understood and can be considered solved, although some work still remains for momentum exchange actuators. On the other hand, despite the recent advances, the complete attitude stabilization problem is not completely understood. For example, all control laws proposed so far are time-varying and local in nature. That is, they work in a sufficiently small neighborhood of the origin. There is no proof of their region of attraction. No globally stabilizing control law has been reported in the literature, as far as the authors know. In addition, due to their periodic (often high frequency)

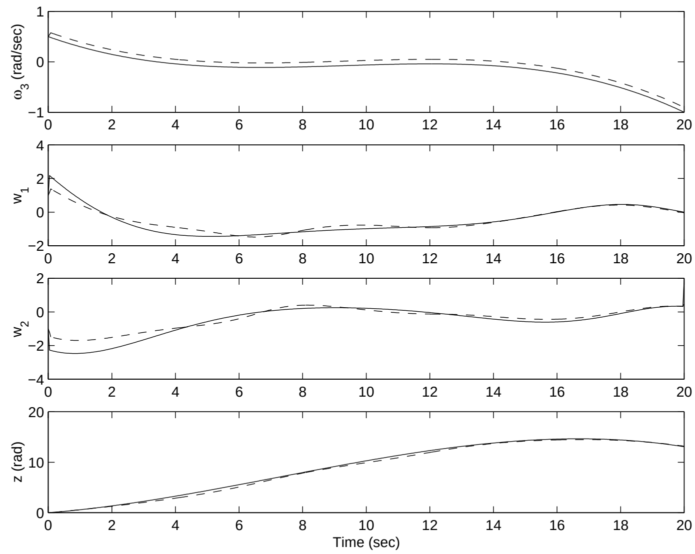
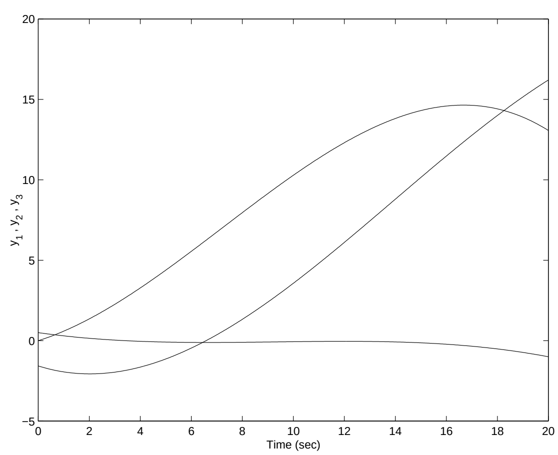
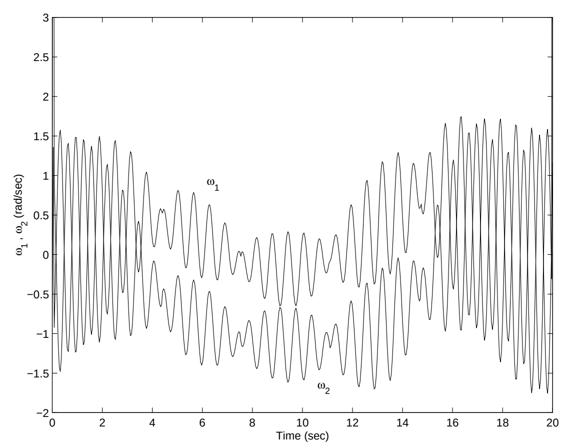


Figure 7: Time history of feasible states ($\varepsilon = 0.2, e = 0.1$).



(a) Time history of the flat outputs.



(b) Time history of the angular velocities ω_{1d} and ω_{2d}.

Figure 8: Time history of flat outputs and corresponding angular velocity commands ($\varepsilon = 0.2, e = 0.1$).

nature these time-varying control laws may not be suitable for spacecraft with flexible parts, antennas, fuel slosh, etc. In such cases, these control laws may induce unacceptable transient response and even instability. Most likely, time-invariant control laws have to be used, instead. This justifies our interest in time-invariant control laws. Our experience seems to indicate that such control laws have to be discontinuous at the origin, a property that rises several practical and theoretical questions. Also, dynamic controllers – with the exception of (25) – have not been sufficiently explored. It should be pointed out here that the time-invariant control laws in (12) and (13) are suggested as possible candidates for the global stabilization based on purely physical intuition and extensive numerical simulations. No formal proof is available at this point.

Another issue of great practical importance is the use of momentum exchange devices (momentum /reaction wheels, or control moment gyros) as actuators. It is well known [26] that for the case of even one such actuator failure the system is not controllable. It may be controllable in a restricted sense, however, i.e., about an equilibrium manifold. What is the most one can expect in such a case? With the exception of Ref. [44] no tracking control laws have been reported in the literature for the underactuated rigid spacecraft problem. Moreover, the robustness question is still open; see Ref. [45] for some early results on this topic.

An equally important problem to actuator failure that has not been addressed in satisfactory detail is that of sensor failure. The only exceptions seem to be Refs. [6, 71] and [72]. The first two references solve the stabilization problem without any angular velocity information (say due to a rate gyro failure). Reference [72] addresses the problem of attitude tracking using only attitude information. Cases of combined rate gyro/attitude sensor failure do no seem to have been addressed. Closely related is the issue of observability for the complete system equations. It would extremely helpful if we had a comprehensive theory that completely characterize the observability question in case of multiple sensor failures. That would also open the door to output feedback control laws, an issue virtually unexplored.

CONCLUSIONS

We have provided a series of new results dealing with the problem of stabilization of a rigid spacecraft when one of the torque actuators has failed. This control problem is challenging since it requires completely nonlinear, non-standard techniques. We have presented time-varying and time-invariant feedback controllers for this problem. We have also addressed the problem of feasible trajectory generation for a spacecraft with two control torques, borrowing ideas from differentially flat systems. These feasible trajectories can be used as reference trajectories in attitude tracking problems. It is hoped that our results will motivate more research in the challenging but important problem of spacecraft control subject to actuator (and, possibly, sensor) failures.

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FIGURE CAPTIONS

Fig. 1: Time history of w, z and ω_3 ($\mu = 3, \lambda = 12, \kappa = 0.05, \varepsilon = 0.2$).
Fig. 2: Time history of angular velocities ($\mu = 3, \lambda = 12, \kappa = 0.05, \varepsilon = 0.2$).
Fig. 3: Time history of states and angular velocities ($\mu = 7, \kappa = 1, \varepsilon = 0.2$).
Fig. 4: Time history of states and angular velocities ($\mu = 2, \kappa = 1, \varepsilon = 0.2$).
Fig. 5: Time history of w_1, w_2, z and ω_3 ($\mu = 2, \kappa = 1, \varepsilon = 0.2, e = 0.1$).
Fig. 6: Time history of angular velocities ($\mu = 2, \kappa = 1, \varepsilon = 0.2, e = 0.1$).
Fig. 7: Time history of feasible states ($\varepsilon = 0.2, e = 0.1$).
Fig. 8: Time history of flat outputs and corresponding angular velocity commands ($\varepsilon = 0.2, e = 0.1$).

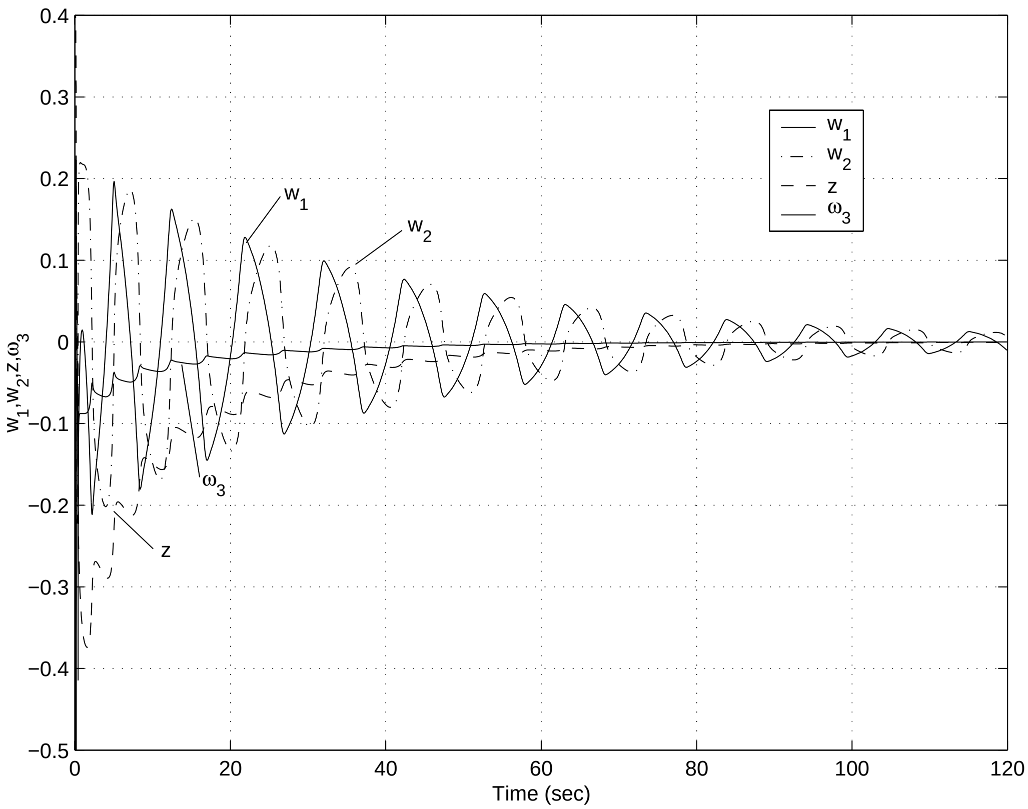
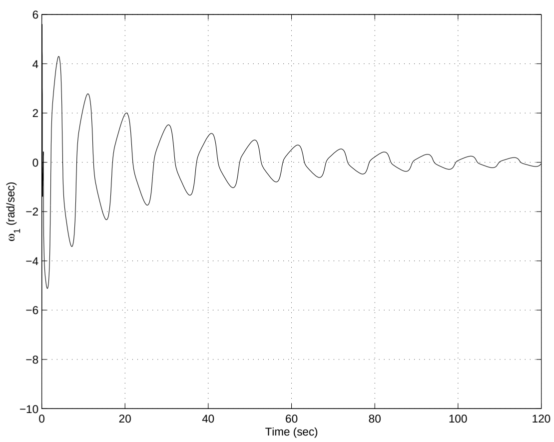
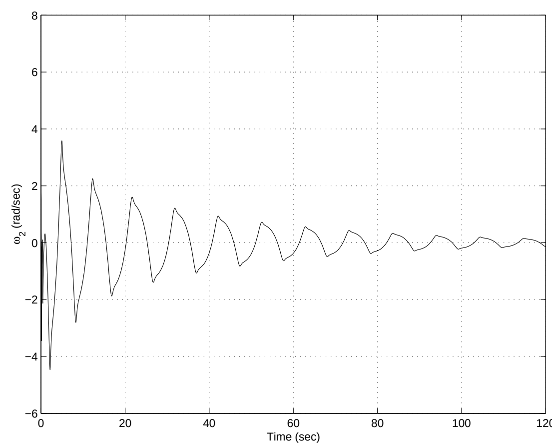


Fig. 1: Time history of w, z and ω_3 ($\mu = 3, \lambda = 12, \kappa = 0.05, \varepsilon = 0.2$).

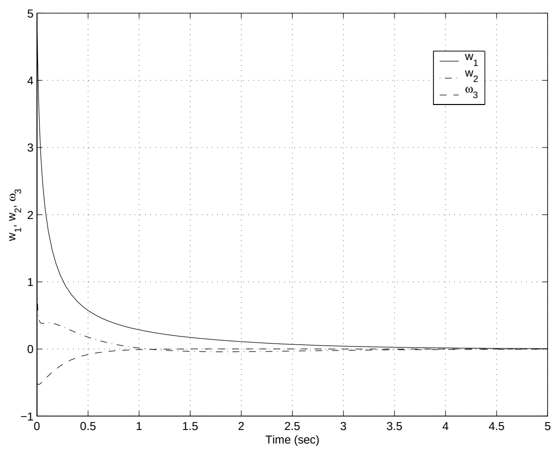


(a) Time history of ω_1.

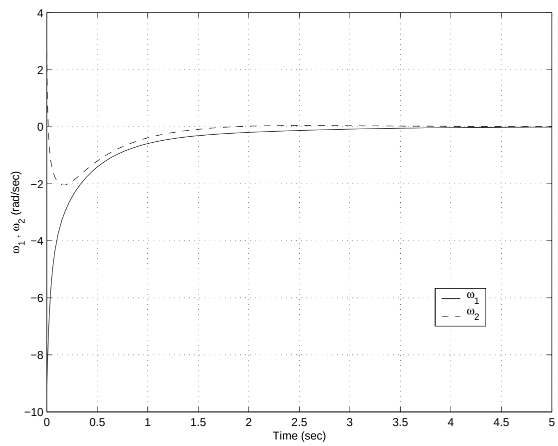


(b) Time history of ω_2.

Fig. 2: Time history of angular velocities $(\mu = 3, \lambda = 12, \kappa = 0.05, \varepsilon = 0.2)$.

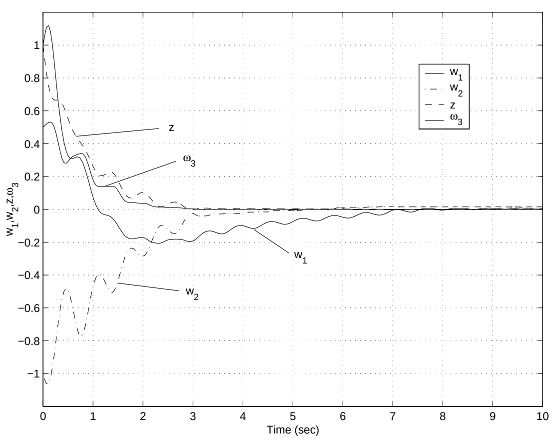


(c) Time history of w_1, w_2 and ω_3.

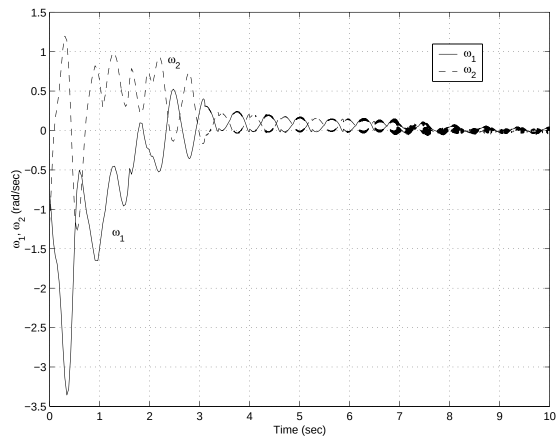


(d) Time history of ω_1 and ω_2.

Fig. 3: Time history of states and angular velocities $(\mu = 7, \kappa = 1, \varepsilon = 0.2)$.



(e) Time history of w_1, w_2, z and ω_3.



(f) Time history of ω_1 and ω_2.

Fig. 4: Time history of states and angular velocities $(\mu = 2, \kappa = 1, \varepsilon = 0.2)$.

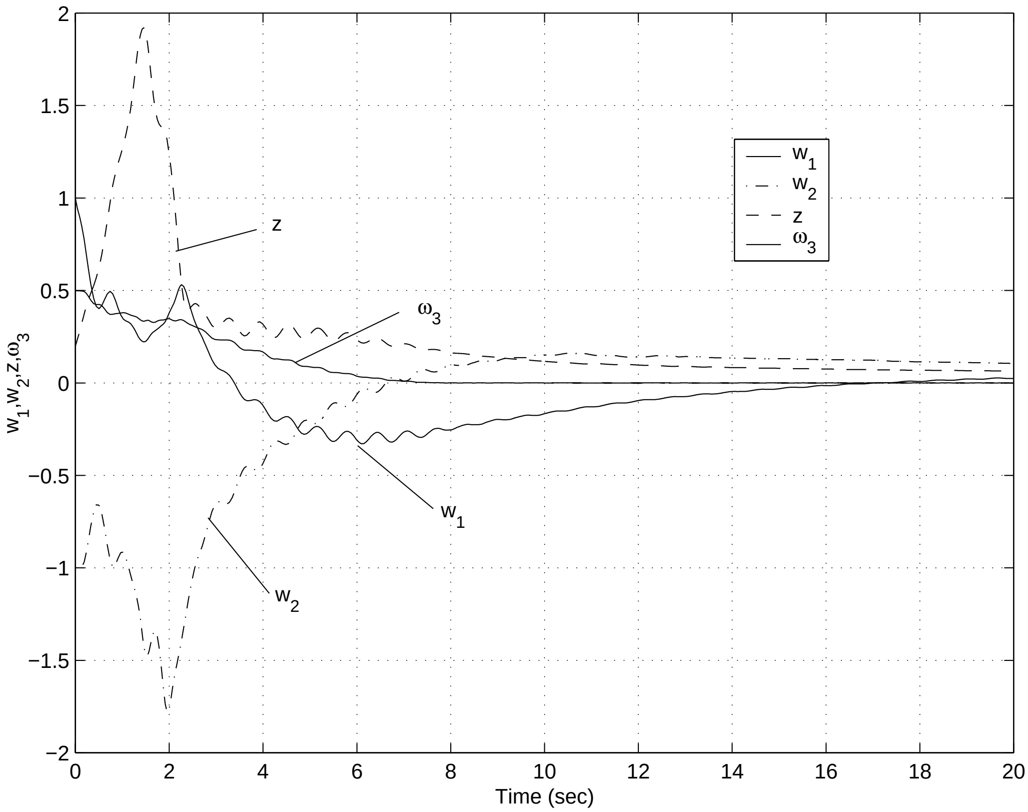
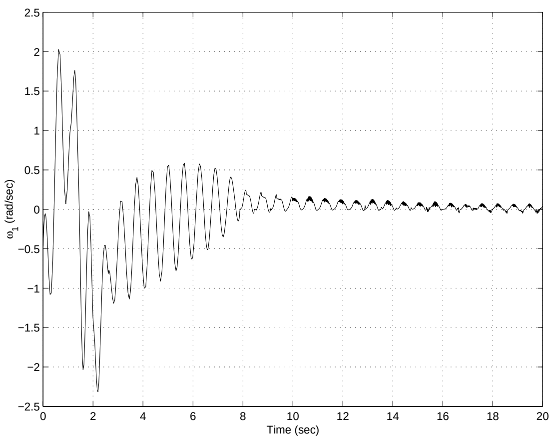
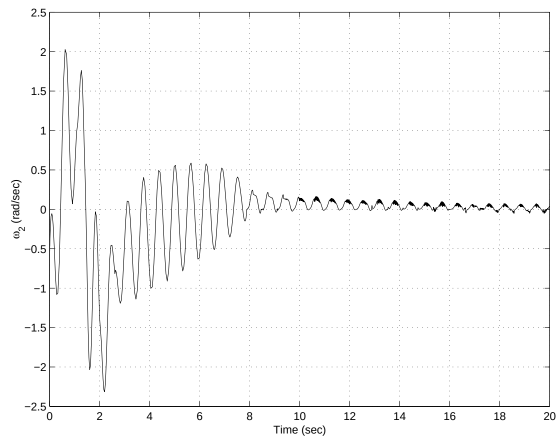


Fig. 5: Time history of w_1, w_2, z and ω_3 ($\mu = 2, \kappa = 1, \varepsilon = 0.2, e = 0.1$).



(g) Time history of ω_1.



(h) Time history of ω_2.

Fig. 6: Time history of angular velocities $(\mu = 2, \kappa = 1, \varepsilon = 0.2, e = 0.1)$.

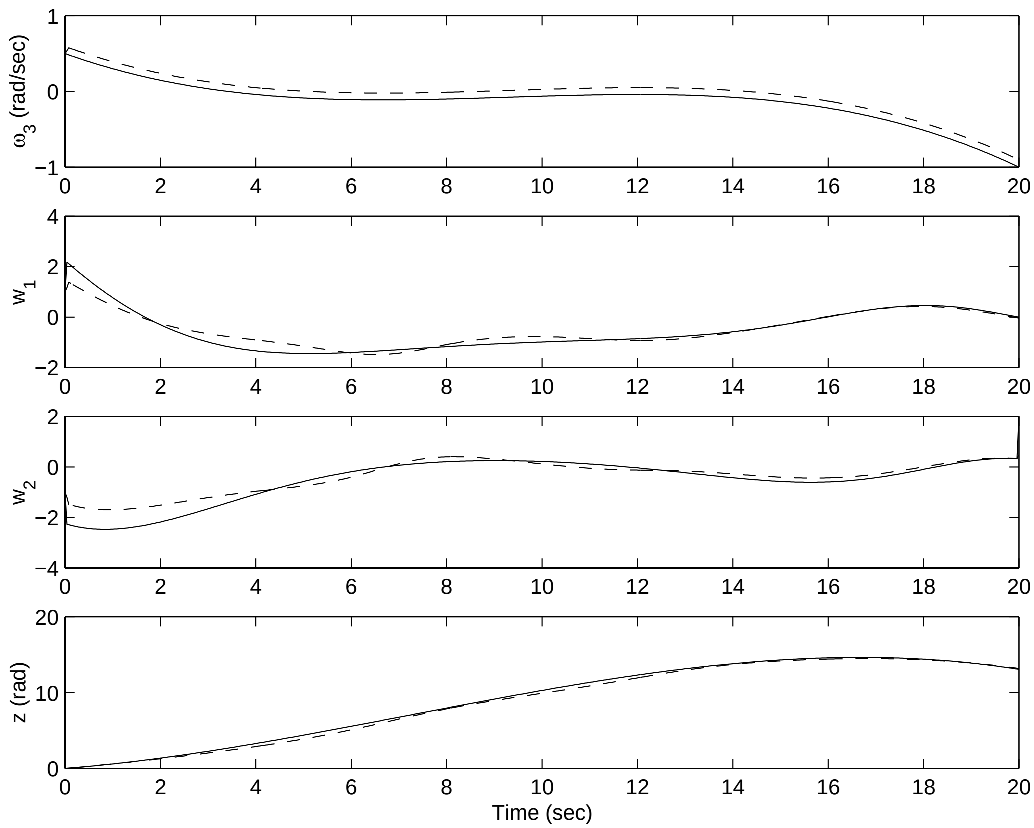
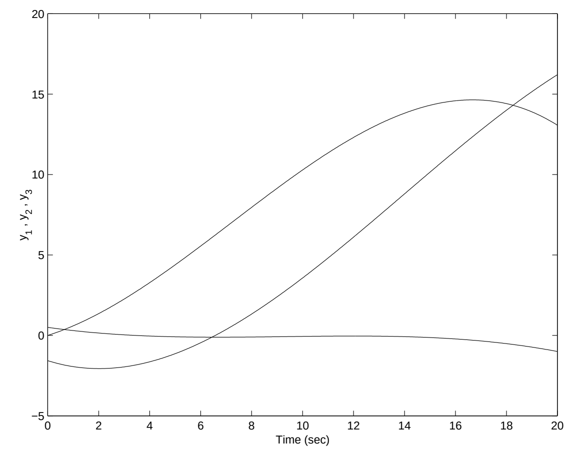
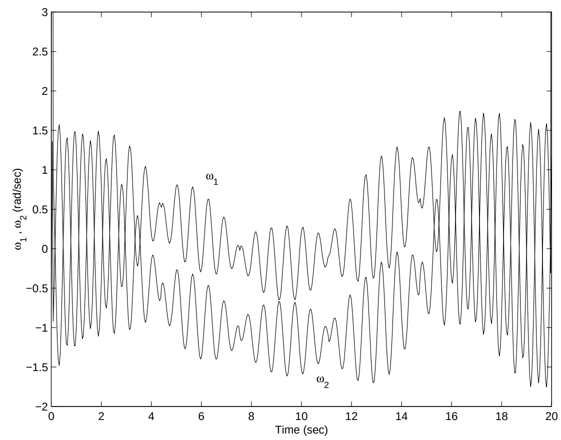


Fig. 7: Time history of feasible states ($\varepsilon = 0.2, e = 0.1$).



(i) Time history of the flat outputs.



(j) Time history of the angular velocities ω_{1d} and ω_{2d}.

Fig. 8: Time history of flat outputs and corresponding angular velocity commands ($\varepsilon = 0.2, e = 0.1$).