VARIABLE SPEED CONTROL MOMENT GYROSCOPE WORKBENCH: A NEW SIMULATION TOOL FOR TOMORROW'S SPACECRAFT

David J. Richie, Panagiotis Tsiotras, Georgia Institute of Technology, Atlanta, GA

Jerry L. Fausz, Air Force Research Laboratory, Kirtland AFB, NM

Abstract

This paper describes the Variable Speed Control Moment Gyroscope (VSCMG) Workbench, a simulation software tool developed at Georgia Tech. This program will provide the Air Force Research Laboratory (AFRL) with a tool for studying spacecraft control system design for vehicles equipped with VSCMG actuators for combined attitude control and energy storage.

Introduction

Space vehicle programs consistently seek to reduce satellite bus mass to increase payload capacity and/or reduce launch and fabrication costs. In addition, satellite system performance demands continually challenge space vehicle designers. Specifically, larger space structures require finer, more accurate three-axis attitude control methods.

One of the most popular methods for this type of control is by employing gas jet thrusters to provide the necessary vehicle control torque. Unfortunately, the plumes of such thrusters often impinge on critical vehicle components such as communications equipment, sensors, and subsystem actuators [1]. For this reason, internal vehicle three-axis control methods with lower mass are needed. Flywheel-based systems providing both energy storage and attitude control address the need for combined energy storage and attitude control. In particular, the Air Force Research Laboratory's Flywheel Attitude Control, Energy Transmission and Storage (FACETS) program will combine all, or part of, the energy storage, attitude control, and power management and distribution (PMAD) subsystems into a single system, significantly decreasing bus mass (and volume) by eliminating the need for conventional chemical batteries [2].

An Integrated Power and Attitude Control System (IPACS) such as FACETS employs flywheels as "mechanical batteries" to perform the attitude control and energy storage functions. The IPACS concept eliminates vehicle mass while improving system performance and lifetime. **Figure 1** reflects the cost benefits of an IPACS system like FACETS. This is attained by eliminating a typical satellite's most expensive subsystem, the batteries. Until now,
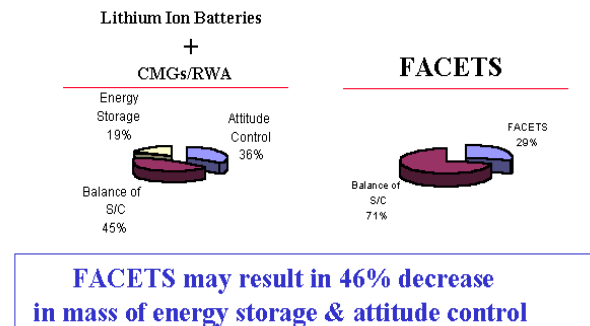

FACETS may result in 46% decrease in mass of energy storage & attitude control

Figure 1. USAF Trade Study

the well-documented IPACS concept has never been implemented, mainly due to high flywheel spin rates (of the order of 40K to 80K RPM -- versus less than 5K RPM for conventional Control Moment Gyroscopes (CMGs) or momentum wheel actuators). At such high speeds, the strength of flywheel material is a major issue. In addition, the actuators quickly wear out mechanical bearings. Additional challenges include flywheel material mass/durability and stiffness inadequacies. Recently, the advent of composite materials and magnetic bearing technology has enabled realistic IPACS developments [2,3].

It has been shown in [4,5] that simultaneous momentum management and power tracking can be accomplished with four or more wheels in momentum wheel (MW) mode. This is done by adjusting the wheel acceleration in the null subspace of the required attitude control torque

dynamics matrix, in such a way as to generate the required vehicle power, while not imparting adverse external torque on the spacecraft. Furthermore, this method was shown to be practical for tracking the required time history profiles for several satellite types [3,4].

AFRL's FACETS program intends to employ Variable Speed CMGs (VSCMGs) on the Advanced Space Structures Technology Research Experiments (ASTREX) test. The ASTREX rests on an air bearing and provides one of the best ground-based test environments for simulating on-orbit satellite motion. **Figure 2** shows the old ASTREX CMG configuration (used in the early 1990s).
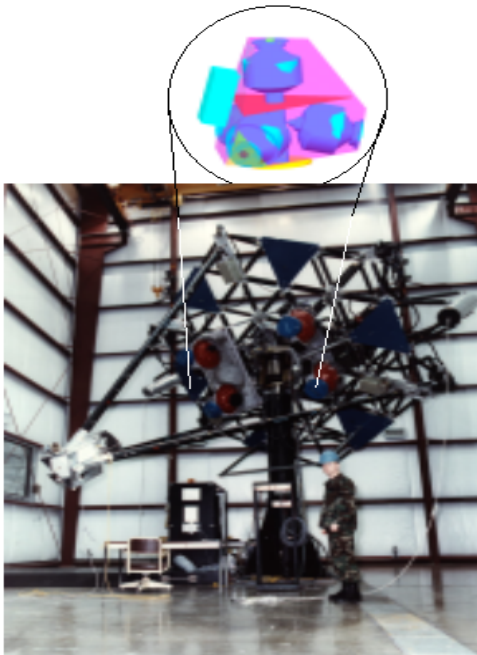

Figure 2. ASTREX Structure

A similar configuration is planned for the new FACETS actuators.

There are different kinds of actuators that could be used for an on-orbit IPACS. Note that the use of VSCMGs instead of momentum wheels may be beneficial in certain applications that require large slew maneuvers. On the other hand, a MW-based IPACS system seems to be better suited for station-keeping applications.

The use of flywheels instead of batteries to store energy on spacecraft was suggested by Roes as early as 1961 [6], when a 17 W hr/kg composite flywheel spinning at 10K to 20K RPM on magnetic bearings was proposed. The configuration included two counter-rotating flywheels, and the author did not mention the possibility of using the momentum for attitude control. This idea grew over the next three decades. References [7-9] are representative of the period from 1970-1977, during which the term IPACS was coined [7] to describe an integrated power storage and attitude control system; this system was envisioned to significantly reduce payload to orbit for shuttle-era satellite programs. Around that time NASA, in collaboration with the Draper Laboratory, completed various concept feasibility studies and even held several working groups in order to investigate potential implementation of IPACS [10-14].

In [15], we investigated the problem of simultaneous attitude and power tracking for a rigid spacecraft using Variable Speed Control Moment Gyroscopes (VSCMGs). The derivation presented in [15] is based on an arbitrary number of VSCMGs and is independent of any particular VSCMG configuration. It includes the exact, nonlinear equations for a spacecraft/VSCMG system. The derived theory uses minimal assumptions and its generality enables direct application of the theory to a wide variety of satellite systems. This creates flexibility for future space systems contemplating an IPACS using VSCMGs for attitude control and energy storage.

In this paper, we present a SIMULINK-based, software tool developed to simulate and implement AFRL's FACETS system using the theory developed in [15]. This tool, the *VSCMG Workbench*, allows analysis of different spacecraft systems that use different attitude control/energy storage actuator types from those mentioned above.

The VSCMG Workbench uses a modularized structure that permits flexibility in implementation. We present several layers of detail for each designed module, as well as the basic program operations. The primary modules in this simulation are identical to those of a standard control loop: the reference attitude and power profile module, the

plant module, the controller module, the actuator module, and the sensor module. These modules have been designed with varying input and output data stream sizes, such that the number of actuators in the system can be changed fairly easily. In addition to setting the number of actuators, the user has freedom to choose between traditional momentum/reaction wheel actuators, classical control moment gyroscopes, or VSCMGs. This allows the simulation to be used by a wider range of potential users.

We analyze the results of several simulation tests in order to study the system parameters and the effects of parameter perturbations on simultaneous spacecraft attitude control and power tracking. In addition, two control laws are applied for both attitude regulation and tracking. These tests are subdivided into six areas: controller parameter effects, system parameter effects, simulation parameter effects, actuator configuration tests, and regulation and tracking with different controllers.

Background

Figure 3 (taken from [21]) shows a CMG actuator and its associated G coordinate frame.
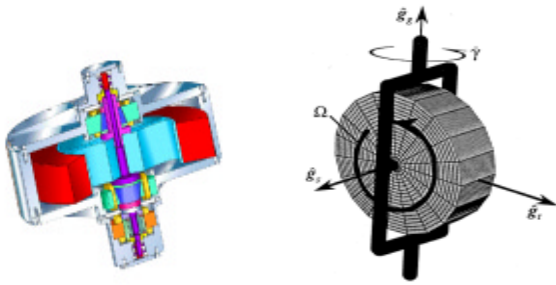

Figure 3. Variable Speed Control Moment Gyro

As shown in **Figure 3**, the frames attached to the n - VSCMGs, G_j, $j = 1 \ldots n$, are characterized by an orthogonal set of unit vectors, $\hat{g}_{sj}$, $\hat{g}_{tj}$, and $\hat{g}_{gj}$, $j = 1 \ldots n$, where the subscripts s, t, and g denote the spin, transverse and gimbal axes, respectively.

For this research, $\ddot{\gamma}$, $\dot{\gamma}$, γ, $\dot{\Omega}$, Ω represent the gimbal acceleration, gimbal rate, gimbal rotation angle, wheel acceleration, and wheel speed.

In CMG mode, an input torque is applied along the gimbal axis through controlling the gimbal acceleration and/or gimbal rate resulting in an output torque about the transverse axis. In MW mode, input torque is applied to the wheel (i.e. affecting the wheel acceleration) imparting an output torque on the vehicle. VSCMGs combine these two methods.

VSCMG Workbench: Layout and Design

The VSCMG workbench is a software program designed for studying the control system design of a spacecraft equipped with VSCMGs for combined attitude control and energy storage. Its modularized structure permits flexibility in implementation. The several layers of the design are described here as well as basic program operation.

Main Program Control Module

The first block the user needs is the Main Program Control Module shown in **Figure 4**. In this module, the user loads the reference data (which currently involves tailoring a MATLAB M-file where the user defines two-column matrices (one column contains the time data, the other contains the time history of that specific parameter) for each of the 11 reference values (these values are the 4-parameter Euler parameter history, 3-parameter vehicle angular velocity (given in the vehicle coordinate reference frame, commonly referred to as p, q and r) and the 3 angular velocity derivatives ($\dot{p}$, $\dot{q}$, $\dot{r}$).

Feedback System Overview Module

Underneath the Program Control Module mask resides the overall feedback control system. Unless the user intentionally reveals the support code by selecting to look under the SIMULINK mask, all underlying subsystems are masked from the user. The underlying system includes modules

representing characteristics inherent in a typical control system -- the plant module, the plant actuators, the controller, the sensor suite, and the reference values.
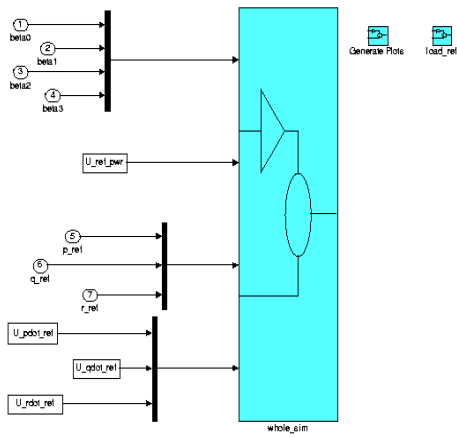

Figure 4. Main Program Control Module

This is reflected in **Figure 5**. Note that each one of these major subsystems is introduced in this section.
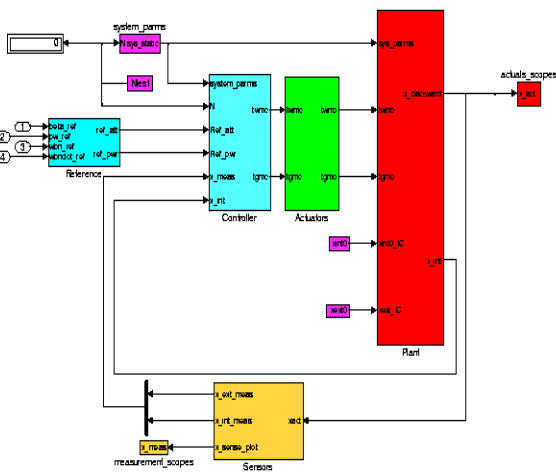

Figure 5. System Schematic

System Parameters Module

The first underlying subsystem is the System Parameters Module (located in the top left potion of **Figure 5**). This includes the two primary parameter sets that drive the system configuration. These are set by the user and depend on the number

of actuators used. These parameters are the reference VSCMG G-frame installation orientation coordinate transformations and the system inertias. One should note that these parameters are constant throughout a simulation run.

Reference Module

Besides setting the system parameters, other key inputs to the control system are the reference attitude and reference power the control system must meet. A schematic of this module is included in **Figure 6**. Design flexibility allows different reference attitude and reference power time histories (in the form of data tables) to be used as simulation inputs. These can represent vehicles in different configurations, different vehicles, different actuator systems, etc.
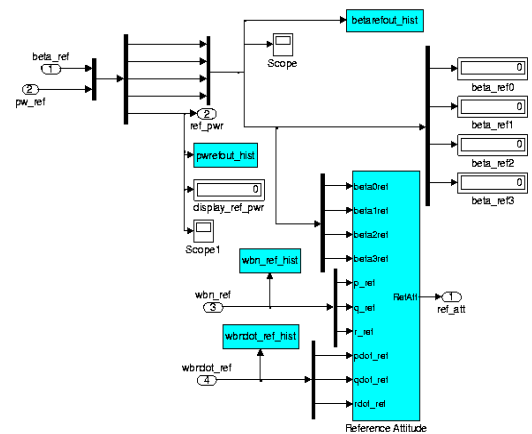

Figure 6. Attitude and Power Reference Module

Controller Module

The central "brain" of the control system is the controller. The crux of the developed theory directly relates to the logic employed in the controller. The controller uses the vehicle configuration mentioned in the previous sections along with the desired (reference) attitude and power profiles to determine the actuator torques that will steer the vehicle to the desired attitude, while simultaneously meeting the power tracking requirements and heeding the gimbal torque amplification constraint described in [16].

4

Actuators Module

The commanded vehicle torque is generated by the system actuators. The actuators are divided into two major portions: the wheel motors and the gimbal motors. Both systems (the wheels and the gimbals) turn commanded motor torque voltages into actuated motor torques. These torques are used to calculate the actuated torque about the combined system center of mass (point O). As in several of the other modules, this module has been designed to allow future designers to add model fidelity. This way, more realistic models of the actuator motors can be added to the simulation in order to analyze different actuator behaviors and concepts of control.

Plant Module

The plant block (shown in **Figure 7**) includes the modeled vehicle inertias, the disturbance model inputs, the actuated torque computation function, and the integrated vehicle state equations (i.e. for vehicle rate and acceleration given the vehicle torques).
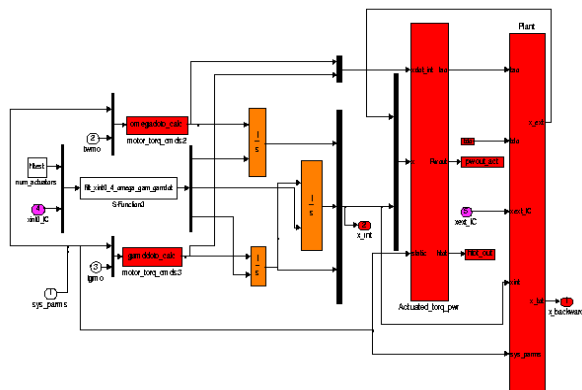

Figure 7. Plant Overview

Sensors Module

Like the actuators module, the sensor module allows much room for the user to add more realistic, higher-fidelity models. Such models may include typical sensor inefficiencies, sensor data non-linear sensor phenomena (such as saturation, hysteresis, and backlash), and sensor fusion/weighting algorithms. All these topics permit much further in-depth analysis of on-orbit spacecraft phenomena previously difficult to study on the ground.

Summary of Program Operation

In order to use these different modules, the user must have a good idea of how the simulation works. As far as basic program operations, the user

- Determines where the actuators are installed (i.e. where the initial gimbal reference axis for each VSCMG is located relative to the body reference frame).
- Sets the vehicle platform plus actuator point-mass inertia matrix as well as the individual actuator inertia matrices (with respect to each G_j frame). These values are set in the system parameters module.
- Configures the controller parameters (as appropriate) in the controller subsystem module
- Sets the desired actuator model in the actuators system module according to the desired fidelity/response of the actuated system.
- Runs the `load ref` function which will load reference data according to the desired scenario
- Starts the simulation in the Master program module by selecting the `play` button.
- Selects `generate plots` in order to create MATLAB output plots, if desired, when the simulation is done. Note that the `load ref` and `generate plots` function should be tailored according to the purpose to which this program is applied.

Simulation Scenarios

Several different simulation tests were performed in order to study simultaneous spacecraft attitude control and power tracking. These tests can be divided into several key categories: controller comparison tests, controller parameter tests, system parameter tests, simulation parameter tests, and actuator model tests.

Before describing these tests in more detail, one must first review some preliminary information on control mode weighting and singularity avoidance found in [15] and then outline the scenarios used for attitude regulation and attitude tracking. Note that in this section, only the test scenarios and parameters are presented - analysis of the tests' results follows in the ensuing section. Furthermore, it is important to understand that the baseline tests described for each test group essentially serve as the "experimental control" variables and thus allow one to compare different parameter changes with the baseline in order to evaluate a parameter's impact on system performance.

Weighting and Singularity Avoidance

Define the weighted generalized inverse C_1^* identical to that used in [16]

$$C_1^* = W C_1^T (C_1 W C_1^T)^{-1}$$

where W is a diagonal RW/CMG mode weighting matrix such that W_{gj0} represents weighting the predominant CMG mode (to capitalize on its torque amplification property) and is constant throughout a maneuver. W_{sj} is the RW mode weight that comes into play near a CMG singularity. W_{sj} is given by

$$W_{sj} = W_{sj0} \exp(-\mu\delta)$$

in which μ and W_{sj0} are chosen by the control system designer to elicit the desired performance. δ is a parameter that describes the proximity to a CMG singularity [16-18]. The expression for δ used here is the minimum singular value of C_1. This is a more accurate way to describe the singularity of the matrix C_1 [19] than the one used in the literature [16].

Attitude Regulation Scenario

The attitude regulation scenario is based on the examples found in [16,18,20]. The Parameters for the attitude regulation scenario, which uses the controller in [15] are shown in **Table 1.**

Table 1. Regulation Scenario Parameter Settings

Sym	Value	Units
N	4	unitless
Θ	54.75	deg
$\omega(0)$	[0 0 0]	rad/sec
$\beta(0)$	[0.5 0.5 0.5 0.5]	unitless
ω_r	[0 0 0]	rad/sec
β_r	[1 0 0 0]	unitless
$\gamma(0)$	$[\dfrac{\pi}{4} \quad -\dfrac{\pi}{4} \quad \dfrac{\pi}{4} \quad -\dfrac{\pi}{4}]$	rad
$\dot{\gamma}(0)$	[0 0 0 0]	rad
$\Omega(0)$	[50000 60000 55000 65000]	RPM
W_{sj0}	40	unitless
W_{gj0}	1	unitless
I_{Wj}	$diag\{0.70, 0.20, 0.20\}$	$kg\,m^2$
I_{Gj}	$diag\{0.10, 0.10, 0.10\}$	$kg\,m^2$
I_{sc}	$diag\{15053, 6510, 11122\}$	$kg\,m^2$
K	$diag\{700, 700, 700\}$	kgm^2/s
k	35.0	kgm^2/s
μ	10^{-4}	unitless
λ	1	unitless

A standard four VSCMG pyramid configuration is used. The pyramid configuration is implemented in order to facilitate comparisons with the related literature even though the theory applies generically to the *n*-actuator case. **Figure 8,** which was taken from reference [21], depicts a standard CMG/VSCMG pyramid configuration. Note that Θ represents the pyramid angle of each VSCMG that is measured from the vehicle's b_1 - b_2 plane to a VSCMG's gimbal axis. In the regulation scenario, the goal is to bring the vehicle from some arbitrary attitude to the reference attitude while tracking a given power profile. The reference attitude in this case is assumed to be a fixed orientation with respect to inertial frame $\mathcal{N}$. Without loss of generality, one can assume this orientation (in the

regulation case) is the origin as is done in [16] and [18] since one can always redefine the origin as being the desired inertial orientation.
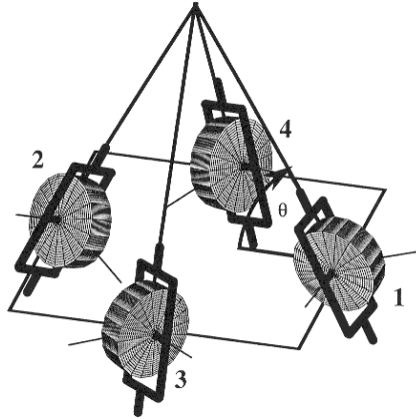

Figure 8. VSCMG Pyramid Configuration

Attitude Tracking Scenario

The tracking scenario is based on the example used in [4]. Similar to the attitude regulation scenario, the goal in the attitude tracking case is to bring the vehicle from an undesired attitude to the reference attitude while tracking a given power profile. The given reference attitude position, velocity, and acceleration correspond to an on-orbit example similar to that used in [4] in which a near-polar orbital satellite has to meet specific sun and ground tracking requirements.

The parameters used in this scenario are virtually identical to those used for the regulation case (see **Table 1**). The only difference is that instead of regulating about a fixed angular position, angular velocity, and angular acceleration are used as references. In this study, the Iridium 25578 orbital parameters were used to generate time varying reference values for angular position, angular velocity, and angular acceleration.

Validation Tests

The first set of tests compared the responses of the two control laws for both scenarios, regulation and tracking. From it, one can see how effective each controller is for each scenario.

Second, several controller parameters can be selected by the user in these control laws. This set of tests outlines the effects that these parameters (e.g., the matrix gain, K, the scalar gain k, the reaction wheel mode weighting, W_{s0}, and the singularity scaling variable, μ), have on the system response.

Third, in order to best understand the spacecraft system, one needs an idea of how the different system variables affect the system response. The primary system parameters tested here are the vehicle body and actuator inertias. These inertias have differing effects on the controlled system response. One might note that another big factor affecting the system is the location of the actuators (i.e. the actuator configuration).

Fourth, tests were performed to investigate how simulation parameters such as integration method, time step, and run time affect the realism of the simulation.

Finally, tests were performed to examine the effects of modeling the actuators. The actuators were modeled both as identity gain and as non-identity gain models. As in the other tests, the results are presented in the next section.

Other Tests Considered

Several other tests were run during this research effort. However, many of them did not appreciably affect the outcome of the simulations and have thus been omitted. This includes issues such as disturbance torque effects, non-diagonal actuator and plant inertia matrices, and gimbal structure inertia changes. The latter of these was tested but had only minor impact on the overall system. The disturbance torques were only initially tested -- a full study of system disturbance rejection could be done in the future..

Simulation Results

As in the previous section, the test results can be broken down into several key areas: controller comparison results, control parameter effects, system parameter effects, simulation parameter effects, and actuator test results.

Controller Comparison Results

Regulation Baseline Results

The first series of plots illustrates the regulation scenario output. *Figures 9*-**10** were created using the control law presented in [15].
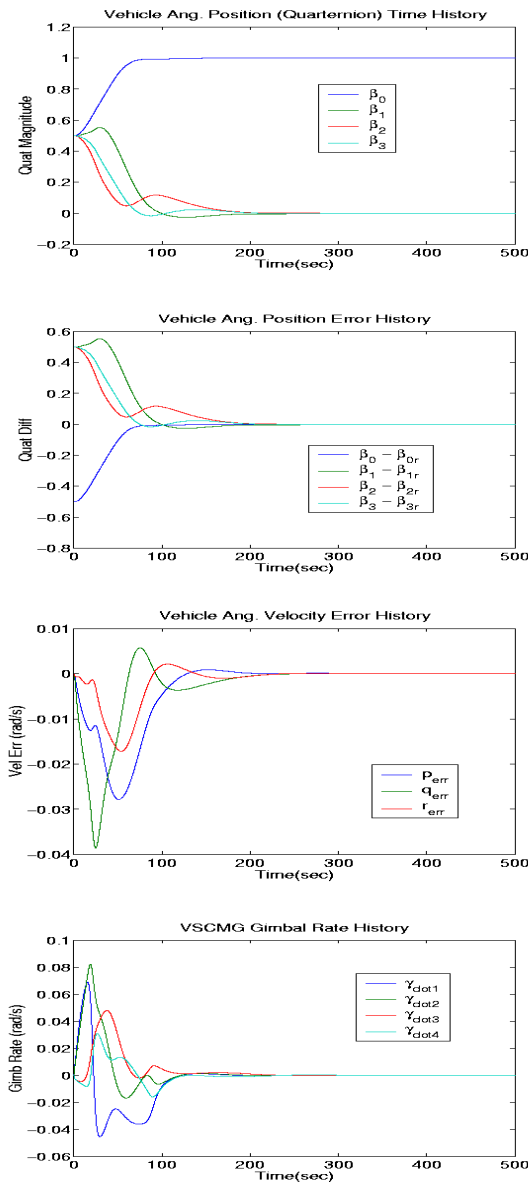
In these plots, one can see the vehicle angular position, vehicle angular position and rate error histories as well as actuator time histories (i.e. wheel speeds, gimbal rates, etc). **Figures 9-10** provide a good demonstration of the typical system behavior based on the given initial conditions.
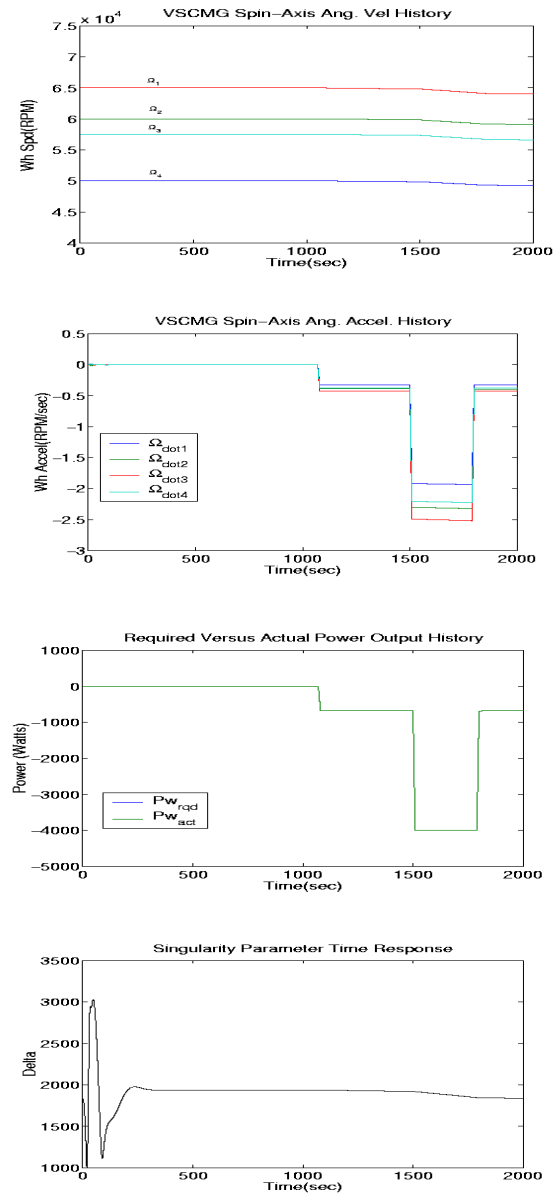
Figure 9. Regulation Baseline Histories: (a) Vehicle Ang. Position, (b) Ang. Position Error, (c) Ang. Velocity Error, (d) Gimbal Rate

Figure 10. Regulation Baseline Histories: (a) Wheel Speed, (b) Wheel Acceleration, (c) Power Required/Actual, (d) Singularity Parameter

These plots show the system going to the desired attitude, while simultaneously tracking the given power profile (shown in **Figure 10(c)**). As expected, power tracking follows the wheel acceleration shown in **Figure 10(b)**. Additionally, one can see through this typical system response the vehicle's response based on different actuator inputs as the spacecraft reorients itself to the proper (inertially fixed in this case) attitude in a relatively short amount of time.

Regulation/Tracking Controller Comparison

The regulation baseline test in the previous section shows the VSCMG Workbench's different outcomes. One can use these outcomes to compare different control laws (i.e. for both the regulation and tracking scenarios described earlier). This type of comparison has already been done as part of this research effort for two different control scenarios (regulation and tracking) using two control law implementations. From this, one can then validate/"fine tune" control laws for his/her application based on the outcome of these plots.

Controller Parameters Comparison

In addition to control response, the control simulation parameters themselves also affect the system. Such parameters include the controller gains, k and K, the wheel spin weighting for CMG singularity avoidance, W_{s0}, and the weighting factor μ. In **Figure 11**, the system response is shown by comparing changes in the scalar gain, k, the norm of the matrix gain, K, the singularity parameter scaling constant μ, and the norm of the momentum wheel spin weighting W_{s0}. Most of the responses are close and therefore difficult to discriminate. The most noticeable effect was that of increasing k. An increase by an order of magnitude (from a value of 35 to 350 in this example) results in oscillatory behavior. On the other hand, increasing the norm of K resulted in a more damped response. Thus, one can manipulate the system performance speed and damping through proper selection of the feedback gains, k and K.
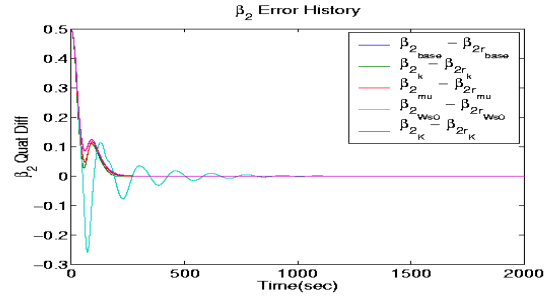

Figure 11. Controller Parameter Impact on Vehicle Angular Position Error

System Parameters Comparison

The next plot illustrates the effects of changes in the system parameters on system response due to an increase in vehicle platform inertia plus actuator point-mass inertia. As shown in **Figure 12** the system response was not significantly affected by inertia uncertainty. The greatest impact was due to increasing wheel spin-axis inertia. Increasing the platform inertia did not have as a great an impact.
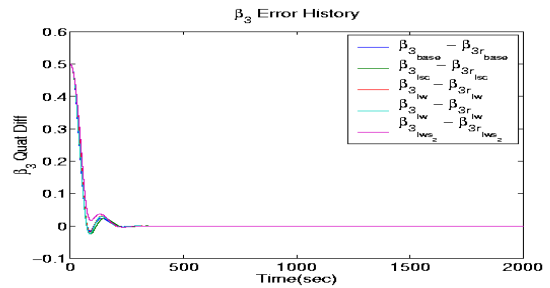

Figure 12. System Parameter Impact

Integration/Simulation Parameter Effects

The simulation parameters also affect the fidelity of the response. Although one might think using the smallest time step is the best solution (it can help precision), it is well known that decreasing the time step can also introduce numerical errors [22] as well greatly increase the run time. **Figure 13** shows the results of different test runs with different fixed integration step sizes. As expected, this plot shows that smaller time steps are more accurate. In this example, the angular momentum magnitude (which is required to be constant as energy is conserved) is compared at three time steps, 0.1 sec, 0.001 sec, and 0.00001 sec with a Runge-Kutta 45 solution method (using

MATLAB's ode45) with a variable step size. The result is that the 0.00001 sec step size yields an accuracy in angular momentum of about 1.8×10^{-6}. For these test cases, the angular momentum was on the order of 7470.0 kgm^2/s.
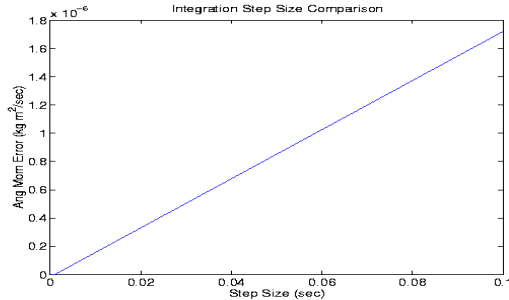

Figure 13. Impact of Integration Step Size

On the other hand, the run time for these runs was greatly increased (for 10 seconds of simulation time run using 0.1 sec took less than 5 minutes whereas a 0.2 second run of the 0.00001 sec time step took over 2 hours to run). Thus, a simple cost benefit analysis shows that there is a balance between accuracy and run time. One should also note that lower time steps do not always yield more accuracy as round-off errors can greatly limit the stability/performance of a simulation run. So, for this case, a time step of 0.1 seconds is adequate for the purpose of the simulation.

In addition, different integration methods were tested using variable time steps. As far as using ode45 (Runge-Kutta), it appears to only affect the result a little but wastes much more state storage space and processing time. Thus, for this case, the Bogacki-Shampine method seems to be the best to use in terms of finding a compromise between storage space, run time, and numerical accuracy/stability.

Features, Benefits, and Limitations

There are benefits and features of the VSCMG Workbench tool with SIMULINK. First, the user is free to set the number of actuators required for applying to a spacecraft system. This gives him/her the flexibility to do tests with differing actuator numbers (and locations) on the vehicle. Second, the modularity of the designed SIMULINK software allows a user to modify equations within the user-defined M-file functions in order to add more fidelity, or he/she can swap out entire modules for completely new ones. This will allow users freedom in testing different control strategies and algorithms. In addition, this software tool will allow testing of different levels of model fidelity using the same program and will allow ground testing of proposed spacecraft subsystems.

The *VSCMG Workbench*, also has some drawbacks. First, the flexibility designed into the program increases the run time of the software. Specifically, the time to run each user defined MATLAB function that is based on a variable number of outputs (it handles a varying number of inputs with ease) is somewhat large. This problem can be drastically improved by translating the modules from MATLAB to C-code. Another limitation is that the code is less efficient since it is not tailored to specific spacecraft/actuator configurations. This increased flexibility (i.e. the ability to address a large class of problems) comes at the expense of computational speed.

Conclusions

A simulation tool has been developed (*The VSCMG Workbench*) to validate advanced algorithms for attitude control and power management for orbiting spacecraft. It is envisioned that this tool will be used by AFRL to test next generation spacecraft currently under development. This will result in testing spacecraft hardware and software systems on the ground, long before use in space. Future work calls for upgrading the software to C-code, adding a near real-time interface system.

greatly indebted to H. Yoon for his help validating the simulation software.

References

[1] Wertz, J. and W. J. Larson, eds., 1999, *Space Mission Analysis and Design*, 3rd Ed, Boston, Kluwer Academic Publishers.

[2] Fausz, J. L. and D. J. Richie, Sept. 25-27, 2000, "Flywheel Simultaneous Attitude Control and Energy Storage using VSCMGs," *IEEE International Conference on Control Applications*, Anchorage, AK, pp. 991-995.

[3] Hall, C., June 4-6, 1997, "High-Speed Flywheels for Integrated Energy Storage and Attitude Control," *Proceedings of the American Control Conference*, Albuquerque, NM, pp. 1894-1898.

[4]Tsiotras, P., H. Shen, and C. Hall, 2001, "Satellite Attitude Control and Power Tracking with Energy/Momentum Wheels," *AIAA Journal of Guidance, Control, and Dynamics*, Vol 24., No. 1, pp. 23-34.

[5] Shen, H. and P. Tsiotras, Aug. 16-18, 1999, "Satellite Attitude Control and Power Tracking with Momentum Wheels," AAS/AIAA *Astrodynamics Specialist Conference, AAS Paper 99-317*, Girdwood, AK.

[6] Roes, J. B., 1961, "An Electro-Mechanical Energy Storage System for Space Application," *Progress in Astronautics and Rocketry*.

[7] Anderson, W. and C. Keckler, 1973, "Integrated Power/Attitude Control System (IPACS) for Space Application," *Proceedings of the 5th IFAC Symposium on Automatic Control in Space*.

[8] Notti, J.E., W. Schmill, W. Klein, and A. Cormack III, 1974, "Integrated Power/Attitude Control System (IPACS) Study: Volume II – Conceptual Designs," *Tech. Rep. NASA CR-2384*, Downey, CA, Rockwell International Space Division.

[9] Anand, D., J.A. Kirk, R.B. Amood, P.A. Studer, and G.E. Rodriguez, 1986, "System Considerations for Magnetically Suspended Flywheel Systems," *Proceedings of the 21st Intersociety Energy Conversion Engineering Conference*, Vol. 3, pp.1829-1833.

[10] Downer, J., D. Eisenhaure, R. Hockney, B. Johnson, and S.O. Odea, 1985, "Magnetic Suspension Design Options for Satellite Attitude Control and Energy Storage," *Proceedings of the 20th Intersociety Energy Conversion Engineering Conference*, Vol. 2, pp. 424-430.

[11] Flatley, T., 1985, "Tetrahedron Array of Reaction Wheels for Attitude Control and Energy Storage," *Proceedings of the 21st Intersociety Energy Conversion Engineering Conference*, Vol. 2, pp. 2353-2360.

[13] Rodriguez, G.E., P.A. Studer, and D.A. Baer, 1983,"Assessment of Flywheel Energy Storage of Spacecraft Power Systems," *Tech. Rep. NASA TM-85061*, NASA Goddard Space Flight Center, Greenbelt, MD.

[14] Simon, W. and K.Van Tassel, 1985, "Inertial Energy Storage for Advanced Space Station Applications," *Proceedings of the 21st Intersociety Energy Conversion Engineering Conference*, Vol. 2, pp. 337-342.

[15] Richie, D. J., P. Tsiotras, and J. L.Fausz, June 25-27, 2001, "Simultaneous Attitude Control and Energy Storage Using VSCMGs: Theory and Simulation," *Proceedings of the American Control Conference*, Arlington, VA, pp. 3973-3979.

[16] Schaub, H., S.R. Vadali, and J.L. Junkins, 1998, "Feedback Control Law for Variable Speed Control Moment Gyros," *Journal of the Astronautical Sciences*, Vol. 46, No. 3, pp.307-328.

[17] Ford, K. A. and Chris Hall, 2000, "Singular Direction Avoidance Steering for Control-Moment Gyros," *AIAA Journal of Guidance, Control, and Dynamics*, Vol 23, No. 4, pp. 648-656.

[18] Oh, H.S. and S. R. Vadali, 1994, "Feedback Control and Steering Laws for Spacecraft Using Single Gimbal Control Moment Gyros," *Journal of the Astronautical Sciences*, Vol. 39, No. 2, pp. 183-203.

[19] Horn, R. and C. R. Johnson, 1985, *Matrix Analysis*, United Kingdom, Cambridge University Press.

[20] Schaub, H. and J.L. Junkins, 2000, "Singularity Avoidance Using Null Motion and Variable-Speed Control Moment Gyros," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 1, pp. 11-16.

[21] Schaub, H., May 1998, *Novel Coordinates for Nonlinear Multibody Motion with Applications to Spacecraft Dynamics and Control*, Ph.D. Thesis, Arlington, TX, Texas A and M University.

[22] Biran, A. and M.Breiner, 1999, *MATLAB 5 for Engineers*, Harlow, England, Addison Wesley.