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ADAPTIVE POSITION AND ATTITUDE TRACKING CONTROLLER FOR SATELLITE PROXIMITY OPERATIONS USING DUAL QUATERNIONS

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In this paper, we propose a nonlinear adaptive position and attitude tracking controller for satellite proximity operations. This controller requires no information about the mass and inertia matrix of the satellite, and takes into account the gravitational force, the gravity-gradient torque, the perturbing force due to Earth's oblateness, and other constant – but otherwise unknown – disturbance forces and torques. We give sufficient conditions on the reference motion for mass and inertia matrix identification. The controller is shown to be *almost* globally asymptotically stable and can handle large error angles and displacements. Unit dual quaternions are used to simultaneously represent the absolute and relative attitude and position of the satellites, resulting in a compact controller representation.

INTRODUCTION

Several organizations around the world are looking at satellite proximity operations as an enabling technology for space missions such as on-orbit satellite inspection, health monitoring, surveillance, servicing, refueling, and optical interferometry.[1,3,4] One of the biggest challenges introduced by this technology is the need to accurately and simultaneously track time-varying relative position and attitude references in order to avoid collisions between the satellites and achieve mission objectives.

The problem of deriving control laws for satellite proximity operations has a long history. Among the several results in this area, here we mention the work in Ref. 4 where, using the vectrix formalism, nonlinear control and adaptation laws were designed ensuring *almost* global asymptotic convergence of the position and attitude errors, despite the presence of unknown mass and inertia parameters. However, the controller in Ref. 4 is a 392nd-order dynamic compensator, which limits its applicability to satellites with limited on-board computational resources. In Ref. 3, three different nonlinear position and attitude controllers for spacecraft formation flying are presented. All controllers require full knowledge of the mass and inertia matrix of the satellite. In Ref. **?**, a relative position and attitude tracking controller that requires no linear and angular velocity measurements and no mass and inertia matrix information is presented. However, as explained in Ref. **?**, if the reference trajectory is not sufficiently exciting, this controller cannot guarantee that the relative position and attitude errors will converge to zero. In Ref. 1, an adaptive terminal sliding-mode *pose* (i.e., position and attitude) tracking controller is proposed based on dual quaternions that does not require full knowledge of the mass and inertia matrix of the spacecraft. This controller takes into account the gravitational force, the gravity-gradient torque, constant – but otherwise unknown – disturbance forces and torques, but not the perturbing force due to J_2. Moreover, this controller requires a priori knowledge of upper bounds on the mass, on the maximum eigenvalue of the inertia matrix, on the constant but otherwise unknown disturbance forces and torques, on the desired relative linear and angular velocity between the spacecraft and its first derivative, on the linear and angular velocity of the chaser spacecraft with respect to the inertial frame, and

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on the position of the chaser spacecraft with respect to the inertial frame. In addition, the convergence region is not specified and no conditions for mass and inertia matrix identification are given.

Similarly to Ref. 1, in this paper we propose an adaptive pose-tracking controller based on dual quaternions.[5] However, unlike Ref. 1, our controller does not require a priori knowledge of any upper bounds on the system parameters or states, and takes into account the perturbing force due to J_2, which is typically the largest perturbing force on a satellite below GEO.[6] In addition, our controller is proven to be *almost* globally asymptotically stable and has only as many states as unknown parameters. Additional analysis allows us to provide sufficient conditions for mass and inertia matrix identification. Although these conditions are not needed for stability, they can be useful to design maneuvers to identify these parameters, if needed (e.g., after a docking maneuver).

Dual quaternions provide a compact way to represent the attitude and position of a body. They are an extension of classical quaternions. Dual quaternions are closely related to Chasles theorem, which states that the general displacement of a rigid body can be represented by a rotation about an axis (called the *screw axis*) and a translation along that axis, creating a screw-like motion.[7,8] A list of successful applications of dual quaternions is given in Ref. 15. Compared to other mathematical representations, such as homogeneous transformations, quaternion/vector pairs, and Lie algebraic methods, dual quaternions have been found to be the most compact and efficient way to represent screw motion in robotic kinematic chains.[7,22,21] Moreover, it has also been shown that combined position and attitude control laws based on dual quaternions automatically take into account the natural coupling between the rotational and translational motion.[12,13] Additionally, dual quaternions allow combined position and attitude control laws to be written compactly as a single control law. However, the property that makes dual quaternions most attractive is the fact that the combined translational and rotational kinematic and dynamic equations of motion written in terms of dual quaternions have the same form as the rotational-only kinematic and dynamic equations of motion written in terms of quaternions. Hence, as has been recently shown,[15,16,5] it is often relatively straightforward to extend an attitude controller with some desirable properties into a combined position *and* attitude controller with equivalent desirable properties, by almost simply substituting quaternions with dual quaternions in the attitude-only quaternion control law.

This technique for developing combined position and attitude controllers from existing attitude controllers has some advantages over techniques based on the special Euclidean group $\mathrm{SE}(3)$, where rotations are represented directly by rotation matrices.[23,24,25] In the latter, asymptotically stability of the combined rotational and translational motion is proven by either defining two different error functions for the position and attitude error[25] or, in two steps, by first proving the asymptotical stability of the rotational motion before the asymptotical stability of the translational motion can be proven[23] (note that the translational motion depends on the rotational motion). In our approach, a single error function, the *error dual quaternion* (defined by analogy to the classical rotation error quaternion), is used to represent the combined position and attitude error. Moreover, the asymptotic stability of the combined rotational and translational motion is proven in one step by using a Lyapunov function with the same form as the Lyapunov function used to prove the asymptotic stability of the rotational controller. On the other hand, whereas quaternions produce two closed-loop equilibrium points (since quaternions cover $\mathrm{SO}(3)$ twice[26]) both representing the identity rotation matrix, rotation matrices produce a minimum of four closed-loop equilibrium points,[24,23] only one of which is the identity rotation matrix. On the negative side, dual quaternions inherit the so-called *unwinding phenomenon* from classical quaternions.[27] Solutions for this problem are known and suggested in this paper. Finally, note that the tracking controllers proposed in Refs. 23 and 24 require the true mass and inertia matrix.

The paper is organized as follows. First, unit quaternions and unit dual quaternions are introduced. Then, the relative kinematic and dynamic equations for satellite proximity operations written in terms of dual quaternions are derived. In the next section, the adaptive attitude and position tracking controller for satellite proximity operations is deduced and proved to be *almost* globally asymptotically stable. Then, sufficient conditions for mass and inertia matrix identification are given. Finally, the proposed controller is analyzed and validated through a numerical example.

MATHEMATICAL PRELIMINARIES

Most of the mathematical preliminaries in this section can be found in Refs. 15, 16, and 5. For the benefit of the reader, we summarize here the main properties of quaternions and dual quaternions that are essential for the developments later on in the paper.

Quaternions

A quaternion is defined as $q = q_1 i + q_2 j + q_3 k + q_4$, where q_1, q_2, q_3, $q_4 \in \mathbb{R}$ and i, j, and k satisfy $i^2 = j^2 = k^2 = -1$, $i = jk = -kj$, $j = ki = -ik$, and $k = ij = -ji$.[12] It can also be represented as the ordered pair $q = (\bar{q}, q_4)$, where $\bar{q} = [q_1\ q_2\ q_3]^\intercal \in \mathbb{R}^3$ is the *vector part* of the quaternion and $q_4 \in \mathbb{R}$ is the *scalar part* of the quaternion. *Vector quaternions* and *scalar quaternions* are quaternions with zero scalar part and vector part, respectively. The set of quaternions, vector quaternions, and scalar quaternions will be denoted by $\mathbb{H} = \{q : \ q = q_1 i + q_2 j + q_3 k + q_4,\ q_1, q_2, q_3, q_4 \in \mathbb{R}\}$, $\mathbb{H}^v = \{q \in \mathbb{H} : \ q_4 = 0\}$, and $\mathbb{H}^s = \{q \in \mathbb{H} : \ q_1 = q_2 = q_3 = 0\}$, respectively.

The basic operations on quaternions are defined as follows:

$$\text{Addition: } a + b = (\bar{a} + \bar{b}, a_4 + b_4),$$
$$\text{Multiplication by a scalar: } \lambda a = (\lambda \bar{a}, \lambda a_4),$$
$$\text{Multiplication: } ab = (a_4 \bar{b} + b_4 \bar{a} + \bar{a} \times \bar{b}, a_4 b_4 - \bar{a} \cdot \bar{b}),$$
$$\text{Conjugation: } a^* = (-\bar{a}, a_4),$$
$$\text{Dot product: } a \cdot b = \tfrac{1}{2}(a^* b + b^* a) = \tfrac{1}{2}(a b^* + b a^*) = (\bar{0}, a_4 b_4 + \bar{a} \cdot \bar{b}),$$
$$\text{Cross product: } a \times b = \tfrac{1}{2}(ab - b^* a^*) = (b_4 \bar{a} + a_4 \bar{b} + \bar{a} \times \bar{b}, 0),$$
$$\text{Norm: } \|a\|^2 = a a^* = a^* a = a \cdot a = (\bar{0}, a_4^2 + \bar{a} \cdot \bar{a}),$$
$$\text{Scalar part: } \mathrm{sc}(a) = (\bar{0}, a_4) \in \mathbb{H}^s,$$
$$\text{Vector part: } \mathrm{vec}(a) = (\bar{a}, 0) \in \mathbb{H}^v,$$

where $a, b \in \mathbb{H}$, $\lambda \in \mathbb{R}$, and $\bar{0} = [0\ 0\ 0]^\intercal$. Note that the quaternion multiplication is not commutative.

The sets $\mathbb{H}$, $\mathbb{H}^v$, and $\mathbb{H}^s$ are isomorphic as vector spaces to $\mathbb{R}^4$, $\mathbb{R}^3$, and $\mathbb{R}$, respectively. Under these isomorphisms, the square of the quaternion norm and the dot product on $\mathbb{H}$ correspond to the square of the Euclidean norm and to the dot (inner) product on $\mathbb{R}^4$, respectively. Also, we will often identify, with a slight abuse of notation, q_4 with $(\bar{0}, q_4)$, when this is clear from the context.

The multiplication of a matrix $M \in \mathbb{R}^{4\times 4}$ with a quaternion $q \in \mathbb{H}$ will be defined as $M * q = (M_{11}\bar{q} + M_{12} q_4, M_{21}\bar{q} + M_{22} q_4) \in \mathbb{H}$, where

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix},$$

$M_{11} \in \mathbb{R}^{3\times 3}$, $M_{12} \in \mathbb{R}^{3\times 1}$, $M_{21} \in \mathbb{R}^{1\times 3}$, and $M_{22} \in \mathbb{R}$. This definition is analogous to the multiplication of a 4-by-4 matrix with a 4-dimensional vector.

It can be shown that the following properties follow from the previous definitions:

$$a \cdot (bc) = b \cdot (a c^*) = c \cdot (b^* a) \in \mathbb{H}^s, \quad a, b, c \in \mathbb{H},$$
$$\|ab\| = \|a\|\|b\|, \quad a, b \in \mathbb{H},$$
$$(M * a) \cdot b = a \cdot (M^\intercal * b), \quad a, b \in \mathbb{H}, \quad M \in \mathbb{R}^{4\times 4},$$
$$a \cdot (b \times c) = b \cdot (c \times a) = c \cdot (a \times b), \quad a, b, c \in \mathbb{H}^v,$$
$$a \times a = 0, \quad a \in \mathbb{H}^v,$$
$$a \times b = -b \times a, \quad a, b \in \mathbb{H}^v,$$
$$\|a^*\| = \|a\|, \quad a \in \mathbb{H},$$
$$|a \cdot b| \leq \|a\|\|b\|, \quad a, b \in \mathbb{H}.$$

Finally, the $\mathcal{L}_\infty$-norm of a function $u : [0,\infty) \to \mathbb{H}$ is defined as $\|u\|_\infty = \sup_{t\geq 0} \|u(t)\|$. Moreover, the quaternion $u \in \mathcal{L}_\infty$, if and only if $\|u\|_\infty < \infty$.

Attitude Representation with Unit Quaternions. The relative orientation of a body frame with respect to the inertial frame can be represented by the *unit* quaternion $q_{\mathrm{B/I}} = \Big(\sin(\frac{\phi}{2})\bar{n}, \cos(\frac{\phi}{2})\Big)$, where the body frame is said to be rotated with respect to the inertial frame about the unit vector $\bar{n}$ by an angle ϕ. Note that $q_{\mathrm{B/I}}$ is a unit quaternion because it belongs to the set $\mathbb{H}^u = \{q \in \mathbb{H} : \; q \cdot q = 1\}$. The body coordinates of a vector, $\bar{v}^{\mathrm{B}}$, can be calculated from the inertial coordinates of that same vector, $\bar{v}^{\mathrm{I}}$, and vice-versa, through $v^{\mathrm{B}} = q_{\mathrm{B/I}}^* v^{\mathrm{I}} q_{\mathrm{B/I}}$ and $v^{\mathrm{I}} = q_{\mathrm{B/I}} v^{\mathrm{B}} q_{\mathrm{B/I}}^*$, where $v^{\mathrm{B}} = (\bar{v}^{\mathrm{B}}, 0)$ and $v^{\mathrm{I}} = (\bar{v}^{\mathrm{I}}, 0)$.

Quaternion Representation of the Rotational Kinematic Equations. The rotational kinematic equations of the body frame and of a frame with some desired attitude, both with respect to the inertial frame and represented by the unit quaternions $q_{\mathrm{B/I}}$ and $q_{\mathrm{D/I}}$, respectively, are given by

$$\dot{q}_{\mathrm{B/I}}=\tfrac{1}{2}q_{\mathrm{B/I}}\omega^{\mathrm{B}}_{\mathrm{B/I}}=\tfrac{1}{2}\omega^{\mathrm{I}}_{\mathrm{B/I}}q_{\mathrm{B/I}} \;\text{ and }\; \dot{q}_{\mathrm{D/I}}=\tfrac{1}{2}q_{\mathrm{D/I}}\omega^{\mathrm{D}}_{\mathrm{D/I}}=\tfrac{1}{2}\omega^{\mathrm{I}}_{\mathrm{D/I}}q_{\mathrm{D/I}}, \tag{1}$$

where $\omega^{\mathrm{X}}_{\mathrm{Y/Z}} = (\bar{\omega}^{\mathrm{X}}_{\mathrm{Y/Z}}, 0)$, and $\bar{\omega}^{\mathrm{X}}_{\mathrm{Y/Z}}$ is the angular velocity of the Y-frame with respect to the Z-frame expressed in the X-frame. The error quaternion

$$q_{\mathrm{B/D}} = q^*_{\mathrm{D/I}} q_{\mathrm{B/I}} \tag{2}$$

is the unit quaternion that rotates the desired frame onto the body frame. By differentiating Eq. (3) and using Eq. (2), the kinematic equations of the error quaternion turn out to be

$$\dot{q}_{\mathrm{B/D}} = \tfrac{1}{2}q_{\mathrm{B/D}}\omega^{\mathrm{B}}_{\mathrm{B/D}} = \tfrac{1}{2}\omega^{\mathrm{D}}_{\mathrm{B/D}}q_{\mathrm{B/D}}, \tag{3}$$

where $\omega^{\mathrm{B}}_{\mathrm{B/D}} = \omega^{\mathrm{B}}_{\mathrm{B/I}} - \omega^{\mathrm{B}}_{\mathrm{D/I}}$ (and $\omega^{\mathrm{D}}_{\mathrm{B/D}} = \omega^{\mathrm{D}}_{\mathrm{B/I}} - \omega^{\mathrm{D}}_{\mathrm{D/I}}$).

Dual Quaternions

A dual quaternion is defined as $\hat{q} = q_r + \epsilon q_d$, where ϵ is the *dual unit* defined by $\epsilon^2 = 0$ and $\epsilon \neq 0$. The quaternions $q_r, q_d \in \mathbb{H}$ are called the *real part* and the *dual part* of the dual quaternion, respectively.

Dual vector quaternions and *dual scalar quaternions* are dual quaternions formed from vector quaternions (i.e., $q_r, q_d \in \mathbb{H}^v$) and scalar quaternions (i.e., $q_r, q_d \in \mathbb{H}^s$), respectively. The set of dual quaternions, dual scalar quaternions, dual vector quaternions, and dual scalar quaternions with zero dual part will be denoted by $\mathbb{H}_d = \{\hat{q} : \; \hat{q} = q_r + \epsilon q_d, \; q_r, q_d \in \mathbb{H}\}$, $\mathbb{H}^s_d = \{\hat{q} : \; \hat{q} = q_r + \epsilon q_d, \; q_r, q_d \in \mathbb{H}^s\}$, $\mathbb{H}^v_d = \{\hat{q} : \; \hat{q} = q_r + \epsilon q_d, \; q_r, q_d \in \mathbb{H}^v\}$, and $\mathbb{H}^r_d = \{\hat{q} : \; \hat{q} = q_r + \epsilon(\bar{0}, 0), \; q_r \in \mathbb{H}^s\}$, respectively.

The basic operations on dual quaternions are defined as follows:[13,1]

$$\begin{gathered}
\text{Addition: } \hat{a} + \hat{b} = (a_r + b_r) + \epsilon(a_d + b_d), \\
\text{Multiplication by a scalar: } \lambda\hat{a} = (\lambda a_r) + \epsilon(\lambda a_d), \\
\text{Multiplication: } \hat{a}\hat{b} = (a_r b_r) + \epsilon(a_r b_d + a_d b_r), \\
\text{Conjugation: } \hat{a}^* = a_r^* + \epsilon a_d^*, \\
\text{Swap: } \hat{a}^{\mathsf{s}} = a_d + \epsilon a_r, \\
\text{Dot product: } \hat{a} \cdot \hat{b}=\tfrac{1}{2}(\hat{a}^*\hat{b} + \hat{b}^*\hat{a})=\tfrac{1}{2}(\hat{a}\hat{b}^* + \hat{b}\hat{a}^*)=a_r \cdot b_r + \epsilon(a_d \cdot b_r + a_r \cdot b_d) \in \mathbb{H}^s_d, \\
\text{Cross product: } \hat{a} \times \hat{b} = \tfrac{1}{2}(\hat{a}\hat{b} - \hat{b}^*\hat{a}^*) = a_r \times b_r + \epsilon(a_d \times b_r + a_r \times b_d) \in \mathbb{H}^v_d, \\
\text{Dual norm: } \|\hat{a}\|_d^2 = \hat{a}\hat{a}^* = \hat{a}^*\hat{a} = \hat{a} \cdot \hat{a} = (a_r \cdot a_r) + \epsilon(2a_r \cdot a_d) \in \mathbb{H}^s_d, \\
\text{Scalar part: } \mathrm{sc}(\hat{a}) = \mathrm{sc}(a_r) + \epsilon\,\mathrm{sc}(a_d) \in \mathbb{H}^s_d, \\
\text{Vector part: } \mathrm{vec}(\hat{a}) = \mathrm{vec}(a_r) + \epsilon\,\mathrm{vec}(a_d) \in \mathbb{H}^v_d,
\end{gathered}$$

where $\hat{a}, \hat{b} \in \mathbb{H}_d$ and $\lambda \in \mathbb{R}$. Note that the dual quaternion multiplication is not commutative.

Since the dot product and dual norm of dual quaternions yield in general a dual number (and not a real number), we define the *dual quaternion norm*[28,1] as $\|\hat{a}\|^2 = \hat{a} \circ \hat{a}$, where $\circ$ denotes the *dual quaternion circle product* given by $\hat{a} \circ \hat{b} = a_r \cdot b_r + a_d \cdot b_d$, where $\hat{a}, \hat{b} \in \mathbb{H}_d$.

The sets $\mathbb{H}_d$, $\mathbb{H}_d^v$, $\mathbb{H}_d^s$, and $\mathbb{H}_d^r$ are isomorphic as vector spaces to $\mathbb{R}^8$, $\mathbb{R}^6$, $\mathbb{R}^2$, and $\mathbb{R}$, respectively. Under these isomorphisms, the square of the dual quaternion norm and the circle product on $\mathbb{H}_d$ correspond to the square of the Euclidean norm and to the dot (inner) product on $\mathbb{R}^8$, respectively. Also, we will often identify, with a slight abuse of notation, q_4 with $(\bar{0}, q_4) + \epsilon(\bar{0}, 0)$, when this is clear from the context.

The multiplication of a matrix $M \in \mathbb{R}^{8\times 8}$ with a dual quaternion $\hat{q} \in \mathbb{H}_d$ will be defined as $M \star \hat{q} = (M_{11} * q_r + M_{12} * q_d) + \epsilon(M_{21} * q_r + M_{22} * q_d)$, where

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \quad M_{11}, M_{12}, M_{21}, M_{22} \in \mathbb{R}^{4\times 4}.$$

This definition is analogous to the multiplication of a 8-by-8 matrix with a 8-dimensional vector.

It can be shown that the following properties[16] follow from the previous definitions:

$$\hat{a}\circ(\hat{b}\hat{c}) = \hat{b}^{\mathsf{s}}\circ(\hat{a}^{\mathsf{s}}\hat{c}^*) = \hat{c}^{\mathsf{s}}\circ(\hat{b}^*\hat{a}^{\mathsf{s}}) \in \mathbb{R}, \quad \hat{a}, \hat{b}, \hat{c} \in \mathbb{H}_d, \tag{4}$$

$$\hat{a}\circ(\hat{b}\times\hat{c}) = \hat{b}^{\mathsf{s}}\circ(\hat{c}\times\hat{a}^{\mathsf{s}}) = \hat{c}^{\mathsf{s}}\circ(\hat{a}^{\mathsf{s}}\times\hat{b}), \quad \hat{a}, \hat{b}, \hat{c} \in \mathbb{H}_d^v, \tag{5}$$

$$\hat{a} \times \hat{a} = 0, \quad \hat{a} \in \mathbb{H}_d^v, \tag{6}$$

$$\hat{a} \times \hat{b} = -\hat{b} \times \hat{a}, \quad \hat{a}, \hat{b} \in \mathbb{H}_d^v, \tag{7}$$

$$\hat{a}^{\mathsf{s}} \circ \hat{b}^{\mathsf{s}} = \hat{a} \circ \hat{b}, \quad \hat{a}, \hat{b} \in \mathbb{H}_d, \tag{8}$$

$$\|\hat{a}^{\mathsf{s}}\| = \|\hat{a}\|, \quad \hat{a} \in \mathbb{H}_d, \tag{9}$$

$$\|\hat{a}^*\| = \|\hat{a}\|, \quad \hat{a} \in \mathbb{H}_d, \tag{10}$$

$$(M \star \hat{a}) \circ \hat{b} = \hat{a} \circ (M^{\mathsf{T}} \star \hat{b}), \ \hat{a}, \hat{b} \in \mathbb{H}_d, \ M \in \mathbb{R}^{8\times 8}, \tag{11}$$

$$|\hat{a} \circ \hat{b}| \leq \|\hat{a}\|\|\hat{b}\|, \quad \hat{a}, \hat{b} \in \mathbb{H}_d, \tag{12}$$

$$\|\hat{a}\hat{b}\| \leq \sqrt{3/2}\,\|\hat{a}\|\|\hat{b}\|, \quad \hat{a}, \hat{b} \in \mathbb{H}_d. \tag{13}$$

Finally, the $\mathcal{L}_\infty$-norm of a function $\hat{u} : [0, \infty) \to \mathbb{H}_d$ is defined as $\|\hat{u}\|_\infty = \sup_{t\geq 0} \|\hat{u}(t)\|$. Moreover, the dual quaternion $\hat{u} \in \mathcal{L}_\infty$, if and only if $\|\hat{u}\|_\infty < \infty$.

Attitude and Position Representation with Unit Dual Quaternions. The position and orientation (i.e., pose) of a body frame with respect to the inertial frame can be represented by a unit quaternion $q_{\text{B/I}} \in \mathbb{H}^u$ and by a translation vector $\bar{r}_{\text{B/I}} \in \mathbb{R}^3$. Alternatively, the pose of the body frame with respect to the inertial frame can be represented more compactly by the *unit* dual quaternion[7] $\hat{q}_{\text{B/I}} = q_{\text{B/I}} + \epsilon\frac{1}{2}r^{\text{I}}_{\text{B/I}}q_{\text{B/I}} = q_{\text{B/I}} + \epsilon\frac{1}{2}q_{\text{B/I}}r^{\text{B}}_{\text{B/I}}$, where $r^{\text{X}}_{\text{Y/Z}} = (\bar{r}^{\text{X}}_{\text{Y/Z}}, 0)$ and $\bar{r}^{\text{X}}_{\text{Y/Z}} = [x^{\text{X}}_{\text{Y/Z}}\ y^{\text{X}}_{\text{Y/Z}}\ z^{\text{X}}_{\text{Y/Z}}]^{\mathsf{T}}$ is the translation vector from the origin of the Z-frame to the origin of the Y-frame expressed in the X-frame. Note that $\hat{q}_{\text{B/I}}$ is a unit dual quaternion because it belongs to the set[15] $\mathbb{H}_d^u = \{\hat{q} \in \mathbb{H}_d : \ \hat{q} \cdot \hat{q} = \hat{q}\hat{q}^* = \hat{q}^*\hat{q} = \|\hat{q}\|_d = 1\}$.

Lemma 1. *The unit dual quaternion* $\hat{q}_{\text{Y/Z}} = q_{\text{Y/Z}} + \epsilon\frac{1}{2}q_{\text{Y/Z}}r^{\text{Y}}_{\text{Y/Z}} \in \mathcal{L}_\infty$*, if and only if* $r^{\text{Y}}_{\text{Y/Z}} \in \mathcal{L}_\infty$.

Proof. If $\hat{q}_{\text{Y/Z}} \in \mathcal{L}_\infty$, then $q_{\text{Y/Z}}r^{\text{Y}}_{\text{Y/Z}} \in \mathcal{L}_\infty$. Note that the unit quaternion $q_{\text{Y/Z}} \in \mathcal{L}_\infty$ by definition. Moreover, since $\|q_{\text{Y/Z}}r^{\text{Y}}_{\text{Y/Z}}\| = \|r^{\text{Y}}_{\text{Y/Z}}\|$, this also implies that $r^{\text{Y}}_{\text{Y/Z}} \in \mathcal{L}_\infty$. On the other hand, it is trivial to see that if $q_{\text{Y/Z}}, r^{\text{Y}}_{\text{Y/Z}} \in \mathcal{L}_\infty$, then $\hat{q}_{\text{Y/Z}} = q_{\text{Y/Z}} + \epsilon\frac{1}{2}q_{\text{Y/Z}}r^{\text{Y}}_{\text{Y/Z}} \in \mathcal{L}_\infty$ as well. □

Dual Quaternion Representation of the Rotational and Translational Kinematic Equations. The rigid body kinematic equations of the body frame and of a frame with some desired position and attitude, both with respect to the inertial frame and represented by the unit dual quaternions $\hat{q}_{\text{B/I}}$ and $\hat{q}_{\text{D/I}} = q_{\text{D/I}} + \epsilon\frac{1}{2}r^{\text{I}}_{\text{D/I}}q_{\text{D/I}} = q_{\text{D/I}} + \epsilon\frac{1}{2}q_{\text{D/I}}r^{\text{D}}_{\text{D/I}}$, respectively, are given by[7]

$$\dot{\hat{q}}_{\text{B/I}} = \tfrac{1}{2}\hat{\omega}^{\text{I}}_{\text{B/I}}\hat{q}_{\text{B/I}} = \tfrac{1}{2}\hat{q}_{\text{B/I}}\hat{\omega}^{\text{B}}_{\text{B/I}} \ \text{ and } \ \dot{\hat{q}}_{\text{D/I}} = \tfrac{1}{2}\hat{\omega}^{\text{I}}_{\text{D/I}}\hat{q}_{\text{D/I}} = \tfrac{1}{2}\hat{q}_{\text{D/I}}\hat{\omega}^{\text{D}}_{\text{D/I}}, \tag{14}$$

where $\hat{\omega}^{\text{X}}_{\text{Y/Z}}$ is the *dual velocity* of the Y-frame with respect to the Z-frame expressed in the X-frame, so that $\hat{\omega}^{\text{X}}_{\text{Y/Z}} = \omega^{\text{X}}_{\text{Y/Z}} + \epsilon(v^{\text{X}}_{\text{Y/Z}} + \omega^{\text{X}}_{\text{Y/Z}} \times r^{\text{X}}_{\text{X/Y}})$, $v^{\text{X}}_{\text{Y/Z}} = (\bar{v}^{\text{X}}_{\text{Y/Z}}, 0)$, and $\bar{v}^{\text{X}}_{\text{Y/Z}}$ is the linear velocity of the Y-frame with respect to the Z-frame expressed in the X-frame.

By direct analogy to Eq. (3), the *dual error quaternion*[1,12] is defined as

$$\hat{q}_{\text{B/D}} \triangleq \hat{q}^*_{\text{D/I}}\hat{q}_{\text{B/I}} = q_{\text{B/D}} + \epsilon\tfrac{1}{2}q_{\text{B/D}}r^{\text{B}}_{\text{B/D}}, \tag{15}$$

where $r^{\text{B}}_{\text{B/D}} = r^{\text{B}}_{\text{B/I}} - r^{\text{B}}_{\text{D/I}}$. As illustrated in Figure 1, the dual error quaternion represents the rotation ($q_{\text{B/D}}$) and the translation ($r^{\text{B}}_{\text{B/D}}$) necessary to align the desired frame with the body frame. It can be shown[15] that $\hat{q}_{\text{B/D}}$ is also a unit dual quaternion. By differentiating Eq. (19) and using Eq. (18), the kinematic equations of the

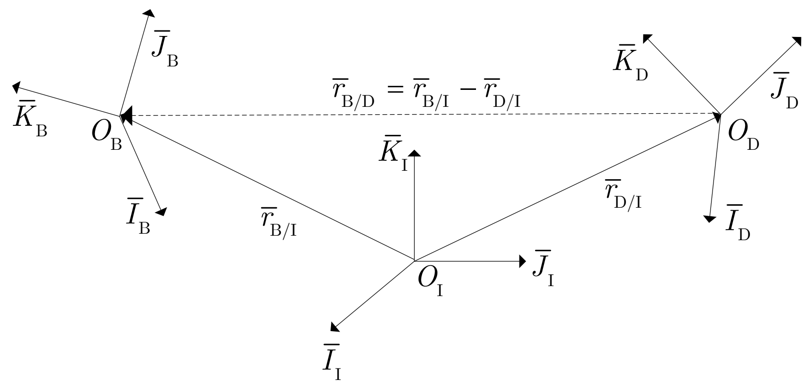

Figure 1. Relation between frames.

dual error quaternion turn out to be[1]

$$\dot{\hat{q}}_{\text{B/D}} = \tfrac{1}{2}\hat{q}_{\text{B/D}}\hat{\omega}^{\text{B}}_{\text{B/D}} = \tfrac{1}{2}\hat{\omega}^{\text{D}}_{\text{B/D}}\hat{q}_{\text{B/D}}, \tag{16}$$

where $\hat{\omega}^{\text{B}}_{\text{B/D}} = \hat{\omega}^{\text{B}}_{\text{B/I}} - \hat{\omega}^{\text{B}}_{\text{D/I}}$ is the *dual relative velocity* between the body frame and the desired frame expressed in the body frame. Note that $\hat{\omega}^{\text{B}}_{\text{D/I}} = \hat{q}^*_{\text{B/D}}\hat{\omega}^{\text{D}}_{\text{D/I}}\hat{q}_{\text{B/D}}$ and $\hat{\omega}^{\text{D}}_{\text{B/I}} = \hat{q}_{\text{B/D}}\hat{\omega}^{\text{B}}_{\text{B/I}}\hat{q}^*_{\text{B/D}}$. Note also that the kinematic equations of the dual error quaternion, Eq. (20), and of the error quaternion, Eq. (4), have the same form.

DUAL QUATERNION REPRESENTATION OF THE RELATIVE DYNAMIC EQUATIONS FOR SATELLITE PROXIMITY OPERATIONS

The dual quaternion representation of the rigid body dynamic equations assuming constant (or slowly varying) mass and inertia matrix is given by[1,16]

$$(\dot{\hat{\omega}}^{\text{B}}_{\text{B/D}})^{\text{s}} = (M^{\text{B}})^{-1}\star\Big(\hat{f}^{\text{B}} - (\hat{\omega}^{\text{B}}_{\text{B/D}} + \hat{\omega}^{\text{B}}_{\text{D/I}})\times\big(M^{\text{B}}\star((\hat{\omega}^{\text{B}}_{\text{B/D}})^{\text{s}} + (\hat{\omega}^{\text{B}}_{\text{D/I}})^{\text{s}})\big) - M^{\text{B}}\star(\hat{q}^*_{\text{B/D}}\dot{\hat{\omega}}^{\text{D}}_{\text{D/I}}\hat{q}_{\text{B/D}})^{\text{s}} - M^{\text{B}}\star(\hat{\omega}^{\text{B}}_{\text{D/I}}\times\hat{\omega}^{\text{B}}_{\text{B/D}})^{\text{s}}\Big), \tag{17}$$

where $\hat{f}^{\text{B}} = f^{\text{B}} + \epsilon\tau^{\text{B}}$ is the total external *dual force* applied to the body about its center of mass expressed in body coordinates, $f^{\text{B}} = (\bar{f}^{\text{B}}, 0)$, $\bar{f}^{\text{B}}$ is the total external force vector applied to the body, $\tau^{\text{B}} = (\bar{\tau}^{\text{B}}, 0)$, and $\bar{\tau}^{\text{B}}$ is the total external moment vector applied to the body about its center of mass. Finally, $M^{\text{B}} \in \mathbb{R}^{8\times 8}$ is the *dual inertia matrix*[15] defined as

$$M^{\text{B}} = \begin{bmatrix} mI_3 & 0_{3\times 1} & 0_{3\times 3} & 0_{3\times 1} \\ 0_{1\times 3} & 1 & 0_{1\times 3} & 0 \\ 0_{3\times 3} & 0_{3\times 1} & \bar{I}^{\text{B}} & 0_{3\times 1} \\ 0_{1\times 3} & 0 & 0_{1\times 3} & 1 \end{bmatrix}, \quad I^{\text{B}} = \begin{bmatrix} \bar{I}^{\text{B}} & 0_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix}, \quad \bar{I}^{\text{B}} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}, \tag{18}$$

$\bar{I}^{\text{B}} \in \mathbb{R}^{3\times 3}$ is the mass moment of inertia of the body about its center of mass written in the body frame, and m is the mass of the body.

Note the similarity between the dual quaternion representation of the combined rotational and translational dynamic equations given by Eq. (21) and the quaternion representation of the rotational(-only) dynamic equations given by[1] $\dot{\omega}^{\rm B}_{\rm B/D}=(I^{\rm B})^{-1}*(\tau^{\rm B}-(\omega^{\rm B}_{\rm B/D}+\omega^{\rm B}_{\rm D/I})\times\big(I^{\rm B}*(\omega^{\rm B}_{\rm B/D}+\omega^{\rm B}_{\rm D/I})\big)-I^{\rm B}*(q^*_{\rm B/D}\dot{\omega}^{\rm D}_{\rm D/I}q_{\rm B/D})-I^{\rm B}*(\omega^{\rm B}_{\rm D/I}\times\omega^{\rm B}_{\rm B/D}))$.

For the case of a spacecraft in Earth orbit, we decompose the total external dual force acting on the body as follows:

$$\hat{f}^{\rm B} = \hat{f}^{\rm B}_g + \hat{f}^{\rm B}_{\nabla g} + \hat{f}^{\rm B}_{J_2} + \hat{f}^{\rm B}_d + \hat{f}^{\rm B}_c, \tag{19}$$

where $\hat{f}^{\rm B}_g = m\hat{a}^{\rm B}_g$, $\hat{a}^{\rm B}_g = a^{\rm B}_g + \epsilon 0$, $a^{\rm B}_g = (\bar{a}^{\rm B}_g, 0)$, $\bar{a}^{\rm B}_g$ is the gravitational acceleration given by

$$\bar{a}^{\rm B}_g = -\mu\frac{\bar{r}^{\rm B}_{\rm B/I}}{\|\bar{r}^{\rm B}_{\rm B/I}\|^3}, \tag{20}$$

$\mu = 398600.4418$ km^3/s^2 is Earth's gravitational parameter,[6] $\hat{f}^{\rm B}_{\nabla g} = 0 + \epsilon\tau^{\rm B}_{\nabla g}$, $\tau^{\rm B}_{\nabla g} = (\bar{\tau}^{\rm B}_{\nabla g}, 0)$, $\bar{\tau}^{\rm B}_{\nabla g}$ is the gravity-gradient torque given by[1]

$$\bar{\tau}^{\rm B}_{\nabla g} = 3\mu\frac{\bar{r}^{\rm B}_{\rm B/I}\times(\bar{I}^{\rm B}\bar{r}^{\rm B}_{\rm B/I})}{\|\bar{r}^{\rm B}_{\rm B/I}\|^5}, \tag{21}$$

$\hat{f}^{\rm B}_{J_2} = m\hat{a}^{\rm B}_{J_2}$, $\hat{a}^{\rm B}_{J_2} = a^{\rm B}_{J_2} + \epsilon 0$, $a^{\rm B}_{J_2} = (\bar{a}^{\rm B}_{J_2}, 0)$, $\bar{a}^{\rm B}_{J_2}$ is the perturbing acceleration due to J_2[29] given by

$$\bar{a}^{\rm I}_{J_2} = -\frac{3}{2}\frac{\mu J_2 R_e^2}{\|\bar{r}^{\rm I}_{\rm B/I}\|^4}\begin{bmatrix}(1-5(\frac{z^{\rm I}_{\rm B/I}}{\|\bar{r}^{\rm I}_{\rm B/I}\|})^2)\frac{x^{\rm I}_{\rm B/I}}{\|\bar{r}^{\rm I}_{\rm B/I}\|}\\(1-5(\frac{z^{\rm I}_{\rm B/I}}{\|\bar{r}^{\rm I}_{\rm B/I}\|})^2)\frac{y^{\rm I}_{\rm B/I}}{\|\bar{r}^{\rm I}_{\rm B/I}\|}\\(3-5(\frac{z^{\rm I}_{\rm B/I}}{\|\bar{r}^{\rm I}_{\rm B/I}\|})^2)\frac{z^{\rm I}_{\rm B/I}}{\|\bar{r}^{\rm I}_{\rm B/I}\|}\end{bmatrix}, \tag{22}$$

$J_2 = 0.0010826267$, $R_e = 6378.137$ km is Earth's mean equatorial radius,[6] $\hat{f}^{\rm B}_d = f^{\rm B}_d + \epsilon\tau^{\rm B}_d$ is the disturbance dual force, and $\hat{f}^{\rm B}_c = f^{\rm B}_c + \epsilon\tau^{\rm B}_c$ is the control dual force. We do not explicitly take into account other disturbance forces and torques due to, for example, atmospheric drag, solar radiation, and third-bodies. Instead, we will assume that $\hat{f}^{\rm B}_d$ is a constant (or slowly varying), but otherwise unknown, dual force that captures all neglected (but small) external forces and torques. For the sake of simplicity and compactness, it is more convenient to write $\hat{f}^{\rm B}_g$, $\hat{f}^{\rm B}_{\nabla g}$, and $\hat{f}^{\rm B}_{J_2}$ in terms of the dual inertia matrix as follows: $\hat{f}^{\rm B}_g = M^{\rm B}\star\hat{a}^{\rm B}_g$, $\hat{f}^{\rm B}_{\nabla g} = \frac{3\mu\hat{r}^{\rm B}_{\rm B/I}}{\|\hat{r}^{\rm B}_{\rm B/I}\|^5}\times(M^{\rm B}\star(\hat{r}^{\rm B}_{\rm B/I})^{\rm s})$, and $\hat{f}^{\rm B}_{J_2} = M^{\rm B}\star\hat{a}^{\rm B}_{J_2}$, where $\hat{r}^{\rm B}_{\rm B/I} = r^{\rm B}_{\rm B/I} + \epsilon 0$.

ADAPTIVE POSITION AND ATTITUDE TRACKING CONTROLLER

The main result of this paper is an adaptive pose-tracking controller for satellite proximity operations that requires no information about the mass and inertia matrix of the satellite. In particular, it requires no bounds on the mass and/or eigenvalues of the inertia matrix. The next theorem presents this controller and shows that it is *almost* globally asymptotically stable.

Theorem 1. *Consider the relative kinematic and dynamic equations given by Eq.* (20) *and Eq.* (21)*. Let the control dual force be defined by the feedback control law*

$$\begin{aligned}\hat{f}^{\rm B}_c =& -\widehat{M^{\rm B}}\star\hat{a}^{\rm B}_g - \frac{3\mu\hat{r}^{\rm B}_{\rm B/I}}{\|\hat{r}^{\rm B}_{\rm B/I}\|^5}\times(\widehat{M^{\rm B}}\star(\hat{r}^{\rm B}_{\rm B/I})^{\rm s}) - \widehat{M^{\rm B}}\star\hat{a}^{\rm B}_{J_2} - \widehat{\hat{f}^{\rm B}_d} - {\rm vec}\big(\hat{q}^*_{\rm B/D}(\hat{q}^{\rm s}_{\rm B/D}-\epsilon)\big) - K_d\star\hat{s}^{\rm s}\\ &+\hat{\omega}^{\rm B}_{\rm B/I}\times(\widehat{M^{\rm B}}\star(\hat{\omega}^{\rm B}_{\rm B/I})^{\rm s}) + \widehat{M^{\rm B}}\star(\hat{q}^*_{\rm B/D}\dot{\hat{\omega}}^{\rm D}_{\rm D/I}\hat{q}_{\rm B/D})^{\rm s} + \widehat{M^{\rm B}}\star(\hat{\omega}^{\rm B}_{\rm D/I}\times\hat{\omega}^{\rm B}_{\rm B/D})^{\rm s} - \widehat{M^{\rm B}}\star\Big(K_p\star\frac{\rm d}{{\rm d}t}(\hat{q}^*_{\rm B/D}(\hat{q}^{\rm s}_{\rm B/D}-\epsilon))\Big)^{\rm s},\end{aligned} \tag{23}$$

where

$$\hat{s} = \hat{\omega}^{\rm B}_{\rm B/D} + (K_p\star(\hat{q}^*_{\rm B/D}(\hat{q}^{\rm s}_{\rm B/D}-\epsilon)))^{\rm s}, \tag{24}$$

$$K_p = \begin{bmatrix}K_r & 0_{4\times4}\\0_{4\times4} & K_q\end{bmatrix},\quad K_d = \begin{bmatrix}K_v & 0_{4\times4}\\0_{4\times4} & K_\omega\end{bmatrix}, \tag{25}$$

$$K_r = \begin{bmatrix}\bar{K}_r & 0_{3\times1}\\0_{1\times3} & 0\end{bmatrix}, K_q = \begin{bmatrix}\bar{K}_q & 0_{3\times1}\\0_{1\times3} & 0\end{bmatrix}, K_v = \begin{bmatrix}\bar{K}_v & 0_{3\times1}\\0_{1\times3} & 0\end{bmatrix}, K_\omega = \begin{bmatrix}\bar{K}_\omega & 0_{3\times1}\\0_{1\times3} & 0\end{bmatrix}, \tag{26}$$

$\bar{K}_r, \bar{K}_q, \bar{K}_v, \bar{K}_\omega \in \mathbb{R}^{3\times 3}$ *are positive definite matrices,* $\widehat{M^{\text{B}}}$ *is an estimate of the dual inertia matrix updated according to*

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{v}(\widehat{M^{\text{B}}}) = K_i\big[\mathrm{h}(\hat{s}^{\mathsf{s}}, -(\hat{q}^*_{\text{B/D}}\dot{\hat{\omega}}^{\text{D}}_{\text{D/I}}\hat{q}_{\text{B/D}})^{\mathsf{s}} - (\hat{\omega}^{\text{B}}_{\text{D/I}} \times \hat{\omega}^{\text{B}}_{\text{B/D}})^{\mathsf{s}} + (K_p \star (\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon)))^{\mathsf{s}} + \hat{a}^{\text{B}}_g + \hat{a}^{\text{B}}_{J_2}) - \mathrm{h}((\hat{s} \times \hat{\omega}^{\text{B}}_{\text{B/I}})^{\mathsf{s}}, (\hat{\omega}^{\text{B}}_{\text{B/I}})^{\mathsf{s}}) + \mathrm{h}((\hat{s} \times \frac{3\mu\hat{r}^{\text{B}}_{\text{B/I}}}{\|\hat{r}^{\text{B}}_{\text{B/I}}\|^5})^{\mathsf{s}}, (\hat{r}^{\text{B}}_{\text{B/I}})^{\mathsf{s}})\big], \tag{27}$$

$K_i \in \mathbb{R}^{7\times 7}$ *is a positive definite matrix,* $\mathrm{v}(M^{\text{B}}) = [I_{11}\ I_{12}\ I_{13}\ I_{22}\ I_{23}\ I_{33}\ m]^{\mathsf{T}}$ *is a vectorized version of the dual inertia matrix* M^{B}*, the function* $\mathrm{h} : \mathbb{H}^v_d \times \mathbb{H}^v_d \to \mathbb{R}^7$ *is defined as* $\hat{a} \circ (M^B \star \hat{b}) = \mathrm{h}(\hat{a}, \hat{b})^{\mathsf{T}}\mathrm{v}(M^B) = \mathrm{v}(M^B)^{\mathsf{T}}\mathrm{h}(\hat{a}, \hat{b})$ *or, equivalently,* $\mathrm{h}(\hat{a}, \hat{b}) = [a_5b_5\ a_6b_5{+}a_5b_6\ a_7b_5{+}a_5b_7\ a_6b_6\ a_7b_6{+}a_6b_7\ a_7b_7\ a_1b_1{+}a_2b_2 {+}a_3b_3]^{\mathsf{T}}$, $\widehat{\hat{f}^{\text{B}}_d}$ *is an estimate of the dual disturbance force updated according to*

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\hat{f}^{\text{B}}_d} = K_j \star \hat{s}^{\mathsf{s}}, \tag{28}$$

$$K_j = \begin{bmatrix} K_f & 0_{4\times 4} \\ 0_{4\times 4} & K_\tau \end{bmatrix}, \quad K_f = \begin{bmatrix} \bar{K}_f & 0_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix}, \quad K_\tau = \begin{bmatrix} \bar{K}_\tau & 0_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix}, \tag{29}$$

and $\bar{K}_f, \bar{K}_\tau \in \mathbb{R}^{3\times 3}$ *are positive definite matrices. Assume that* $\hat{\omega}^{\text{D}}_{\text{D/I}}, \dot{\hat{\omega}}^{\text{D}}_{\text{D/I}} \in \mathcal{L}_\infty$ *and* $\hat{r}^{\text{B}}_{\text{B/I}} \neq 0$*. Then, for all initial conditions,* $\lim_{t\to\infty} \hat{q}_{\text{B/D}} = \pm 1$ *(i.e.,* $\lim_{t\to\infty} q_{\text{B/D}} = \pm 1$ *and* $\lim_{t\to\infty} r^{\text{B}}_{\text{B/D}} = 0$*),* $\lim_{t\to\infty} \hat{\omega}^{\text{B}}_{\text{B/D}} = 0$ *(i.e.,* $\lim_{t\to\infty} \omega^{\text{B}}_{\text{B/D}} = 0$ *and* $\lim_{t\to\infty} v^{\text{B}}_{\text{B/D}} = 0$*), and* $\mathrm{v}(\widehat{M^{\text{B}}}), \widehat{\hat{f}^{\text{B}}_d} \in \mathcal{L}_\infty$.

Proof. First, define the dual inertia matrix and dual disturbance force estimation errors as

$$\Delta M^{\text{B}} = \widehat{M^{\text{B}}} - M^{\text{B}} \ \text{ and } \ \Delta\hat{f}^{\text{B}}_d = \widehat{\hat{f}^{\text{B}}_d} - \hat{f}^{\text{B}}_d. \tag{30}$$

Note that $\hat{q}_{\text{B/D}} = \pm 1$, $\hat{\omega}^{\text{B}}_{\text{B/D}} = 0$, $\mathrm{v}(\Delta M^{\text{B}}) = 0$, and $\Delta\hat{f}^{\text{B}}_d = 0$ are the equilibrium conditions of the closed-loop system formed by Eqs. (21), (24), (20), (35), and (39). Consider now the following candidate Lyapunov function for the equilibrium point $\hat{q}_{\text{B/D}} = +1$, $\hat{\omega}^{\text{B}}_{\text{B/D}} = 0$, $\mathrm{v}(\Delta M^{\text{B}}) = 0$, and $\Delta\hat{f}^{\text{B}}_d = 0$: $V(\hat{q}_{\text{B/D}}, \hat{s}, \mathrm{v}(\Delta M^{\text{B}}), \Delta\hat{f}^{\text{B}}_d) = (\hat{q}_{\text{B/D}} - 1) \circ (\hat{q}_{\text{B/D}} - 1) + \frac{1}{2}\hat{s}^{\mathsf{s}} \circ (M^{\text{B}} \star \hat{s}^{\mathsf{s}}) + \frac{1}{2}\mathrm{v}(\Delta M^{\text{B}})^{\mathsf{T}}K_i^{-1}\mathrm{v}(\Delta M^{\text{B}}) + \frac{1}{2}\Delta\hat{f}^{\text{B}}_d \circ (K_j^{-1} \star \Delta\hat{f}^{\text{B}}_d)$. Note that V is a valid candidate Lyapunov function since $V(\hat{q}_{\text{B/D}} = 1, \hat{s} = 0, \mathrm{v}(\Delta M^{\text{B}}) = 0, \Delta\hat{f}^{\text{B}}_d = 0) = 0$ and $V(\hat{q}_{\text{B/D}}, \hat{s}, \mathrm{v}(\Delta M^{\text{B}}), \Delta\hat{f}^{\text{B}}_d) > 0$ for all $(\hat{q}_{\text{B/D}}, \hat{s}, \mathrm{v}(\Delta M^{\text{B}}), \Delta\hat{f}^{\text{B}}_d) \in \mathbb{H}^u_d \times \mathbb{H}^v_d \times \mathbb{R}^7 \times \mathbb{H}^v_d \backslash \{1, 0, 0, 0\}$. The time derivative of V is equal to $\dot{V} = 2(\hat{q}_{\text{B/D}} - 1) \circ \dot{\hat{q}}_{\text{B/D}} + \hat{s}^{\mathsf{s}} \circ (M^{\text{B}} \star \dot{\hat{s}}^{\mathsf{s}}) + \mathrm{v}(\Delta M^{\text{B}})^{\mathsf{T}}K_i^{-1}\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{v}(\Delta M^{\text{B}}) + \Delta\hat{f}^{\text{B}}_d \circ (K_j^{-1} \star \frac{\mathrm{d}}{\mathrm{d}t}\Delta\hat{f}^{\text{B}}_d)$. Then, since from Eq. (20), $\hat{\omega}^{\text{B}}_{\text{B/D}} = 2\hat{q}^*_{\text{B/D}}\dot{\hat{q}}_{\text{B/D}}$, Eq. (32) can be rewritten as $\dot{\hat{q}}_{\text{B/D}} = \frac{1}{2}\hat{q}_{\text{B/D}}\hat{s} - \frac{1}{2}\hat{q}_{\text{B/D}}(K_p \star (\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon)))^{\mathsf{s}}$, which can then by plugged into $\dot{V}$, together with the time derivative of Eq. (32), to yield $\dot{V}{=}(\hat{q}_{\text{B/D}} - 1) \circ (\hat{q}_{\text{B/D}}\hat{s} - q_{\text{B/D}}(K_p \star (\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon)))^{\mathsf{s}}) + \hat{s}^{\mathsf{s}} \circ (M^{\text{B}} \star (\dot{\hat{\omega}}^{\text{B}}_{\text{B/D}})^{\mathsf{s}}){+}\hat{s}^{\mathsf{s}} \circ (M^{\text{B}} \star (K_p \star \frac{\mathrm{d}(\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon))}{\mathrm{d}t})^{\mathsf{s}}) + \mathrm{v}(\Delta M^{\text{B}})^{\mathsf{T}}K_i^{-1}\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{v}(\Delta M^{\text{B}}) + \Delta\hat{f}^{\text{B}}_d \circ (K_j^{-1} \star \frac{\mathrm{d}}{\mathrm{d}t}\Delta\hat{f}^{\text{B}}_d)$. Applying Eq. (7) and inserting Eq. (21) yields $\dot{V}{=}\hat{s}^{\mathsf{s}} \circ (\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon)) - (K_p \star (\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon))) \circ (\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon)){+}\hat{s}^{\mathsf{s}} \circ (\hat{f}^{\text{B}}{-}\hat{\omega}^{\text{B}}_{\text{B/I}}{\times}\big(M^{\text{B}} \star (\hat{\omega}^{\text{B}}_{\text{B/I}})^{\mathsf{s}}\big){-}M^{\text{B}}{\star}(\hat{q}^*_{\text{B/D}}\dot{\hat{\omega}}^{\text{D}}_{\text{D/I}}\hat{q}_{\text{B/D}})^{\mathsf{s}}{-}M^{\text{B}}{\star}(\hat{\omega}^{\text{B}}_{\text{D/I}}{\times}\hat{\omega}^{\text{B}}_{\text{B/D}})^{\mathsf{s}}){+}\hat{s}^{\mathsf{s}}{\circ}(M^{\text{B}}{\star}(K_p{\star}\frac{\mathrm{d}(\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon))}{\mathrm{d}t})^{\mathsf{s}}){+}\mathrm{v}(\Delta M^{\text{B}})^{\mathsf{T}}K_i^{-1}\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{v}(\Delta M^{\text{B}}) {+}\Delta\hat{f}^{\text{B}}_d \circ (K_j^{-1} \star \frac{\mathrm{d}}{\mathrm{d}t}\Delta\hat{f}^{\text{B}}_d)$. By introducing the feedback control law, Eq. (31), and using Eq. (8), $\dot{V}$ becomes $\dot{V}{=} - (\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon)) \circ (K_p \star (\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon))) + \hat{s}^{\mathsf{s}} \circ (\hat{\omega}^{\text{B}}_{\text{B/I}} \times (\Delta M^{\text{B}} \star (\hat{\omega}^{\text{B}}_{\text{B/I}})^{\mathsf{s}}) + \Delta M^{\text{B}} \star (\hat{q}^*_{\text{B/D}}\dot{\hat{\omega}}^{\text{D}}_{\text{D/I}}\hat{q}_{\text{B/D}})^{\mathsf{s}} + \Delta M^{\text{B}} \star (\hat{\omega}^{\text{B}}_{\text{D/I}} \times \hat{\omega}^{\text{B}}_{\text{B/D}})^{\mathsf{s}} - \Delta M^{\text{B}} \star (K_p \star \frac{\mathrm{d}(\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon))}{\mathrm{d}t})^{\mathsf{s}} - \Delta M^{\text{B}} \star \hat{a}^{\text{B}}_g - \frac{3\mu\hat{r}^{\text{B}}_{\text{B/I}}}{\|\hat{r}^{\text{B}}_{\text{B/I}}\|^5} \times (\Delta M^{\text{B}} \star (\hat{r}^{\text{B}}_{\text{B/I}})^{\mathsf{s}}) - \Delta M^{\text{B}} \star \hat{a}^{\text{B}}_{J_2} - \Delta\hat{f}^{\text{B}}_d) - \hat{s}^{\mathsf{s}} \circ (K_d \star \hat{s}^{\mathsf{s}}) + \mathrm{v}(\Delta M^{\text{B}})^{\mathsf{T}}K_i^{-1}\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{v}(\Delta M^{\text{B}}) + \Delta\hat{f}^{\text{B}}_d \circ (K_j^{-1} \star \frac{\mathrm{d}}{\mathrm{d}t}\Delta\hat{f}^{\text{B}}_d) = -(\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon)) \circ (K_p{\star}(\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon))){+}(\hat{s} \times \hat{\omega}^{\text{B}}_{\text{B/I}})^{\mathsf{s}} \circ (\Delta M^{\text{B}} \star (\hat{\omega}^{\text{B}}_{\text{B/I}})^{\mathsf{s}}){+}\hat{s}^{\mathsf{s}} \circ (\Delta M^{\text{B}}{\star}(\hat{q}^*_{\text{B/D}}\dot{\hat{\omega}}^{\text{D}}_{\text{D/I}}\hat{q}_{\text{B/D}})^{\mathsf{s}} + \Delta M^{\text{B}}{\star}(\hat{\omega}^{\text{B}}_{\text{D/I}}{\times}\hat{\omega}^{\text{B}}_{\text{B/D}})^{\mathsf{s}}{-}\Delta M^{\text{B}} \star (K_p \star \frac{\mathrm{d}(\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}} - \epsilon))}{\mathrm{d}t})^{\mathsf{s}}{-}\Delta M^{\text{B}} \star \hat{a}^{\text{B}}_g{-}\Delta M^{\text{B}} \star \hat{a}^{\text{B}}_{J_2} - \Delta\hat{f}^{\text{B}}_d){-}(\hat{s} \times \frac{3\mu\hat{r}^{\text{B}}_{\text{B/I}}}{\|\hat{r}^{\text{B}}_{\text{B/I}}\|^5})^{\mathsf{s}} \circ (\Delta M^{\text{B}} \star (\hat{r}^{\text{B}}_{\text{B/I}})^{\mathsf{s}}){-}\hat{s}^{\mathsf{s}} \circ (K_d \star \hat{s}^{\mathsf{s}}) + \mathrm{v}(\Delta M^{\text{B}})^{\mathsf{T}}K_i^{-1}\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{v}(\Delta M^{\text{B}}) + \Delta\hat{f}^{\text{B}}_d \circ (K_j^{-1} \star \frac{\mathrm{d}}{\mathrm{d}t}\Delta\hat{f}^{\text{B}}_d)$. Therefore, if $\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{v}(\Delta M^{\text{B}})$ is defined as in Eq. (35) and $\frac{\mathrm{d}}{\mathrm{d}t}\Delta\hat{f}^{\text{B}}_d$ is defined as in Eq. (39), it follows that $\dot{V}{=}{-}(\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}}{-}\epsilon)){\circ}(K_p{\star}(\hat{q}^*_{\text{B/D}}(\hat{q}^{\mathsf{s}}_{\text{B/D}}{-}\epsilon))){-}\hat{s}^{\mathsf{s}}{\circ}(K_d{\star}\hat{s}^{\mathsf{s}}) \leq 0$, for all $(\hat{q}_{\text{B/D}}, \hat{s}, \mathrm{v}(\Delta M^{\text{B}}), \Delta\hat{f}^{\text{B}}_d) \in \mathbb{H}^u_d \times \mathbb{H}^v_d \times \mathbb{R}^7 \times \mathbb{H}^v_d \backslash \{1, 0, 0, 0\}$. Hence, $\hat{q}_{\text{B/D}}$, $\hat{s}$, $\mathrm{v}(\Delta M^{\text{B}})$, and $\Delta\hat{f}^{\text{B}}_d$

are uniformly bounded, i.e., $\hat{q}_{\rm B/D}, \hat{s}, {\rm v}(\Delta M^{\rm B}), \Delta\hat{f}_d^{\rm B} \in \mathcal{L}_\infty$. Moreover, from Eqs. (32) and (**??**), this also means that $\hat{\omega}_{\rm B/D}^{\rm B}, {\rm v}(\widehat{M^{\rm B}}), \widehat{\hat{f}_d^{\rm B}} \in \mathcal{L}_\infty$. Since $V \geq 0$ and $\dot{V} \leq 0$, $\lim_{t\to\infty} V(t)$ exists and is finite. Hence, $\lim_{t\to\infty}\int_0^t \dot{V}(\tau)\,{\rm d}\tau = \lim_{t\to\infty} V(t) - V(0)$ also exists and is finite. Since $\hat{q}_{\rm B/D}, \hat{s}, {\rm v}(\Delta M^{\rm B}), \Delta\hat{f}_d^{\rm B}, \hat{\omega}_{\rm B/D}^{\rm B}$, ${\rm v}(\widehat{M^{\rm B}}), \widehat{\hat{f}_d^{\rm B}}, \dot{\hat{\omega}}_{\rm D/I}^{\rm D}, \hat{\omega}_{\rm D/I}^{\rm B} \in \mathcal{L}_\infty$ and $\hat{r}_{\rm B/I}^{\rm B} \neq 0$, then from Eqs. (20), (31), and (21) and from Lemma 1, $\dot{\hat{q}}_{\rm B/D}, \hat{f}^{\rm B}, \dot{\hat{\omega}}_{\rm B/D}^{\rm B}$, $\dot{\hat{s}} \in \mathcal{L}_\infty$. Hence, by Barbalat's lemma, ${\rm vec}\big(\hat{q}_{\rm B/D}^*(\hat{q}_{\rm B/D}^{\sf s} - \epsilon)\big) \to 0$ and $\hat{s} \to 0$ as $t \to \infty$. In Ref. 15, it is shown that ${\rm vec}\big(\hat{q}_{\rm B/D}^*(\hat{q}_{\rm B/D}^{\sf s} - \epsilon)\big) \to 0$ is equivalent to $\hat{q}_{\rm B/D} \to \pm 1$. Finally, calculating the limit as $t \to \infty$ of both sides of Eq. (32) yields $\hat{\omega}_{\rm B/D}^{\rm B} \to 0$. □

Remark 1. Theorem 1 states that $\hat{q}_{\rm B/D}$ converges to either $+1$ or -1. Note that $\hat{q}_{\rm B/D} = +1$ and $\hat{q}_{\rm B/D} = -1$ represent the same physical relative position and attitude between frames, so either equilibrium is acceptable. However, this can lead to the so-called *unwinding phenomenon* where a large rotation (greater than 180 degrees) is performed, despite the fact that a smaller rotation to the equilibrium (less than 180 degrees) exits. This problem of quaternions is well documented and possible solutions exist in literature.[13,27,30,1]

Remark 2. It can be easily shown that if the control law given by Eq. (31) is replaced by its nonadaptive model-dependent version, where the estimates of the dual inertia matrix and of the dual disturbance force are replaced by its true values in Theorem 1, i.e.,

$$\begin{aligned}\hat{f}_c^{\rm B} =& -M^{\rm B}\star\hat{a}_g^{\rm B} - \frac{3\mu\hat{r}_{\rm B/I}^{\rm B}}{\|\hat{r}_{\rm B/I}^{\rm B}\|^5}\times(M^{\rm B}\star(\hat{r}_{\rm B/I}^{\rm B})^{\sf s}) - M^{\rm B}\star\hat{a}_{J_2}^{\rm B} - \hat{f}_d^{\rm B} - {\rm vec}\big(\hat{q}_{\rm B/D}^*(\hat{q}_{\rm B/D}^{\sf s} - \epsilon)\big) - K_d\star\hat{s}^{\sf s} \\ & + \hat{\omega}_{\rm B/I}^{\rm B}\times(M^{\rm B}\star(\hat{\omega}_{\rm B/I}^{\rm B})^{\sf s}) + M^{\rm B}\star(\hat{q}_{\rm B/D}^*\dot{\hat{\omega}}_{\rm D/I}^{\rm D}\hat{q}_{\rm B/D})^{\sf s} + M^{\rm B}\star(\hat{\omega}_{\rm D/I}^{\rm B}\times\hat{\omega}_{\rm B/D}^{\rm B})^{\sf s} - M^{\rm B}\star(K_p\star\frac{\rm d}{{\rm d}t}(\hat{q}_{\rm B/D}^*(\hat{q}_{\rm B/D}^{\sf s} - \epsilon)))^{\sf s},\end{aligned} \tag{31}$$

then it is still true that, for all initial conditions, $\lim_{t\to\infty}\hat{q}_{\rm B/D} = \pm 1$ and $\lim_{t\to\infty}\hat{\omega}_{\rm B/D}^{\rm B} = 0$.

SUFFICIENT CONDITIONS FOR MASS AND INERTIA MATRIX IDENTIFICATION

In this section, we give sufficient conditions on the reference motion that guarantee that the estimate of the dual inertia matrix will converge to the true dual inertia matrix. Note however that the result presented in Theorem 1 does not depend on the convergence of this estimate. In other words, the controller proposed in Theorem 1 guarantees almost global asymptotical stability of the closed-loop system even without estimate convergence. Nevertheless, identification of the mass and inertia matrix of the chaser satellite might be important for fuel consumption estimation, calculation of re-entry trajectories and terminal velocities, state estimation, fault-detecting-and-isolation systems, and docking/undocking scenarios.

Proposition 1. *Let the dual disturbance force be exactly known or estimated so that* $\widehat{\hat{f}_d^{\rm B}}$ *can be replaced by* $\hat{f}_d^{\rm B}$ *in Eq.* (31). *Moreover, assume that* $\hat{\omega}_{\rm D/I}^{\rm D}, \dot{\hat{\omega}}_{\rm D/I}^{\rm D}, \ddot{\hat{\omega}}_{\rm D/I}^{\rm D} \in \mathcal{L}_\infty$, $\hat{r}_{\rm B/I}^{\rm B} \neq 0$, *and* $\hat{q}_{\rm D/I}$ *is periodic. Furthermore, let* ${\rm W} : [0,\infty) \to \mathbb{R}^{8\times 7}$ *be defined as*

$$\begin{aligned}{\rm W}(t){\rm v}(\Delta M^{\rm B}) =& \hat{\omega}_{\rm D/I}^{\rm D}(t)\times(\Delta M^{\rm B}\star(\hat{\omega}_{\rm D/I}^{\rm D}(t))^{\sf s}) + \Delta M^{\rm B}\star(\dot{\hat{\omega}}_{\rm D/I}^{\rm D}(t))^{\sf s} \\ & - \Delta M^{\rm B}\star\hat{a}_g^{\rm D} - \frac{3\mu\hat{r}_{\rm D/I}^{\rm D}}{\|\hat{r}_{\rm D/I}^{\rm D}\|^5}\times(\Delta M^{\rm B}\star(\hat{r}_{\rm D/I}^{\rm D})^{\sf s}) - \Delta M^{\rm B}\star\hat{a}_{J_2}^{\rm D}\end{aligned} \tag{32}$$

or, equivalently, ${\rm W}(t) = {\rm W}_{rb}(t) + {\rm W}_g(t) + {\rm W}_{\nabla g}(t) + {\rm W}_{J_2}(t)$, *where*

$${\rm W}_{rb}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dot{u}+qw-rv \\ 0 & 0 & 0 & 0 & 0 & 0 & \dot{v}-pw+ru \\ 0 & 0 & 0 & 0 & 0 & 0 & \dot{w}+pv-qu \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dot{p} & \dot{q}-pr & \dot{r}+pq & -qr & q^2-r^2 & qr & 0 \\ pr & \dot{p}+qr & -p^2+r^2 & \dot{q} & \dot{r}-pq & -pr & 0 \\ -pq & p^2-q^2 & \dot{p}-qr & pq & \dot{q}+pr & \dot{r} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{33}$$

$$\mathrm{W}_g(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu x}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu y}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu z}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{W}_{J_2}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{34}$$

$$\mathrm{W}_{\nabla g}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3\mu xz}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & -\frac{3\mu xy}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & \frac{3\mu yz}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & -\frac{3\mu(y^2-z^2)}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & -\frac{3\mu yz}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & 0 \\ -\frac{3\mu xz}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & -\frac{3\mu yz}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & -\frac{3\mu(z^2-x^2)}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & 0 & \frac{3\mu xy}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & \frac{3\mu xz}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & 0 \\ \frac{3\mu xy}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & -\frac{3\mu(x^2-y^2)}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & \frac{3\mu yz}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & -\frac{3\mu xy}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & -\frac{3\mu xz}{\|\bar{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{35}$$

where $\bar{v}^{\mathrm{D}}_{\mathrm{D/I}} = [u\ v\ w]^\mathsf{T}$, $\bar{\omega}^{\mathrm{D}}_{\mathrm{D/I}} = [p\ q\ r]^\mathsf{T}$, $\bar{r}^{\mathrm{D}}_{\mathrm{D/I}} = [x\ y\ z]^\mathsf{T}$, *and* $\bar{a}^{\mathrm{D}}_{J_2} = [a_1\ a_2\ a_3]^\mathsf{T}$. *Let also* $0 \le t_1 \le t_2 \le ... \le t_n$ *be such that*

$$\operatorname{rank} \begin{bmatrix} \mathrm{W}(t_1) \\ \vdots \\ \mathrm{W}(t_n) \end{bmatrix} = 7. \tag{36}$$

Then, under the control law given by Eq. (31), $\lim_{t\to\infty} \widehat{M^{\mathrm{B}}} = M^{\mathrm{B}}$.

Proof. We start by showing that $\lim_{t\to\infty} \dot{\hat{\omega}}^{\mathrm{B}}_{\mathrm{B/D}} = 0$. (Note that $\lim_{t\to\infty} \hat{\omega}^{\mathrm{B}}_{\mathrm{B/D}} = 0$ does not imply that $\lim_{t\to\infty} \dot{\hat{\omega}}^{\mathrm{B}}_{\mathrm{B/D}} = 0$.) First, note that $\lim_{t\to\infty} \int_0^t \dot{\hat{\omega}}^{\mathrm{B}}_{\mathrm{B/D}}(\tau)\,\mathrm{d}\tau = \lim_{t\to\infty} \hat{\omega}^{\mathrm{B}}_{\mathrm{B/D}}(t) - \hat{\omega}^{\mathrm{B}}_{\mathrm{B/D}}(0) = -\hat{\omega}^{\mathrm{B}}_{\mathrm{B/D}}(0)$ exits and is finite. Furthermore, since $\hat{\omega}^{\mathrm{D}}_{\mathrm{D/I}}, \dot{\hat{\omega}}^{\mathrm{D}}_{\mathrm{D/I}}, \ddot{\hat{\omega}}^{\mathrm{D}}_{\mathrm{D/I}}$, $\dot{\hat{q}}_{\mathrm{B/D}}, \ddot{\hat{q}}_{\mathrm{B/D}}, \dot{\hat{\omega}}^{\mathrm{B}}_{\mathrm{B/D}}$, $\frac{\mathrm{dv}(\widehat{M^{\mathrm{B}}})}{\mathrm{d}t}$, $\frac{\mathrm{d}\widehat{f}^{\mathrm{B}}_d}{\mathrm{d}t} \in \mathcal{L}_\infty$ and $\hat{r}^{\mathrm{B}}_{\mathrm{B/I}} \neq 0$, we have that $\ddot{\hat{\omega}}^{\mathrm{B}}_{\mathrm{B/D}} \in \mathcal{L}_\infty$ by differentiating Eq. (21). Hence, by Barbalat's lemma, $\lim_{t\to\infty} \dot{\hat{\omega}}^{\mathrm{B}}_{\mathrm{B/D}} = 0$. Now, we calculate the limit as $t \to \infty$ of both sides of Eq. (21) to get

$$\begin{aligned} 0 = \lim_{t\to\infty} (\hat{\omega}^{\mathrm{D}}_{\mathrm{D/I}}(t) \times (\varDelta M^{\mathrm{B}} \star (\hat{\omega}^{\mathrm{D}}_{\mathrm{D/I}}(t))^{\mathsf{s}}) + \varDelta M^{\mathrm{B}} \star (\dot{\hat{\omega}}^{\mathrm{D}}_{\mathrm{D/I}}(t))^{\mathsf{s}} \\ - \varDelta M^{\mathrm{B}} \star \hat{a}^{\mathrm{D}}_g - \frac{3\mu \hat{r}^{\mathrm{D}}_{\mathrm{D/I}}}{\|\hat{r}^{\mathrm{D}}_{\mathrm{D/I}}\|^5} \times (\varDelta M^{\mathrm{B}} \star (\hat{r}^{\mathrm{D}}_{\mathrm{D/I}})^{\mathsf{s}}) - \varDelta M^{\mathrm{B}} \star \hat{a}^{\mathrm{D}}_{J_2} - \varDelta \hat{f}^{\mathrm{B}}_d) \end{aligned} \tag{37}$$

Note that under the conditions of Proposition 1, $\varDelta \hat{f}^{\mathrm{B}}_d = 0$. Moreover, if $\hat{q}_{\mathrm{D/I}}$ is periodic with period T, so is $\mathrm{W}(t)$. Finally, noting that $\lim_{t\to\infty} \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{v}(\widehat{M^{\mathrm{B}}}) = 0$ from Eq. (35) and Theorem 1, under the conditions of Proposition 1, Eq. (53) implies that $\lim_{t\to\infty} \mathrm{v}(\varDelta M^{\mathrm{B}}) = 0$ or, equivalently, $\lim_{t\to\infty} \widehat{M^{\mathrm{B}}} = M^{\mathrm{B}}$. □

Remark 3. In practice, the true disturbance dual force $\hat{f}^{\mathrm{B}}_d$ is never known. Moreover, we cannot guarantee that the estimate of the disturbance dual force will converge to its true value. Hence, in practice, the estimate of the mass and inertia matrix of the spacecraft will only be as good as the estimate of the disturbance dual force.

Remark 4. An alternative, and more general, sufficient condition than Eq. (52) for dual inertia matrix identification, which does not require $\hat{q}_{\mathrm{D/I}}$ to be periodic, is that the 7×7 matrix $\int_t^{t+T_2} \mathrm{W}^\mathsf{T}(t)\mathrm{W}(t)\,\mathrm{d}t$, is positive definite for all $t \ge T_1$ for some $T_1 \ge 0$ and $T_2 > 0$.[?,?]

SIMULATION RESULTS

In this section, we demonstrate how our controller can be used in a conceivable satellite proximity operations scenario. We assume that the target spacecraft is in a highly eccentric Molniya orbit with orbital elements given in Table 1 and nadir pointing. Inspired by linear results for circular target orbits,[31] we would like our chaser satellite to perform a full elliptical revolution around the target satellite while always pointing to it. We would like this elliptical motion to take a full orbital period, to always be in a plane perpendicular to the angular velocity of the target satellite, and to describe a 2-by-1 ellipse. This elliptical motion is illustrated in Figure **??** through a series of snapshots.

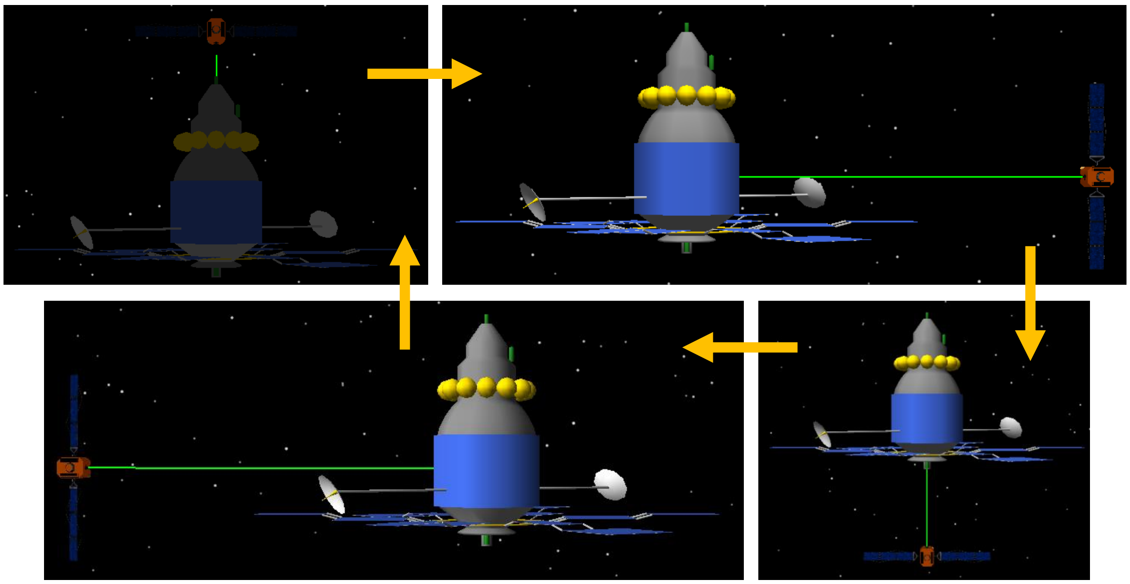

Figure 2. Snapshots of the elliptical motion.

For this problem, we define four reference frames: the inertial frame, the target frame, the desired frame, and the body frame. The inertial frame is the Earth-Centered-Inertial (ECI) frame. The body frame is some frame fixed to the chaser satellite and centered at its center of mass. The target frame and the desired frame are defined in Table 2, where

$$\bar{\omega}_{\mathrm{T/I}} = \frac{\bar{r}_{\mathrm{T/I}} \times \bar{v}_{\mathrm{T/I}}}{\|\bar{r}_{\mathrm{T/I}}\|^2} \tag{38}$$

is calculated from the angular momentum of the target spacecraft with respect to the inertial frame given by $\bar{h}_{\mathrm{T/I}} = m\|\bar{r}_{\mathrm{T/I}}\|^2\bar{\omega}_{\mathrm{T/I}} = \bar{r}_{\mathrm{T/I}} \times m\bar{v}_{\mathrm{T/I}}$. The target satellite is assumed to be fixed to the target frame. The objective of the control law is to superimpose the body frame to the desired frame. The relationship between the different frames is illustrated in Figure 2.

Table 1. Orbital elements of target satellite.

Perigee altitude	813.2 km
Eccentricity	0.7
Inclination	63.4 deg
Argument of perigee	270 deg
RAAN	329.6 deg
True anomaly	180 deg

Table 2. Target and desired frames.

Target frame	Desired frame
$\bar{I}_{\mathrm{T}} = \frac{\bar{r}_{\mathrm{T/I}}}{\|\bar{r}_{\mathrm{T/I}}\|}$	$\bar{I}_{\mathrm{D}} = \frac{\bar{r}_{\mathrm{D/T}}}{\|\bar{r}_{\mathrm{D/T}}\|}$
$\bar{J}_{\mathrm{T}} = \bar{K}_{\mathrm{T}} \times \bar{I}_{\mathrm{T}}$	$\bar{J}_{\mathrm{D}} = \bar{K}_{\mathrm{D}} \times \bar{I}_{\mathrm{D}}$
$\bar{K}_{\mathrm{T}} = \frac{\bar{\omega}_{\mathrm{T/I}}}{\|\bar{\omega}_{\mathrm{T/I}}\|}$	$\bar{K}_{\mathrm{D}} = \frac{\bar{\omega}_{\mathrm{D/T}}}{\|\bar{\omega}_{\mathrm{D/T}}\|}$

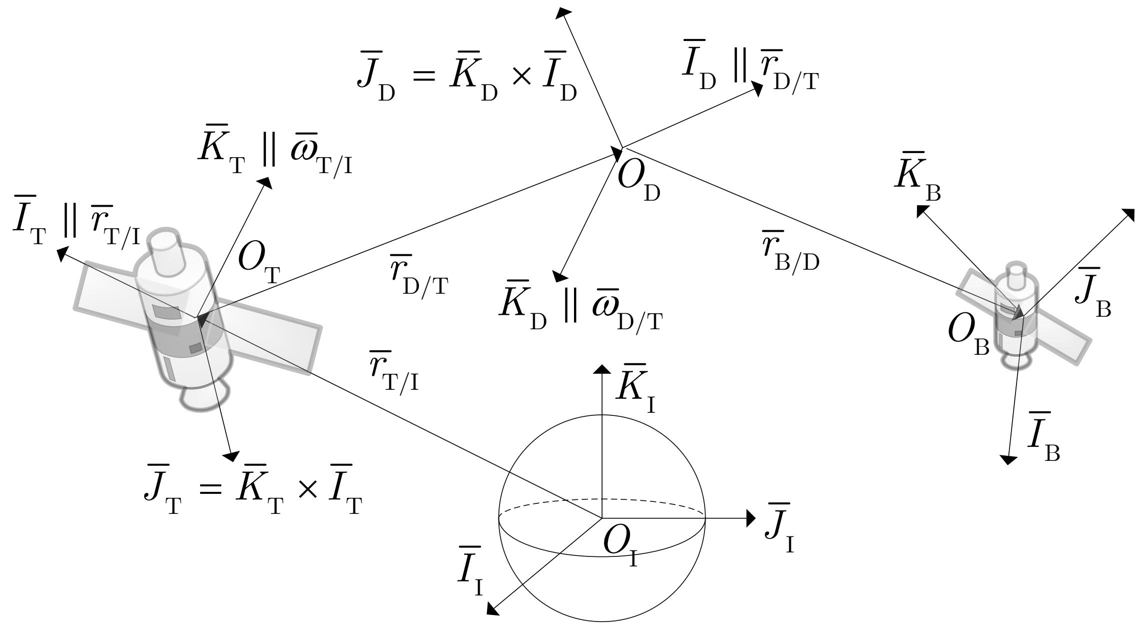

Figure 3. Reference frames.

The linear velocity of the target satellite with respect to the inertial frame is calculated by numerically integrating the gravitational acceleration and also the perturbing acceleration due to J_2 acting on the target satellite. On the other hand, the angular acceleration of the target satellite with respect to the inertial frame is calculated analytically through

$$\alpha^{\mathrm{I}}_{\mathrm{T/I}} = \dot{\omega}^{\mathrm{I}}_{\mathrm{T/I}} = \frac{(r^{\mathrm{I}}_{\mathrm{T/I}} \times a^{\mathrm{I}}_{\mathrm{T/I}})\|r^{\mathrm{I}}_{\mathrm{T/I}}\|^2 - (r^{\mathrm{I}}_{\mathrm{T/I}} \times v^{\mathrm{I}}_{\mathrm{T/I}})2(r^{\mathrm{I}}_{\mathrm{T/I}} \cdot v^{\mathrm{I}}_{\mathrm{T/I}})}{\|r^{\mathrm{I}}_{\mathrm{T/I}}\|^4}. \tag{39}$$

Equation (55) is the analytical time derivative of Eq. (54). Note that the J_2 perturbation changes the direction of the target's angular velocity with respect to the inertial frame. However, this change is relatively small in our scenario due to the critical inclination of the Molniya orbit. The rotational and translational kinematic equations of the target frame with respect to the inertial frame are written in terms of dual quaternions as in Eq. (18).

In this scenario, the angular and linear velocity of the desired frame with respect to the target frame are defined as $\bar{\omega}^{\mathrm{T}}_{\mathrm{D/T}} = [0,\ 0,\ n]^{\mathsf{T}}$ rad/s and $\bar{v}^{\mathrm{T}}_{\mathrm{D/T}} = [-a_e n \sin(nt),\ b_e n \cos(nt),\ 0]^{\mathsf{T}}$ m/s, where $a_e = 10$ m and $b_e = 20$ m are the semi-minor and the semi-major axes of the 2-by-1 ellipse around the target satellite, $n = \sqrt{\frac{\mu}{a^3}}$ is the mean motion of the target satellite (assuming no J_2 perturbation), and a is the semi-major axis of the target satellite (assuming no J_2 perturbation). The initial position of the desired frame with respect to the target frame is set as $\bar{r}^{\mathrm{T}}_{\mathrm{D/T}} = [a_e,\ 0,\ 0]^{\mathsf{T}}$ m. The rotational and translational kinematic equations of the desired frame with respect to the target frame are written in terms of dual quaternions as in Eq. (20).

The control law given by Eq. (31) is a function of $\hat{\omega}^{\rm D}_{\rm D/I}$ and $\dot{\hat{\omega}}^{\rm D}_{\rm D/I}$. We calculate these variables in terms of dual quaternions as follows:

$$\hat{\omega}^{\rm D}_{\rm D/I} = \hat{\omega}^{\rm D}_{\rm T/I} + \hat{\omega}^{\rm D}_{\rm D/T} = \hat{q}^*_{\rm D/I}\hat{\omega}^{\rm I}_{\rm T/I}\hat{q}_{\rm D/I} + \hat{q}^*_{\rm D/T}\hat{\omega}^{\rm T}_{\rm D/T}\hat{q}_{\rm D/T}, \tag{40}$$

$$\dot{\hat{\omega}}^{\rm D}_{\rm D/I} = \hat{q}^*_{\rm D/I}\hat{\alpha}^{\rm I}_{\rm T/I}\hat{q}_{\rm D/I} - \hat{\omega}^{\rm D}_{\rm D/I} \times \hat{\omega}^{\rm D}_{\rm T/I} + \hat{q}^*_{\rm D/T}\hat{\alpha}^{\rm T}_{\rm D/T}\hat{q}_{\rm D/T}, \tag{41}$$

where $\hat{\alpha}^{\rm T}_{\rm D/T} = \dot{\hat{\omega}}^{\rm T}_{\rm D/T} = \alpha^{\rm T}_{\rm D/T} + \epsilon(a^{\rm T}_{\rm D/T} - \alpha^{\rm T}_{\rm D/T} \times r^{\rm T}_{\rm D/T} - \omega^{\rm T}_{\rm D/T} \times v^{\rm T}_{\rm D/T})$ and $\hat{\alpha}^{\rm I}_{\rm T/I} = \dot{\hat{\omega}}^{\rm I}_{\rm T/I} = \alpha^{\rm I}_{\rm T/I} + \epsilon(a^{\rm I}_{\rm T/I} - \alpha^{\rm I}_{\rm T/I} \times r^{\rm I}_{\rm T/I} - \omega^{\rm I}_{\rm T/I} \times v^{\rm I}_{\rm T/I})$. Eq. (57) is calculated by differentiating Eq. (56) and using the dual quaternion counterpart of the classical transport theorem.[16] Note that instead of calculating $\hat{\omega}^{\rm D}_{\rm D/I}$ and $\dot{\hat{\omega}}^{\rm D}_{\rm D/I}$ in terms of dual quaternions, one could calculate instead $\omega^{\rm D}_{\rm D/I}$, $\dot{\omega}^{\rm D}_{\rm D/I}$, $v^{\rm D}_{\rm D/I}$, and $\dot{v}^{\rm D}_{\rm D/I}$ using the traditional equations for a point moving with respect to a rotating rigid body. However, this would require the calculation of four parameters instead of just two and significant more work to calculate $v^{\rm D}_{\rm D/I}$ and $\dot{v}^{\rm D}_{\rm D/I}$, whose expressions are coupled with the rotational motion. Thus, Eqs. (56) and (57) are another good example of the benefits in terms of compactness and simplicity of using dual quaternions.

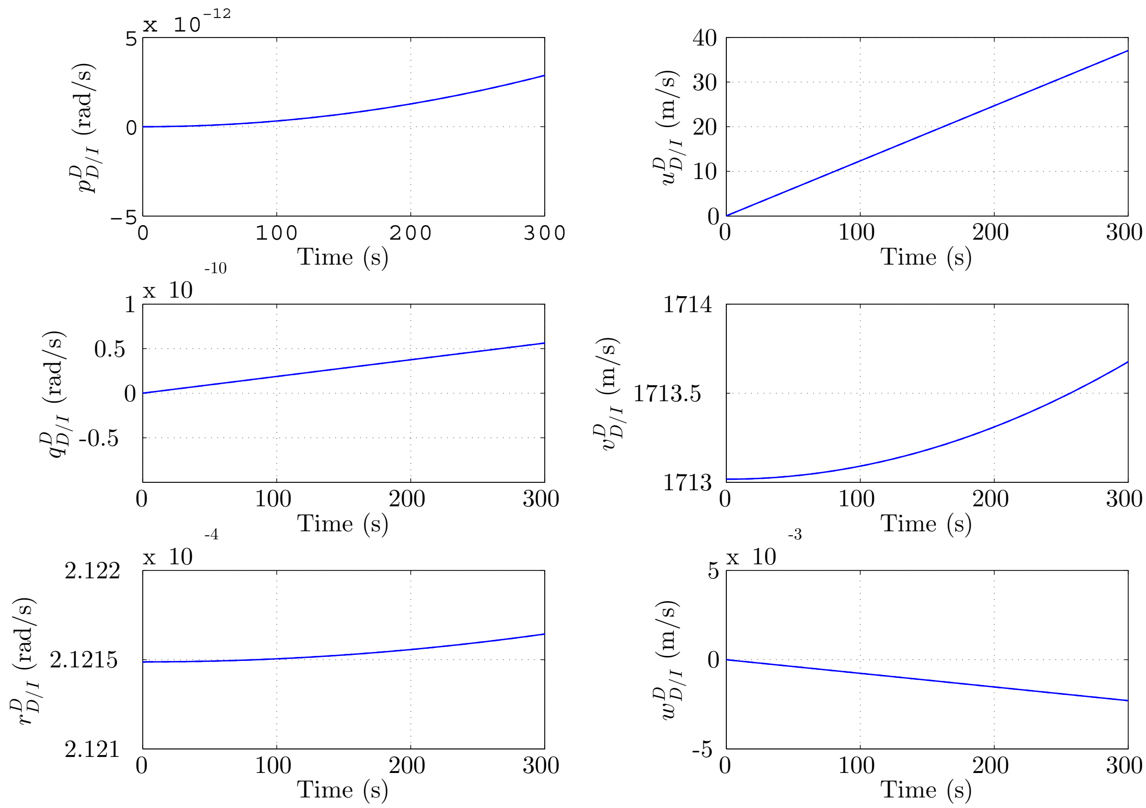

Figure 4. Desired linear and angular velocity expressed in the desired frame.

The inertia matrix and mass of the chaser satellite are assumed to be[1]

$$\bar{I}^{\rm B} = \begin{bmatrix} 22 & 0.2 & 0.5 \\ 0.2 & 20 & 0.4 \\ 0.5 & 0.4 & 23 \end{bmatrix} \ \text{Kg.m}^2$$

and $m = 100$ kg, respectively. The constant disturbance force and torque acting on the chaser satellite are set to $\bar{f}^{\rm B}_d = [0.005,\ 0.005,\ 0.005]^{\rm T}$ N and $\bar{\tau}^{\rm B}_d = [0.005,\ 0.005,\ 0.005]^{\rm T}$ N.m, respectively. The origin of the body frame (coincident with the center of mass of the chaser satellite) is positioned relatively to the origin of the desired frame at $\bar{r}^{\mathrm{B}}_{\mathrm{B/D}}$=$[10,\ 10,\ 10]^{\mathsf{T}}$ m. The initial error quaternion, relative linear velocity, and relative angular velocity of the body frame with respect to the desired frame are set to $q_{\mathrm{B/D}} = [q_{\mathrm{B/D}1}\ q_{\mathrm{B/D}2}\ q_{\mathrm{B/D}3}\ q_{\mathrm{B/D}4}]^{\mathsf{T}} = [0.4618,\ 0.1917,\ 0.7999,\ 0.3320]^{\mathsf{T}}$, $\bar{v}^{\mathrm{B}}_{\mathrm{B/D}} = [u^{\mathrm{B}}_{\mathrm{B/D}}\ v^{\mathrm{B}}_{\mathrm{B/D}}\ w^{\mathrm{B}}_{\mathrm{B/D}}]^{\mathsf{T}} = [0.1,\ 0.1,\ 0.1]^{\mathsf{T}}$ m/s, and $\bar{\omega}^{\mathrm{B}}_{\mathrm{B/D}} = [p^{\mathrm{B}}_{\mathrm{B/D}}\ q^{\mathrm{B}}_{\mathrm{B/D}}\ r^{\mathrm{B}}_{\mathrm{B/D}}]^{\mathsf{T}} = [0.1,\ 0.1,\ 0.1]^{\mathsf{T}}$ rad/s, respectively.

The initial estimates for the mass, inertia matrix, and disturbance dual force are set to zero, whereas the control gains are chosen to be $\bar{K}_r = 0.1I_3$, $\bar{K}_q = 0.5I_3$, $\bar{K}_v = 8I_3$, $\bar{K}_w = 8I_3$, $K_i = \mathrm{diag}([100,\ 100,\ 100,\ 100,\ 100,\ 100,\ 1])$, $\bar{K}_f = 0.8I_3$, and $\bar{K}_\tau = 0.8I_3$.

The linear and angular velocity of the desired frame with respect to the inertial frame expressed in the desired frame, which form the reference for the controller, are illustrated in Figure 3.

The relative position and attitude of the body frame with respect to the desired frame using the controller given by Eq. (31) (adaptive) and the controller given by Eq. (46) (nonadaptive) are compared in Figure 4. In both cases, $q_{\mathrm{B/D}} \to 1$ and $\bar{r}^{\mathrm{B}}_{\mathrm{B/D}} \to 0$ as $t \to \infty$, as expected. Figure 5 shows the relative linear and angular

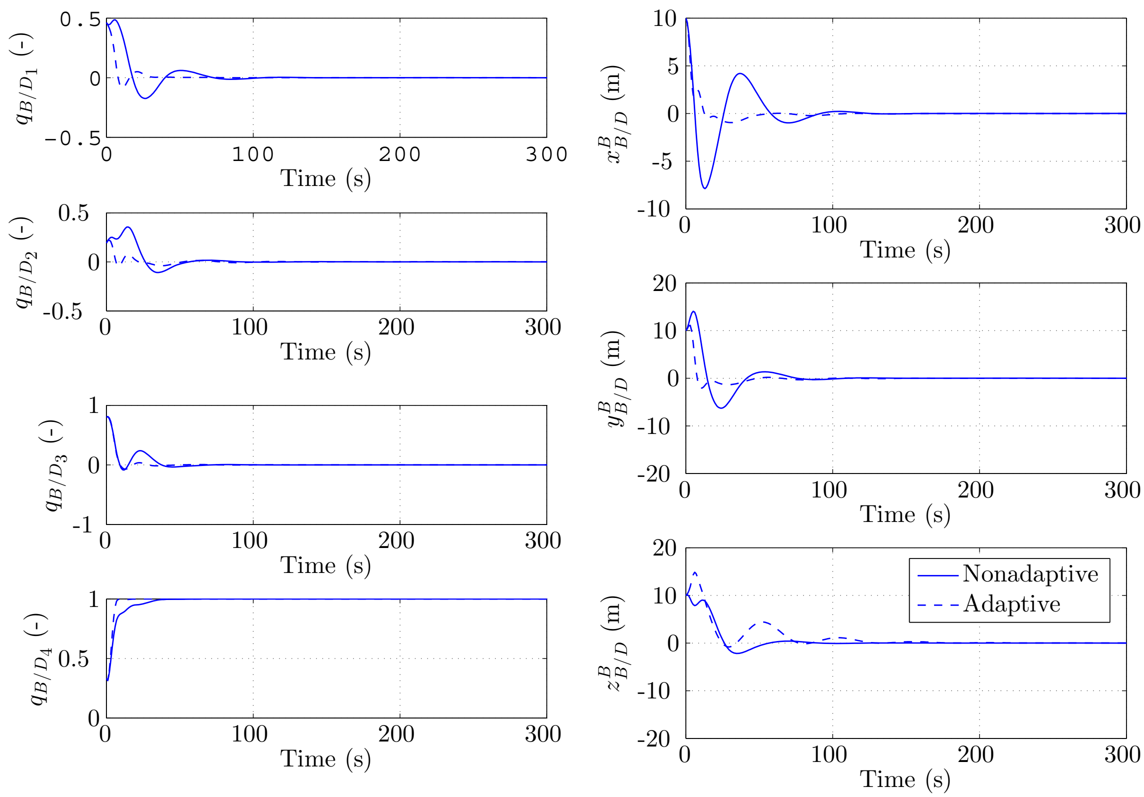

Figure 5. Relative attitude and position.

velocity of the body frame with respect to the desired frame for the same two cases studied in Figure 4. As predicted, $\bar{\omega}^{\mathrm{B}}_{\mathrm{B/D}} \to 0$ and $\bar{v}^{\mathrm{B}}_{\mathrm{B/D}} \to 0$ as $t \to \infty$. Figure 6 shows that the adaptive controller is unable to identify the true mass and inertia matrix of the chaser satellite for this reference motion (which does not even cover a full orbital period). Nevertheless, the adaptive controller is still able to track the reference motion. As a matter of fact, the similarities between the responses obtained with the adaptive controller (which has zero information about the true mass, inertia matrix, and disturbance dual force) and the nonadaptive controller (which knowns the true mass, inertia matrix, and disturbance dual force) are quite remarkable. Note that the minimum singular value of the matrix in Eq. (52) for $t_1 = 0$, $t_2 = 1$, ..., $t_{300} = 300$ s is $1.2e^{-9}$. Figure 7

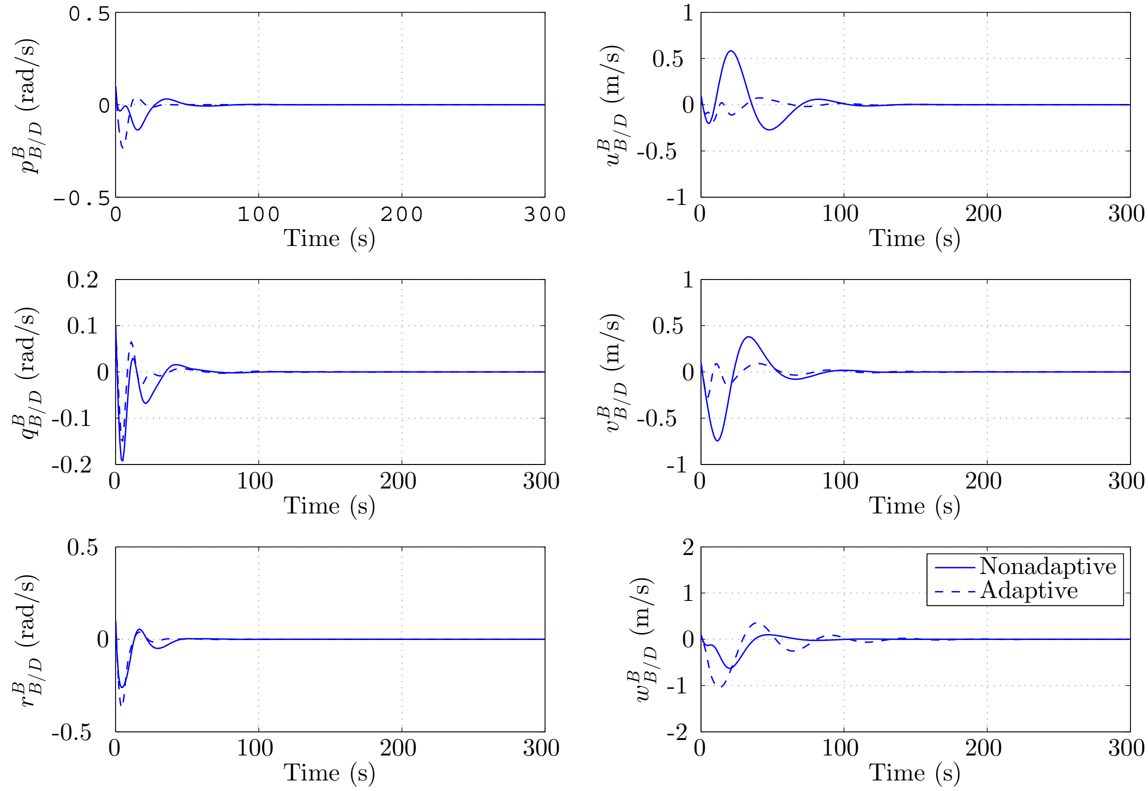

Figure 6. Relative linear and angular velocity expressed in the body frame.

shows that for this reference motion, even though the adaptive controller is unable to exactly identify the true disturbance dual force, it converges to values of the same order of magnitude. Note that Theorem 1 only guarantees that these estimates will be uniformly bounded. For completeness, Figure 8 shows the control force, $\bar{f}^{\mathrm{B}}_c = [f^{\mathrm{B}}_{c1}\ f^{\mathrm{B}}_{c2}\ f^{\mathrm{B}}_{c3}]^{\mathsf{T}}$, and the control torque, $\bar{\tau}^{\mathrm{B}}_c = [\tau^{\mathrm{B}}_{c1}\ \tau^{\mathrm{B}}_{c2}\ \tau^{\mathrm{B}}_{c3}]^{\mathsf{T}}$, produced by the adaptive and nonadaptive controllers during the initial transient response. These relatively high values of control force and torque are required to eliminate the initial relative position, attitude, linear and angular velocity errors that we arbitrarily set between the body frame and the desired frame. More interestingly, Figure 9 shows the control force and torque required to perform a full ellipse around the target satellite in a orbital period. The relatively low values of required control force are in line with those produced by the most common electric engines used in spacecraft, namely, ion drives and Hall thrusters,[32] which can produce up to 250 and 600 mN of thrust, respectively. Note that the control torques, on the other hand, can be implemented by electrical reaction wheels and/or control moment gyros without the use of on-board fuel.

Finally, to demonstrate that the adaptive control law can estimate the mass and inertia matrix of the chaser satellite given appropriate reference signals, we redefine the dual velocity of the desired frame with respect to the target frame as $\bar{\omega}^{\mathsf{T}}_{\mathrm{D/T}} = \bar{v}^{\mathsf{T}}_{\mathrm{D/T}} = [0.1,\ 0.2,\ 0.3]^{\mathsf{T}} \cos([3000n,\ 2000n,\ 1000n]^{\mathsf{T}} t + \frac{\pi}{180}[0,\ 20,\ 40]^{\mathsf{T}})$ rad/s and m/s, respectively, and true anomaly of the target satellite as 0 deg. For this reference motion, the minimum singular value of the matrix in Eq. (52) for $t_1 = 0$, $t_2 = 1$, ..., $t_{600} = 600$ s is $8.6e^{-1}$. The mass and inertia matrix identification is shown in Figure 10. The control forces required to follow such rather unrealistic reference motion are too high for practical applications. However, note that the mass and inertia matrix identification shown in Figure 10 occurs even for a non-zero and unknown disturbance dual force.

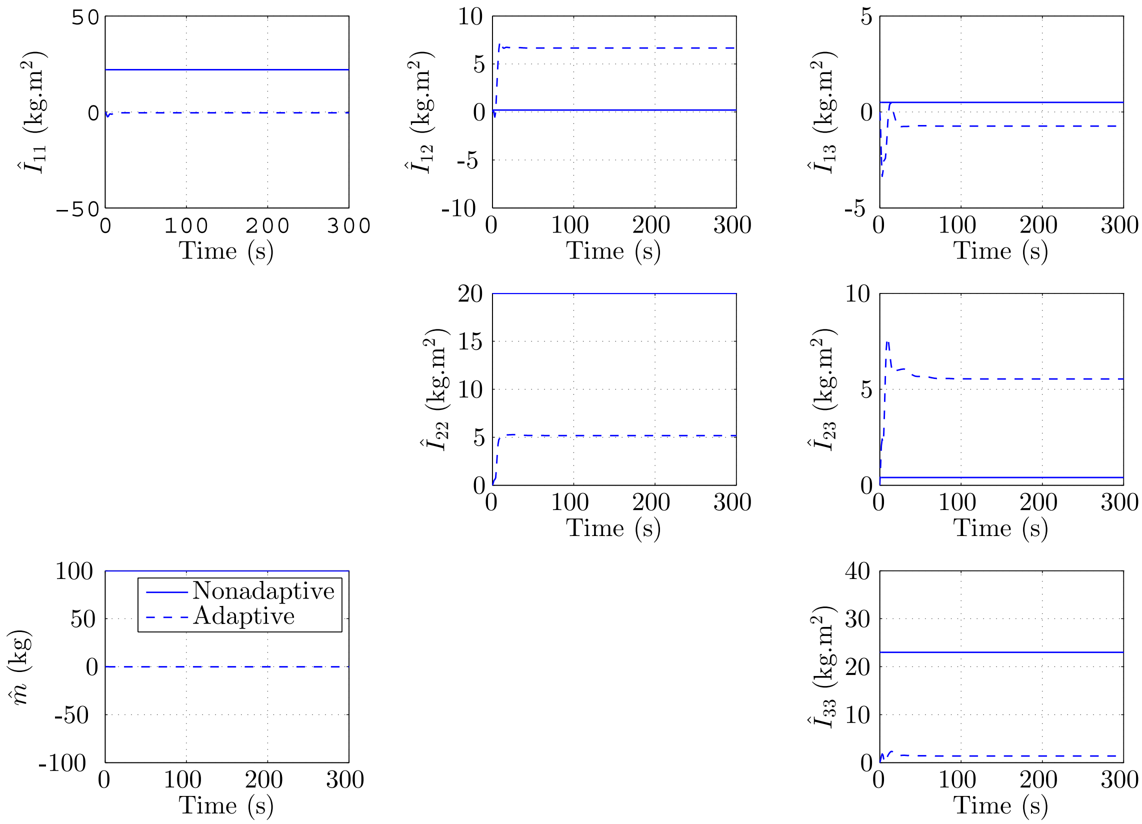

Figure 7. Mass and inertia matrix estimation for low-exciting reference motion.

CONCLUSION

An adaptive position and attitude tracking controller for satellite proximity operations is presented in this paper. The controller requires no information about the mass and inertia matrix of the chaser satellite and takes into account the gravitational force, the perturbing force due to J_2, and the gravity-gradient torque. This controller is shown to be *almost* globally asymptotically stable, even in the presence of constant unknown disturbance forces and torques. Sufficient conditions for mass and inertia matrix identification are given. The numerical simulations of the controller indicate that it could be implemented using electric propulsion. Future work includes combining the proposed controller with the velocity-free pose-tracking controller described in Ref. 16, in order to create a model-independent velocity-free pose-tracking controller.

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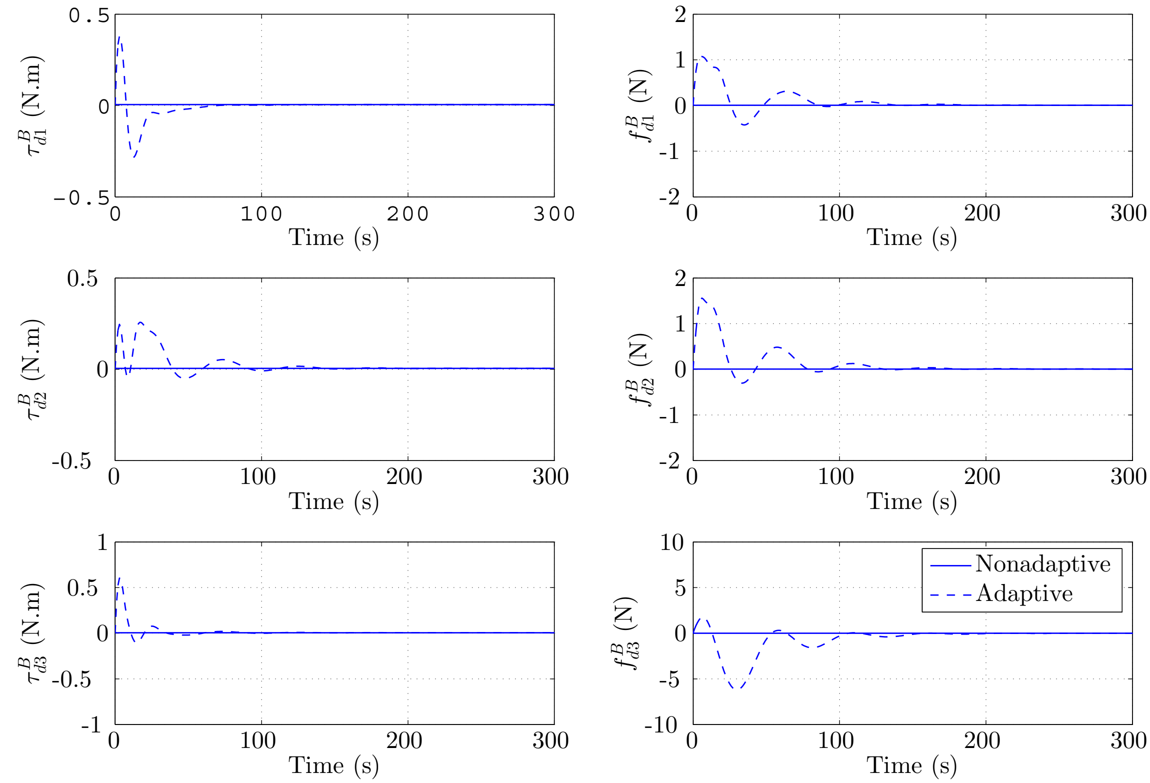

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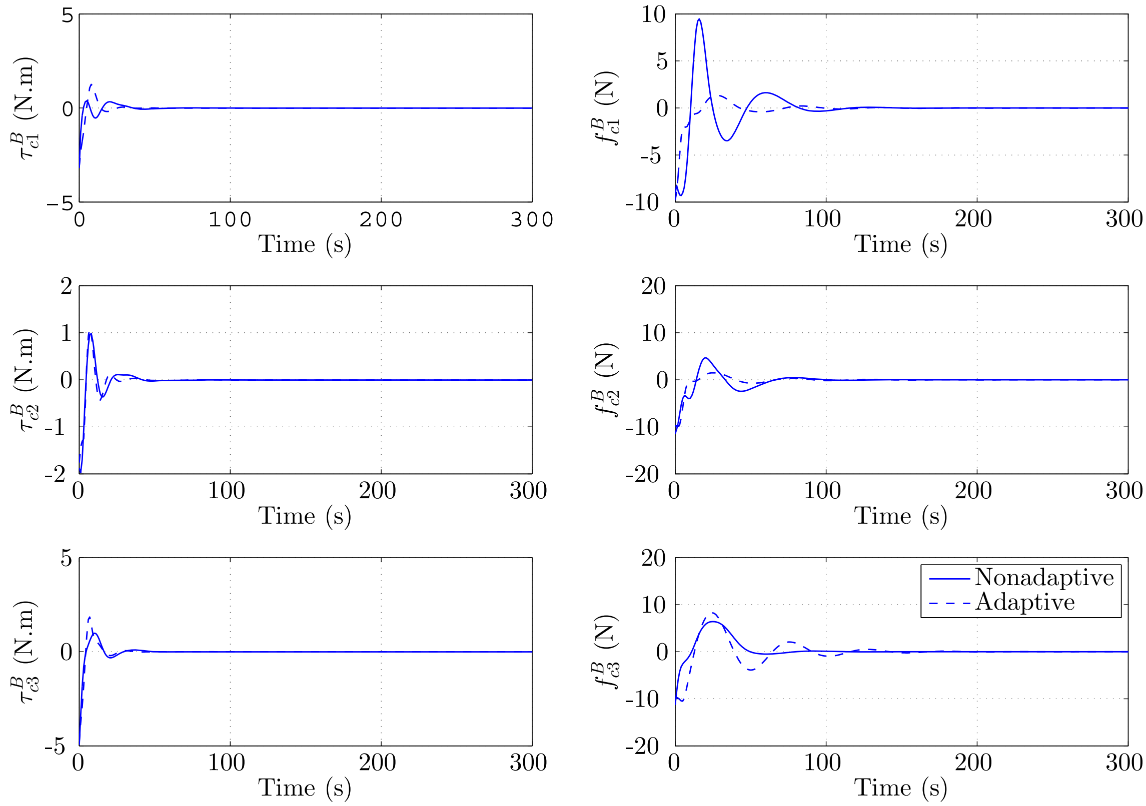

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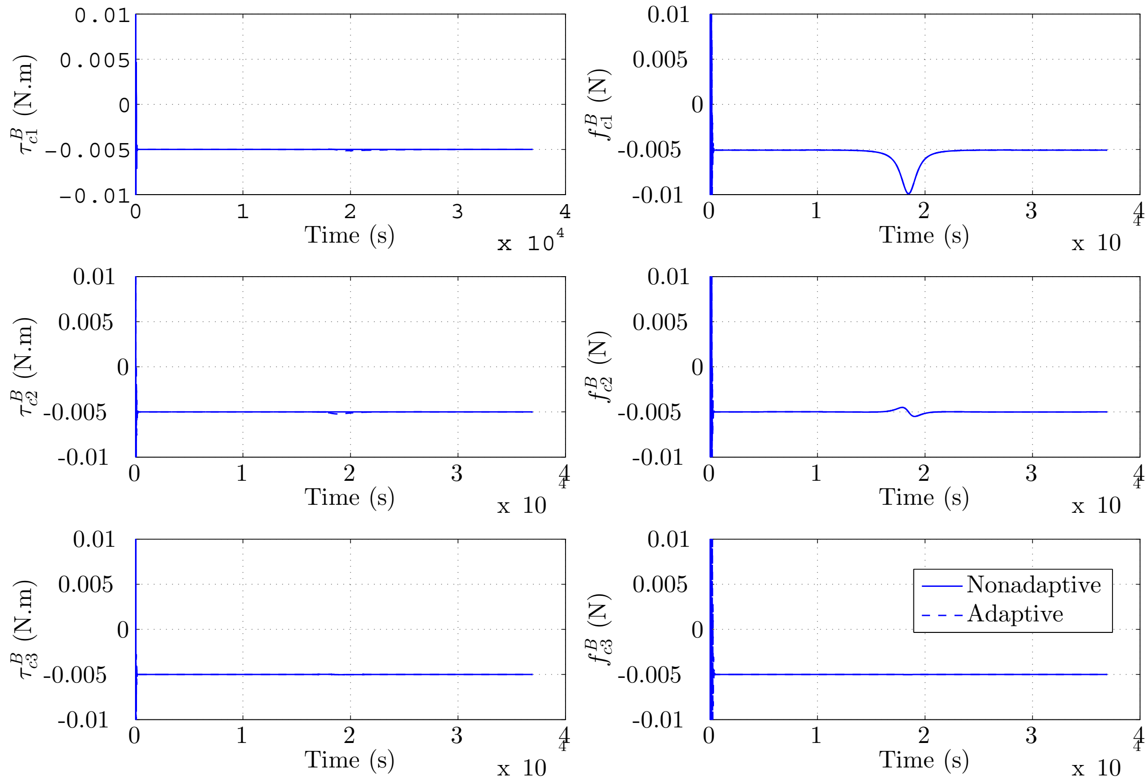

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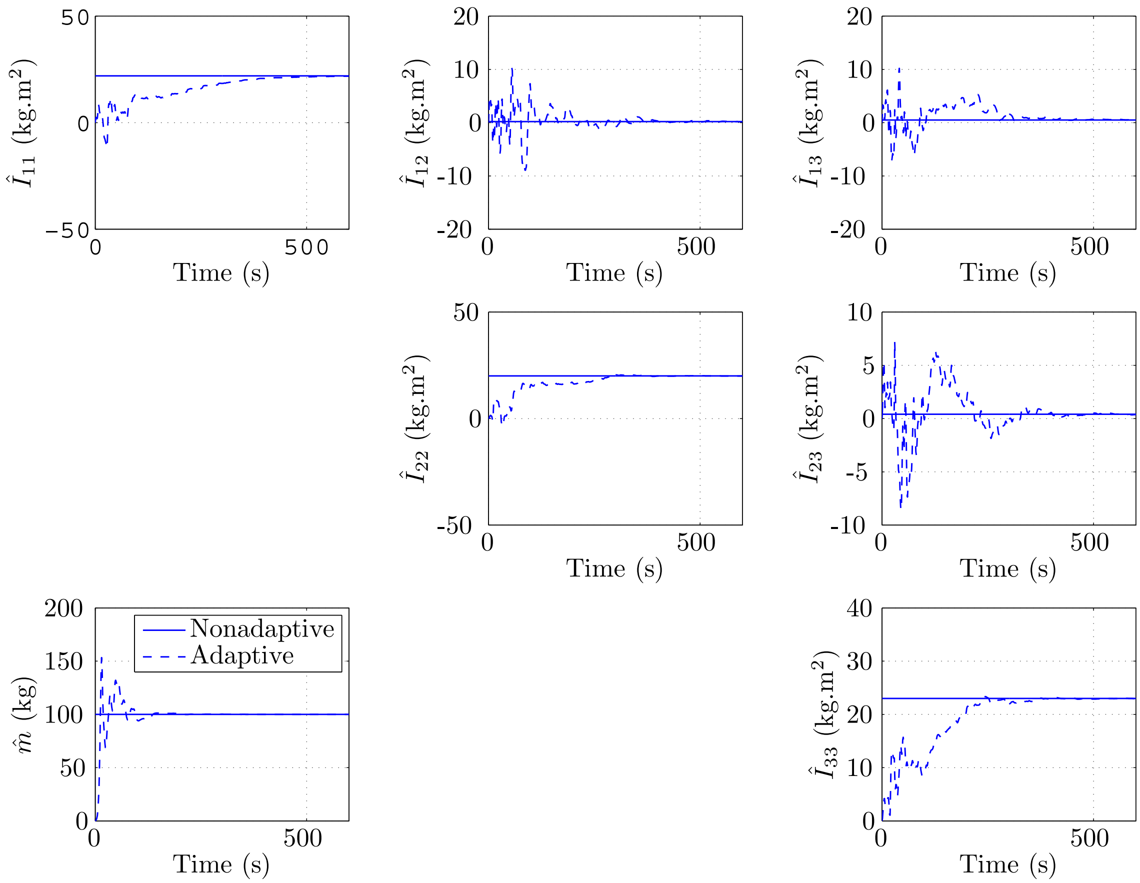

Figure 11. Mass and inertia matrix identification for high-exciting reference motion.