AAS 01-379

SPACECRAFT ADAPTIVE ATTITUDE CONTROL AND POWER TRACKING WITH SINGLE-GIMBALLED VSCMGs AND WHEEL SPEED EQUALIZATION

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A control law for an integrated power/attitude control system (IPACS) for a satellite using variable speed single-gimbal control moment gyros (VSCMG) is introduced. While the wheel spin rates of the conventional CMGs are controlled to be constant, the rates of VSCMGs are allowed to have variable speeds. Therefore, VSCMGs have extra degrees of freedom and can be used for additional objectives such as energy storage as well as attitude control. In this paper VSCMGs are used for an IPACS. The gimbal rates are used to provide the reference-tracking torques, while the wheel accelerations are used both for attitude reference tracking and power tracking. The latter objective is achieved by storing or releasing the kinetic energy in the wheels. The control algorithm performs both attitude and power tracking goals simultaneously. A model-based control and an indirect adaptive control for a spacecraft with uncertain inertia properties are developed. Moreover, control laws for equalization of the wheel speeds are also proposed. Wheel speed equalization minimizes the possibility of wheel speed saturation and avoids zero-speed singularities. Finally, a numerical example for a satellite in a low Earth near-polar orbit is provided to test the proposed IPACS algorithm.

INTRODUCTION

Most spacecraft use chemical batteries to store excess energy generated by the solar panels during periods of exposure to the sun. The batteries are used to provide power for the spacecraft subsystems during the eclipse and are re-charged when the spacecraft is in the sunlight. However, the use of chemical batteries has some problems such as a limited life cycle, shallow discharge depth (approximately 20-30 % of their rated energy-storage capacity), large weight and strict temperature limits (at or below 20°C in a low-Earth orbit). As a matter of fact, these limitations often drive the entire spacecraft thermal design. Moreover, the use of chemical batteries requires additional system mass for controlling the charging and discharging cycles.

An alternative to chemical batteries is the use of flywheels to store energy. The use of flywheels as "mechanical batteries" has the benefit of increased efficiency (up to 90 % depth of discharge with essentially unlimited life), and ability to operate in a relatively hot (up to 40°C) environment. Most

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importantly, flywheels offer the potential to combine the energy-storage and the attitude-control functions into a single device, thus increasing reliability and significantly reducing the overall weight and spacecraft size. This means increasing payload capacity and reducing launch and fabrication costs significantly. This concept, termed the integrated power and attitude-control system (IPACS) has been studied since the 1960s, but it has been particularly popular during the last decade. In fact, the use of flywheels instead of batteries to store energy on spacecraft was suggested as early as 1961 in the paper by Roes,[1] when a 17Wh/kg composite flywheel spinning at 10,000-20,000 rpm on magnetic bearings was proposed. The configuration included two counter-rotating flywheels, and the author did not mention the possibility of using the momentum for attitude control. This idea grew over the next three decades. Refs. 2–4 are representative of the period from 1970-1977, during which the term IPACS was coined.[2] A complete survey on IPACS is given in Ref. 5 and 6.

Up until now, however, this well-documented IPACS concept has never been implemented due to high flywheel spin rates required for an IPACS system[7] (on the order of 40,000 to 80,000 rpm versus less than 5,000 rpm for conventional control moment gyros or momentum wheel actuators). At such high speeds, the actuators quickly wear out mechanical bearings. Additional challenges include flywheel material mass/durability and stiffness inadequacies. Recently, the advent of advanced composite materials and magnetic bearing technology promises to enable a realistic IPACS development.

Since the flywheels are typically used onboard orbiting satellites to control the attitude, a suitable algorithm must be used to simultaneously meet the attitude torques and the power requirements. In Ref. 5 a control law was presented for an IPACS with momentum wheels. In the present article, a control law for an IPACS using variable speed single-gimballed control moment gyros (VSCMG)[8] is introduced. While the wheel spin rates of the conventional CMGs are kept constant, the wheel speeds of the VSCMGs are allowed to vary continuously. Therefore, VSCMGs have extra degrees of freedom and can be used for additional objectives such as energy storage, as well as attitude control. In addition, single-gimbal VSCMG's still have the capability of producing large torques due to their torque amplification property. This makes them ideal for several commercial and military missions.

The VSCMG system stores the kinetic energy by spinning up its wheels during exposure to the sunlight. It provides power for the satellite subsystems by despinning the wheels during the eclipse. The spinning-up/spinning-down operation has to be coordinated in such a manner that the generated torques do not disturb the attitude.

Conventionally, most control designs for the IPACS problem use the linearized equations of motion. In this article, we use the complete, nonlinear equations with minimal assumptions. The derived equations of motion used in this paper for a cluster of VSCMGs are similar to those in Refs. 8 and 9. The only mild assumptions made in deriving these equations are that the spacecraft, flywheels, and gimbal frames are rigid and that the flywheels and gimbals are balanced. In addition, Ref. 8 imposes the additional assumption that the gimbal frame inertia is negligible. No such assumption is necessary in our derivations. Without loss of generality, the gimbal angle rates and reaction wheel accelerations are taken as control inputs to the VSCMG system. That is, a velocity steering law is assumed. This implies that the gimbal angle acceleration is small. The gimbal angle acceleration is then calculated so that the actual gimbal angle rate converges to the desired rate. This is somewhat different than commanding directly gimbal accelerations (i.e., acceleration steering law) that typically results in excessive gimbal torque commands.[8,10]

In addition to the model-based attitude and power tracking control law, an adaptive control concept is also derived to deal with the uncertainty of the inertia properties of spacecraft. For exact attitude tracking, the inertia of spacecraft should be known. However, the inertia of spacecraft may change considerably due to docking, releasing a payload, retrieving a satellite, slushing and/or consumption of fuel etc., so an adaptive control scheme is required for precise attitude tracking control. Several adaptive control laws for the attitude tracking problem have been reported in the literature.[11–16] Most of the previous results use variable thrust gas jets, momentum or reaction wheels or conventional CMGs as actuators. As far as the authors know there are no results for adaptive attitude control for a VSCMG system. There have been a few results of adaptive control for a conventional CMG system, but most of them use the linearized or simplified equations of motion.[12,13] Of particular interest is Ref. 14 where adaptation is used to control a double-gimballed CMG with uncertain inertia properties. The present article offers the first design of an adaptive control using the complete nonlinear equations of motion for a rigid spacecraft with a VSCMG cluster.

One of the difficulties encountered in the use of traditional CMGs is the possibility of singularity (gimbal lock) when control torques cannot be generated along certain directions. In addition, conventional momentum wheels have to deal with wheel speed saturation issues and momentum damping. Control laws for VSCMGs must address both the CMG singularity as well as the momentum wheel saturation problem. Moreover, since the VSCMGs are used as energy storage devices, it is important that none of the VSCMGs despins completely. To avoid this from happening, an algorithm to equalize the wheel speeds of the VSCMG cluster is proposed. Speed equalization is desirable because it can also reduce the possibility of actuator saturation and/or singularity. Two techniques to equalize the wheel speeds are introduced. The merits and pitfalls of each method are discussed in detail. A comparison via numerical examples is provided at the end of the paper.

SYSTEM MODEL

Dynamics

Consider a rigid spacecraft with a cluster of N single-gimbal VSCMGs to provide internal torques. The definition of the axes is shown in Fig. 1.

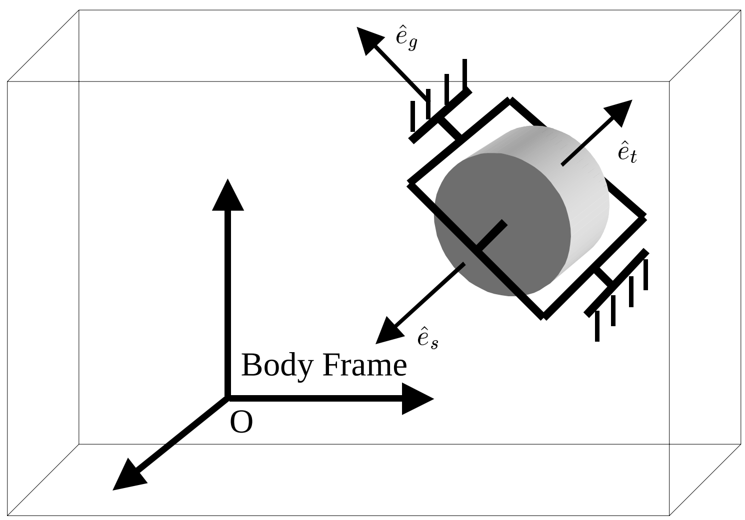


Figure 1: Spacecraft Body with a Single VSCMG

The total angular momentum of a spacecraft with a VSCMG cluster consisting of N devices can be expressed in the spacecraft body frame as

$$h = J\omega + A_g I_{cg} \dot{\gamma} + A_s I_{ws} \Omega \tag{1}$$

where $\gamma = (\gamma_1, \ldots, \gamma_N)^T \in \mathbb{R}^N$ and $\Omega = (\Omega_1, \ldots, \Omega_N)^T \in \mathbb{R}^N$ are column vectors whose elements are the gimbal angles and the wheel speeds of the VSCMGs, respectively. In (1) the matrix J, is the inertia matrix of the whole spacecraft, defined as

$$J(\gamma) = {}^B I + A_s I_{cs} A_s^T + A_t I_{ct} A_t^T + A_g I_{cg} A_g^T \tag{2}$$

where ${}^{B}I$ is the combined matrix of inertia of the spacecraft platform and the point-masses of the VSCMGs. The matrices $I_{c\star}$ and $I_{w\star}$ are diagonal with elements the values of the inertias of the gimbal plus wheel structure and wheels only structure of the VSCMGs, respectively. Specifically, $I_{c\star} = I_{g\star} + I_{w\star}$ where $I_{g\star} = \mathrm{diag}[I_{c\star 1}, \ldots, I_{c\star N}]$ and $I_{w\star} = \mathrm{diag}[I_{w\star 1}, \ldots, I_{w\star N}]$, where $\star$ is g, s or t.

The matrices $A_\star \in \mathbb{R}^{3\times N}$ have as columns the gimbal, spin and transverse directional unit vectors expressed in the body-frame. Thus,

$$A_\star = [e_{\star 1}, \cdots, e_{\star N}]$$

where $e_{\star j}$ is the unit column vector for the jth VSCMG along the direction of the gimbal, spinning, or transverse axis. Note that $A_s = A_s(\gamma)$ and $A_g = A_g(\gamma)$ and thus are both functions of the gimbal angles. Consequently, the inertia matrix $J = J(\gamma)$ is also a function of the gimbal angles γ, whereas the matrix ${}^{B}I$ is constant.

The equations of motion are derived by taking the time derivative of the total angular momentum of the system. If h_c is defined as

$$h_c = A_g I_{cg} \dot{\gamma} + A_s I_{ws} \Omega \tag{3}$$

then $h = J\omega + h_c$ and the time derivative of h with respect to the body B-frame is

$$\dot{h} = \dot{J}\omega + J\dot{\omega} + \dot{h}_c = -[\omega^\times]\, h + g_e \tag{4}$$

where g_e is an external torque (assumed here to be zero for simplicity), and where for any vector $x = [x_1, x_2, x_3]^T \in \mathbb{R}^3$, the notation $[x^\times]$ denotes the matrix

$$[x^\times] = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

The matrices A_g, A_s, and A_t can be written using their initial values at time $t = 0$, A_{g0}, A_{s0}, A_{t0} and the gimbal angles as

$$A_g = A_{g0} \tag{5}$$
$$A_s = A_{s0}[\cos\gamma]^d + A_{t0}[\sin\gamma]^d \tag{6}$$
$$A_t = A_{t0}[\cos\gamma]^d - A_{s0}[\sin\gamma]^d \tag{7}$$

where $\cos\gamma = [\cos\gamma_1, \ldots, \cos\gamma_N]^T \in \mathbb{R}^N$ and $\sin\gamma = [\sin\gamma_1, \ldots, \sin\gamma_N]^T \in \mathbb{R}^N$, and where $[x]^d \in \mathbb{R}^{N\times N}$ denotes a diagonal matrix with its elements the components of the vector $x \in \mathbb{R}^N$,

$$[x]^d = \begin{bmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & x_N \end{bmatrix}$$

Using Eqs. (5)-(7), a simple calculation shows that

$$\dot{A}_s = A_t[\dot{\gamma}]^d, \qquad \dot{A}_t = -A_s[\dot{\gamma}]^d$$

The time derivatives of J and h_c in Eq. (4) are then calculated as

$$\begin{aligned} \dot{h}_c &= A_g I_{cg}\ddot{\gamma} + \dot{A}_s I_{ws}\Omega + A_s I_{ws}\dot{\Omega} \\ &= A_g I_{cg}\ddot{\gamma} + A_t I_{ws}[\Omega]^d\dot{\gamma} + A_s I_{ws}\dot{\Omega} \end{aligned} \tag{8}$$

and

$$\dot{J} = A_t[\dot{\gamma}]^d(I_{cs} - I_{ct})A_s^T + A_s[\dot{\gamma}]^d(I_{cs} - I_{ct})A_t^T \tag{9}$$

where we have made use of the obvious fact that $[\dot\gamma]^d\Omega = [\Omega]^d\dot\gamma$. Finally, the dynamic equations take the form[9]

$$\begin{aligned}(10)\qquad &\Big(A_t[\dot\gamma]^d(I_{cs}-I_{ct})A_s^T + A_s[\dot\gamma]^d(I_{cs}-I_{ct})A_t^T\Big)\omega + J\dot\omega \\ &\quad + A_g I_{cg}\ddot\gamma + A_t I_{ws}[\Omega]^d\dot\gamma + A_s I_{ws}\dot\Omega + [\omega^\times]\Big(J\omega + A_g I_{cg}\dot\gamma + A_s I_{ws}\Omega\Big) = 0\end{aligned}$$

Note that the equations for a VSCMG system can also be applied to a reaction/momemtum wheel system by letting the gimbal angles γ be constant. They can be applied to a conventional CMG system by letting the wheel rotation speeds Ω be constant.

Kinematics

The so-called modified Rodrigues parameters (MRPs) given in Refs. 17, 18 and 19 are chosen to describe the attitude kinematics error of the spacecraft. The MRPs are defined in terms of the Euler principal unit vector η and angle ϕ by

$$\sigma = \eta\tan(\phi/4)$$

The MRPs have the advantage of being well defined for the whole range for rotations,[17,18,20] i.e., $\phi \in [0, 2\pi)$. The differential equation that governs the kinematics in terms of the MRPs is given by

$$(11)\qquad \dot\sigma = G(\sigma)\omega$$

where

$$(12)\qquad G(\sigma) = \frac{1}{2}\Big(\mathbf{I} + [\sigma^\times] + \sigma\sigma^T - [(1+\sigma^T\sigma)/2]\,\mathbf{I}\Big)$$

and $\mathbf{I}$ is the 3×3 identity matrix.

MODEL-BASED ATTITUDE TRACKING CONTROLLER

In this section a control law based on Lyapunov stability theory is derived for the attitude tracking problem. In the sequel, it is assumed that the spacecraft and VSCMGs inertia properties are exactly known.

Lyapunov Stability Condition for Attitude Tracking

Assume that attitude to track is given in terms of the dynamics and kinematics of a desired reference frame (D-frame) in terms of σ_d, ω_d and $\dot\omega_d$. Here σ_d is the MRP vector presenting the attitude of the D-frame w.r.t the inertial frame (N-frame) and ω_d is the angular velocity of the D-frame w.r.t the N-frame expressed in the B-frame. Let ω_d^D denote the angular velocity of the D-frame expressed in its own frame, and let $\dot\omega_d^D$ denote the time derivative w.r.t. the D-frame, assumed to be known. Then the following relationships hold

$$\begin{aligned}\omega_d &= C_D^B\,\omega_d^D \\ \dot\omega_d &= C_D^B\,\dot\omega_d^D - [\omega^\times]\,C_D^B\,\omega_d^D\end{aligned}$$

The angular-velocity tracking error written in the body frame (B-frame) is defined as

$$\omega_e = \omega - \omega_d$$

and σ_e is the error Modified Rodrigues Parameters vector between the reference frame and the body frame calculated from

$$C_D^B(\sigma_e) = C_N^B(\sigma)C_D^N(\sigma_d)$$

The kinematics of the error MRP is then

$$\dot{\sigma}_e = G(\sigma_e)\,\omega_e$$

A feedback control law to render $\omega_e \to 0$ and $\sigma_e \to 0$ is found using the following Lyapunov function[5]

$$V = \frac{1}{2}\omega_e^T J \omega_e + 2k_0 \ln\left(1 + \sigma_e^T \sigma_e\right) \tag{13}$$

where $k_0 > 0$. This function is positive definite and radially unbounded in terms of the tracking errors ω_e and σ_e. The time derivative of V is

$$\begin{aligned} \dot{V} &= \frac{1}{2}(\omega-\omega_d)^T \dot{J}(\omega-\omega_d) + (\omega-\omega_d)^T J(\dot{\omega}-\dot{\omega}_d) + 2k_o \frac{2\sigma_e^T \dot{\sigma}_e}{1+\sigma_e^T\sigma_e} \\ &= -(\omega-\omega_d)^T \left\{ -\frac{1}{2}\dot{J}(\omega-\omega_d) - J(\dot{\omega}-\dot{\omega}_d) - k_0\sigma_e \right\} \end{aligned}$$

The previous equation suggests that for Lyapunov stability, the choice

$$-\frac{1}{2}\dot{J}(\omega-\omega_d) - J(\dot{\omega}-\dot{\omega}_d) - k_0\sigma_e = K_1(\omega-\omega_d) \tag{14}$$

where K_1 is a 3×3 positive definite matrix results in global asymptotic stability of the closed-loop system[§]. Equation (4) then implies that

$$\dot{h}_c + \frac{1}{2}\dot{J}(\omega+\omega_d) = K_1(\omega-\omega_d) + k_0\sigma_e - J\dot{\omega}_d - [\omega^\times]\left(J\omega + A_s I_{ws}\Omega\right) \tag{15}$$

Recall that the lhs of the previous equation contains the control inputs $\dot{\gamma}$ and $\dot{\Omega}$. In particular, it can be shown that

$$\dot{h}_c + \frac{1}{2}\dot{J}(\omega+\omega_d) = B\ddot{\gamma} + C\dot{\gamma} + D\dot{\Omega} = L_{\rm rm}$$

where

$$\begin{aligned} B &= A_g I_{cg} && (16) \\ C &= A_t I_{ws}[\Omega]^d + [\omega^\times]A_g I_{cg} && (17) \\ &\quad + \frac{1}{2}[(e_{s1}e_{t1}^T + e_{t1}e_{s1}^T)(\omega+\omega_d), \cdots, (e_{sN}e_{tN}^T + e_{tN}e_{sN}^T)(\omega+\omega_d)](I_{cs}-I_{ct}) && \\ D &= A_s I_{ws} && (18) \end{aligned}$$

By denoting the rhs of Eq. (15) as the required control torque $L_{\rm rm}$ for attitude tracking

$$L_{\rm rm} = K_1(\omega-\omega_d) + k_0\sigma_e - J\dot{\omega}_d - [\omega\times]\left(J\omega + A_s I_{ws}\Omega\right)$$

one obtains that the control inputs must be chosen as

$$B\ddot{\gamma} + C\dot{\gamma} + D\,\dot{\Omega} = L_{\rm rm} \tag{19}$$

Velocity-Based Steering Law for Attitude Tracking

Typically, the gimbal acceleration term $B\ddot{\gamma}$ can be ignored since the matrix B is small compared to the matrices C and D.[8] In this case $\dot{\gamma}$ and $\dot{\Omega}$ can be used as control inputs instead of $\ddot{\gamma}$ and $\dot{\Omega}$. This

[§]Strictly speaking, the choice of Eq. (14) proves only Lyapunov stability. Asymptotic convergence to the origin follows from a straightforward argument using La Salle's invariant set theory; see, for instance, Ref. 5.

is referred to in the literature as (gimbal) velocity-based steering law. Letting $B\ddot{\gamma} \approx 0$ in Eq. (19) the condition for stabilization then becomes

$$[C \quad D]\begin{bmatrix} \dot{\gamma} \\ \dot{\Omega} \end{bmatrix} = L_{\mathrm{rm}} \tag{20}$$

Since I_{ws} and $[\Omega]^d$ are diagonal matrices and the second and third terms in the rhs of Eq. (17) are relatively small, it follows that the column vectors of the C matrix are almost parallel to the transverse axes of the gimbal structure and the column vectors of the D matrix are parallel to the spin axes of the gimbal structure. Therefore, if there are at least two VSCMGs and their (fixed) gimbal axes are not parallel to each other, and if none of the wheel spin rates becomes zero, the column vectors of C and D always span the 3-dimensional space. It follows that this VSCMG system can generate control torques along an arbitrary direction. In other words, such a VSCMG system never falls into the singularity (gimbal lock) of a conventional CMG system owing to the extra degrees of freedom provided by the wheel speed control. Moreover, if we have three or more VSCMGs, Eq. (30) is underdetermined and there exist null-motion solutions which do not have any effect on the generated control torque.[9, 21] Therefore, we can use this null-motion for power tracking and/or wheel speed equalization. This is discussed in the 'Power Tracking' section below.

ADAPTIVE ATTITUDE TRACKING CONTROLLER

In this section, we design a control law to deal with the uncertainty associated with the spacecraft inertia matrix. Several research results have been published on adaptive attitude control of spacecraft, but most of these results use gas jets and/or reaction/momentum wheels as actuators. In all these cases, the spacecraft inertia matrix J is constant. As previously stated, however, a difficulty arises from the fact that in the VSCMG case the spacecraft inertia matrix J is not constant because it dependents on the gimbal angles γ.

Next, we propose an adaptive control law for the VSCMG case. The approach follows arguments that are similar (but not the same) to standard adaptive control design techniques. In the sequel we assume that the VSCMG cluster inertia properties are exactly known.

Adaptive Control with VSCMGs

In the VSCMG mode, the inertia matrix J is not constant because it depends on the gimbal angles γ. However, the *derivative* of J is known since it is determined by the control gimbal commands $\dot{\gamma}$. In this section we use this important observation to design an adaptive control law which uses estimates of the elements of J. Although indirect adaptive schemes that do not identify the moments of inertia are also possible, knowledge of the inertia matrix is often required to meet other mission objectives. We do not pursue such indirect adaptive schemes in this work. Of course, as with all typical adaptive control schemes, persistency of excitation of the trajectory is required to identify the correct values of the inertia matrix. Nonetheless, in all cases it is shown that the controller stabilizes the system.

First we re-write the equations of the system (10) as follows

$$\frac{1}{2}\dot{J}\omega + J\dot{\omega} + [\omega^\times](J\omega + A_s I_{sw}\Omega) + B\ddot{\gamma} + \tilde{C}\dot{\gamma} + D\dot{\Omega} = 0 \tag{21}$$

where B and D as in Eqs. (16) and (18) and where

$$\begin{aligned}\tilde{C} &= A_t I_{ws}[\Omega]^d + [\omega^\times]A_g I_{cg} \\ &+ \frac{1}{2}[(e_{s1}e_{t1}^T + e_{t1}e_{s1}^T)\omega, \cdots, (e_{sN}e_{tN}^T + e_{tN}e_{sN}^T)\omega](I_{cs} - I_{ct})\end{aligned} \tag{22}$$

We again make the assumption that the term $B\ddot{\gamma}$ can be neglected and hence the system dynamics reduce to

$$\frac{1}{2}\dot{J}\,\omega + J\dot{\omega} + [\omega^{\times}](J\omega + A_s I_{sw}\Omega) + \tilde{C}\dot{\gamma} + D\dot{\Omega} = 0 \tag{23}$$

By differentiating now Eq. (11), one obtains

$$\begin{aligned}\omega &= G^{-1}(\sigma)\dot{\sigma}\\ \ddot{\sigma} &= G(\sigma)\dot{\omega} + \dot{G}(\sigma,\dot{\sigma})\,\omega\end{aligned}$$

and using Eq. (23),

$$\begin{aligned}JG^{-1}(\sigma)\ddot{\sigma} &= JG^{-1}(\sigma)\dot{G}(\sigma,\dot{\sigma})\omega + J\dot{\omega}\\ &= JG^{-1}(\sigma)\dot{G}(\sigma,\dot{\sigma})\omega - [\omega^{\times}](J\omega + A_s I_{sw}\Omega) - \tilde{C}\dot{\gamma} - D\dot{\Omega} - \frac{1}{2}\dot{J}\,\omega\end{aligned}$$

Let now $h_1 = J\omega$ and $h_2 = A_s I_{sw}\Omega$. The equation of the system can then be written in the standard form,

$$H^*(\sigma)\,\ddot{\sigma} + C^*(\sigma,\dot{\sigma})\,\dot{\sigma} = F \tag{24}$$

where

$$\begin{aligned}H^*(\sigma) &= G^{-T}(\sigma)JG^{-1}(\sigma)\\ C^*(\sigma,\dot{\sigma}) &= -G^{-T}(\sigma)JG^{-1}(\sigma)\dot{G}(\sigma,\dot{\sigma})G^{-1}(\sigma) - G^{-T}(\sigma)[h_1^{\times}]G^{-1}(\sigma)\\ F &= G^{-T}(\sigma)[h_2^{\times}]\,\omega - G^{-T}(\sigma)\,(\tilde{C}\dot{\gamma} + D\dot{\Omega}) - \frac{1}{2}G^{-T}(\sigma)\dot{J}\,\omega\end{aligned}$$

Note that the lhs of Eq. (24) is linear in terms of the elements of J which are the unknown parameters to be estimated.

The term $\dot{G}(\sigma,\dot{\sigma})$ can be derived by differentiating Eq. (12) as

$$\dot{G}(\sigma,\dot{\sigma}) = \frac{1}{2}\Big([\dot{\sigma}^{\times}] + \dot{\sigma}\sigma^T + \sigma\dot{\sigma}^T - \dot{\sigma}^T\sigma\,\mathbf{I}\Big) \tag{25}$$

Using now the fact that $\frac{d}{dt}(G^{-1}) = -G^{-1}\dot{G}G^{-1}$ we have

$$\dot{H}^* - 2C^* = \frac{d}{dt}(G^{-T})JG^{-1} - G^{-T}J\frac{d}{dt}(G^{-1}) + 2G^{-T}[h_1^{\times}]G^{-1} + G^{-T}(\sigma)\dot{J}G^{-1}(\sigma)$$

which implies that the matrix $(\dot{H}^* - 2C^* - G^{-T}\dot{J}G^{-1})$ is skew-symmetric.

The remaining procedure follows one of the standard adaptive control design methods.[15] To this end, let $a \in \mathbb{R}^6$ be the parameter vector defined by

$$a = [J_{11}\ J_{12}\ J_{13}\ J_{22}\ J_{23}\ J_{33}]^T \tag{26}$$

and let $\hat{a}$ be the parameter vector estimate. The parameter estimation error is $\tilde{a} = \hat{a} - a$ and $\tilde{\sigma} = \sigma - \sigma_d$ is the attitude tracking error. Consider now the Lyapunov-like function

$$V_a = \frac{1}{2}s^T H^*(\sigma)s + \frac{1}{2}\tilde{a}^T\Gamma^{-1}\tilde{a}$$

where Γ is a strictly positive constant matrix and where $s = \dot{\tilde{\sigma}} + \lambda\tilde{\sigma} = \dot{\sigma} - \dot{\sigma}_r$ $(\lambda > 0)$ is a measure of the attitude tracking error. Note that $\dot{\sigma}_r = \dot{\sigma}_d - \lambda\tilde{\sigma}$ is the reference velocity vector. Differentiating V, and using the skew-symmetry of the matrix $(\dot{H}^* - 2C^* - G^{-T}\dot{J}G^{-1})$, one obtains

$$\dot{V}_a = s^T\Big(F - H^*(\sigma)\ddot{\sigma}_r - C^*(\sigma,\dot{\sigma})\dot{\sigma}_r + \frac{1}{2}G^{-T}(\sigma)\dot{J}G^{-1}(\sigma)s\Big) + \tilde{a}^T\Gamma^{-1}\dot{\hat{a}}$$

Let a control law such that

$$F = \hat{H}^*(\sigma)\ddot{\sigma}_r + \hat{C}^*(\sigma,\dot{\sigma})\dot{\sigma}_r - K_D s - \frac{1}{2}G^{-T}(\sigma)\dot{J}G^{-1}(\sigma)s \tag{27}$$

where $\hat{H}^* = G^{-T}\hat{J}G^{-1}$ and $\hat{C}^* = -G^{-T}\hat{J}G^{-1}\dot{G}G^{-1} - G^{-T}[\hat{h}_1^\times]G^{-1}$ and where K_D is a symmetric positive definite matrix. Then it follows that

$$\dot{V}_a = s^T\left(\tilde{H}^*(\sigma)\ddot{\sigma}_r + \tilde{C}^*(\sigma,\dot{\sigma})\dot{\sigma}_r - K_D s\right) + \tilde{a}^T\Gamma^{-1}(\dot{\hat{a}} - \dot{a})$$

where $\tilde{H}^*(\sigma) = \hat{H}^*(\sigma) - H^*(\sigma)$ and $\tilde{C}^*(\sigma,\dot{\sigma}) = \hat{C}^*(\sigma,\dot{\sigma}) - C^*(\sigma,\dot{\sigma})$. Note that Eq. (9) implies that $\dot{a}$ is known if $\dot{\gamma}$ is known.

The linear parameterization of the dynamics allows us to define a *known* matrix $Y^*(\sigma,\dot{\sigma},\dot{\sigma}_r,\ddot{\sigma}_r)$ such that

$$\tilde{H}^*(\sigma)\ddot{\sigma}_r + \tilde{C}^*(\sigma,\dot{\sigma})\dot{\sigma}_r = Y^*(\sigma,\dot{\sigma},\dot{\sigma}_r,\ddot{\sigma}_r)\,\tilde{a} \tag{28}$$

Choosing the adaptation law as

$$\dot{\hat{a}} = -\Gamma(Y^*)^T s + \dot{a} \tag{29}$$

yields $\dot{V}_a = -s^T K_D s \le 0$. The last inequality implies boundedness of s and $\tilde{a}$ and, in addition, that $s \to 0$. Using standard arguments[15, 22] it follows that $\sigma \to \sigma_d$. Therefore, global asymptotic stability of the attitude tracking error is guaranteed.

From Eq. (27) it follows that the required control inputs are obtained by solving

$$[C \quad D]\begin{bmatrix}\dot{\gamma}\\ \dot{\Omega}\end{bmatrix} = L_{\mathrm{ra}} \tag{30}$$

where $D = A_s I_{ws}$ and

$$\begin{aligned} C &= A_t I_{ws}[\Omega]^d + [\omega^\times]A_g I_{cg} \\ &\quad + \frac{1}{2}[(e_{s1}e_{t1}^T + e_{t1}e_{s1}^T)(\omega + G^{-1}\dot{\sigma}_r), \cdots, (e_{sN}e_{tN}^T + e_{tN}e_{sN}^T)(\omega + G^{-1}\dot{\sigma}_r)](I_{cs} - I_{ct}) \end{aligned} \tag{31}$$

and where

$$L_{\mathrm{ra}} = -G^T(\sigma)\left(\hat{H}^*(\sigma)\ddot{\sigma}_r + \hat{C}^*\dot{\sigma}_r - K_D s\right) + [h_2^\times]\,\omega \tag{32}$$

Once $\dot{\gamma}$ is known from the solution of Eq. (30), it can be substituted in the adaptive control law in Eq. (29).

POWER TRACKING

In Ref. 5 a solution to the simultaneous attitude and power tracking problem was given for the case of a rigid spacecraft with N momentum wheels. In this section we extend these results to the case of N VSCMGs. By setting the gimbal angles to a constant value, we can retrieve the results of Ref. 5 as a special case.

The total (useful) kinetic energy stored in the momentum wheels is

$$T = \frac{1}{2}\Omega^T I_{ws}\Omega$$

Hence, the power (rate of change of the energy) is given by

$$\begin{aligned} P &= \frac{\mathrm{d}T}{\mathrm{d}t} = \Omega^T I_{ws}\dot{\Omega} \\ &= \begin{bmatrix} 0 & \Omega^T I_{ws} \end{bmatrix}\begin{bmatrix}\dot{\gamma}\\ \dot{\Omega}\end{bmatrix} \end{aligned} \tag{33}$$

This equation is augmented to the attitude tracking equation (20) or (30), to obtain the equation for IPACS with VSCMG as follows

$$Qu = L_{\mathrm{rp}} \tag{34}$$

where

$$u = \begin{bmatrix} \dot{\gamma} \\ \dot{\Omega} \end{bmatrix}, \qquad Q = \begin{bmatrix} C_{3\times N} & D_{3\times N} \\ 0_{1\times N} & (\Omega^T I_{ws})_{1\times N} \end{bmatrix}, \qquad L_{\mathrm{rp}} = \begin{bmatrix} L_r \\ P \end{bmatrix} \tag{35}$$

and P is the required power and L_{r} is either L_{rm} or L_{ra}, depending on the attitude controller used.

Solution of Velocity Steering Law for IPACS

If Q is full row rank the solution of Eq. (34) can be calculated from

$$u = Q^T (QQ^T)^{-1} L_{\mathrm{rp}} \tag{36}$$

If the C matrix in Eq. (35) has rank 3 (is full row rank), then the matrix Q is also full row rank and the control can be calculated from Eq. (36). If, however, the C matrix has rank two or one then Q may be rank deficient and a solution can be calculated from $u = Q^\dagger L_{\mathrm{rp}}$, where $Q^\dagger$ is the Moore-Penrose inverse of Q. In this case, simultaneous attitude and power tracking is not possible, except in very special cases.[5]

Although the rank deficiency of the C matrix can be reduced using more VSCMGs, the possibility of a singularity still remains. Moreover, if the minimum norm solution of Eq. (34) is used for control, this solution tends to steer the gimbals toward the rank deficiency states.[23–25] This happens because the projection of the generated torques along the required torque direction is maximum when the transverse axis of the gimbal (the axis along which torque can be generated in CMG mode) is close to the required torque. Thus, the minimum norm solution tends to use the gimbals whose configuration is far from the rank deficiency states. There exist research results which propose methods for keeping the C matrix full rank using null-motion.[8,9,21,25] In particular, Schaub and Junkins[8] suggested a singularity avoidance method using a VSCMG system.

It is advantageous for the VSCMGs to act as conventional CMGs in order to make the most out of the torque amplification effect, which is the most significant merit of the CMGs. A weighted minimum norm solution, which minimizes the weighted cost

$$\mathcal{J}_2 = u^T W^{-1} u \tag{37}$$

can be used to operate between the CMG and MW modes.[8] For example, if the weighting matrix W is defined as

$$W = \begin{bmatrix} w_1 e^{-w_2 \sigma_c} \mathbf{I}_N & 0_N \\ 0_N & \mathbf{I}_N \end{bmatrix} \tag{38}$$

where σ_c is the condition number of C (the ratio of the largest to the smallest singular value) and w_1 and w_2 are positive gains chosen by the user, the weighted minimum norm solution control law is given by

$$u = WQ^T (QWQ^T)^{-1} L_{\mathrm{rp}} \tag{39}$$

In case Q is not full row rank, the solution can be obtained from

$$u = W^{\frac{1}{2}} (QW^{\frac{1}{2}})^\dagger L_{\mathrm{rp}} \tag{40}$$

Note that according to the condition number of the matrix C, the VSCMG can operate either as a MW (close to a CMG singularity, i.e., when σ_c is large) or as a regular CMG (away from a singularity i.e., when σ_c is small). As a CMG singularity is approached, the VSCMG's will smoothly switch to a momentum wheel mode. As a result, this method can also handle temporary rank deficiencies of the matrix C.[8] In this work, the condition number of matrix C is used as a measure of closeness of the matrix C to being rank-deficient. Larger condition numbers mean a more "singular" matrix C. This is a more reliable measure of rank-deficiency of a matrix than, say, the determinant of the matrix.[26]

Notice that a purely MW mode can be enforced by letting W in (39) be

$$W = \begin{bmatrix} 0_N & 0_N \\ 0_N & \mathbf{I}_N \end{bmatrix}$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix. A conventional CMG operation is enforced if W in (39) is chosen as

$$W = \begin{bmatrix} \mathbf{I}_N & 0_N \\ 0_N & 0_N \end{bmatrix}$$

WHEEL SPEED EQUALIZATION

If some of the wheel spin rates become too small, a change of the gimbal angle cannot generate the required torque. If this is the case, then the remaining degrees of freedom may not be enough to allow exact attitude and power tracking. On the other hand, if some of the wheel spin rates become too large, some of the wheels may saturate. Desaturation of the wheels requires thruster firing, thus depleting valuable fuel. To minimize the possibility of singularity and/or the saturation problem, it is desired to equalize the wheel spinning rates of the VSCMGs, whenever possible. Next we propose two control laws to achieve wheel speed equalization for a VSCMG-based IPACS.

The first method adds an extra constraint that forces the wheel speeds to converge to the average wheel speed of the cluster. By introducing

$$\mathcal{J}_{w1} = \frac{1}{2}\sum_{i=1}^{N}(\Omega_i - \bar{\Omega})^2 \tag{41}$$

where $\bar{\Omega} = \frac{1}{N}\sum_{i=1}^{N}\Omega_i$, the condition for equalization is expressed as the requirement that

$$\frac{d}{dt}\mathcal{J}_{w1} = \nabla\mathcal{J}_{w1}\,\dot{\Omega} = \sum_{i=1}^{N}\frac{\partial\mathcal{J}_{w1}}{\partial\Omega_i}\dot{\Omega}_i = -k_2\mathcal{J}_{w1}$$

where $k_2 > 0$. This condition is augmented in Eq. (34) and the control input u is calculated from this augmented equation. Summarizing, the control law that achieves attitude and power tracking with wheel speed equalization is given by

$$\begin{bmatrix} C & D \\ 0 & \Omega^T I_{ws} \\ 0 & \nabla\mathcal{J}_{w1} \end{bmatrix}\begin{bmatrix} \dot{\gamma} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} L_{\mathrm{r}} \\ P \\ -k_2\mathcal{J}_{w1} \end{bmatrix} = L_{\mathrm{rp}} \tag{42}$$

and the use of (39).

The second method uses a modified cost of (37) in which the directions of wheel speed *changes* are considered. The cost to be minimized in this case is expressed as

$$\mathcal{J}_{w2} = u^T\,W^{-1}\,u + R\,u \tag{43}$$

The weighting matrix R is determined so that wheels which rotate faster or slower than the average wheel speed are suitably penalized. For instance, one may choose

$$R = \begin{bmatrix} 0_{1\times N} & k_3\Omega_e^T \end{bmatrix} \tag{44}$$

where $\Omega_e = \Omega - \bar{\Omega}\mathbf{1}_{N\times 1}$, $k_3 > 0$, and $\mathbf{1}_{N\times 1}$ is $N \times 1$ vector whose elements are 1's. The motivation for this choice for R stems from the following observation. Notice that with R as in (44) we have $Ru = \sum_{i=1}^{N}(\Omega_i - \bar{\Omega})\dot{\Omega}_i$. If $\Omega_i > \bar{\Omega}$ for some i, then Ru is minimized by choosing $\dot{\Omega}_i < 0$, i.e, by making Ω_i tend closer to $\bar{\Omega}$. If, on the other hand $\Omega_i < \bar{\Omega}$ then Ru is minimized by choosing $\dot{\Omega}_i > 0$ forcing again Ω_i towards $\bar{\Omega}$. Of course, the linear term Ru does not have an unconstrained minimum hence a quadratic term is included in (43) to ensure that the minimization problem has a finite solution.

The solution that minimizes the cost (43) subject to the equality constraint (34) is

$$u = W\left(Q^T(QWQ^T)^{-1}(L_{\mathrm{rp}} + QWR^T) - R^T\right) \tag{45}$$

In case Q is not full row rank, the equation

$$u = W^{\frac{1}{2}}(QW^{\frac{1}{2}})^{\dagger}(L_{\mathrm{rp}} + QWR^T) - WR^T$$

can be used, instead. Note that this method is identical with the control law without wheel speed equalization if $k_3 = 0$.

Each of the previous two wheel speed equalization algorithms has their own merits and pitfalls. The first one guarantees exact equalization for the IPACS. However, the first method uses an additional degree of freedom since one has to solve the augmented liner system (42). The second method, on the other hand, shows a tendency for wheel speed equalization but it does not guarantee perfect equalization of wheel speeds, in general. The wheel speeds tend to become equal away from the CMG singularity, but they exhibit a bifurcation near the singularity, since the torques for attitude control must be generated from changes of wheel speeds. However, this method does not use any additional degrees of freedom. If some other objectives such as a singularity avoidance strategy is desired, the second method may be preferable.

NUMERICAL EXAMPLES

A numerical example for a satellite in a low Earth orbit is provided to test the proposed IPACS algorithm. Similarly to Refs. 7,8, we use a standard four VSCMG pyramid configuration. In this configuration the VSCMGs are installed so that the four gimbal axes form a pyramid with respect to the body. The angle of the pyramid sides to the base is given by θ. Table 1 contains the parameters used for the simulation. These parameters closely parallel those used in Refs. 7,8.

A simulation scenario is presented to demonstrate the validity of the adaptive IPACS and speed equalization control algorithms given in the previous sections. In the first scenario a near-polar orbital satellite (the orbital data are chosen as in Ref. 5). The satellite's boresight axis is required to track a ground station, and the satellite is required to rotate about its boresight axis so that the solar panel axis is perpendicular to the satellite-sun axis in order to maximize the efficiency of the panel. During the eclipse, the nominal power requirement is 680 W, with additional requirement of 4-kW power for 5 min. During sunlight, the wheels are charged with a power level of 1 kW until the total energy stored in the wheels reaches 1.5 kWh. These attitude and power tracking requirements are the same as in Ref. 5. The details of the method used to generated the required attitude, body rate and body acceleration are also given in the same reference. In this scenario, the spacecraft body frame is initially aligned with the inertia frame. The control gains are chosen as

$$\begin{array}{llll} K_D = 4\times 10^3\,\mathbf{I}_{3\times 3}, & \Gamma = 1\times 10^7\,\mathbf{I}_{6\times 6}, & \lambda = 0.01, & k_2 = 2\times 10^{-3} \\ k_3 = 2\times 10^{-3}, & K_4 = 2\,\mathbf{I}_{4\times 4}, & w_1 = 1\times 10^{-4}, & w_2 = 1 \end{array}$$

All of the initial parameter estimates are chosen to be zero, which means no initial information about the inertia matrix is available. The results of the numerical simulations are shown below. Figure 2 shows that the spacecraft attitude tracks the desired attitude exactly after a short period of

Table 1: SIMULATION PARAMETERS

Symbol	Value	Units
N	4	–
θ	54.75	deg
$\omega(0)$	$[0,0,0]^T$	rad/sec
$\dot{\omega}(0)$	$[0,0,0]^T$	rad/sec^2
$\sigma(0)$	$[0,0,0]^T$	–
$\gamma(0)$	$[\pi/2,-\pi/2,-\pi/2,\pi/2]^T$	rad
$\dot{\gamma}(0)$	$[0,0,0]^T$	rad/sec^2
${}^B I$	$\begin{bmatrix} 15053 & 3000 & -1000 \\ 3000 & 6510 & 2000 \\ -1000 & 2000 & 11122 \end{bmatrix}$	kg m^2
I_{ws}	diag{0.7, 0.7, 0.7, 0.7}	kg m^2
I_{wt}, I_{wg}	diag{0.4, 0.4, 0.4, 0.4}	kg m^2
I_{gs}, I_{gt}, I_{gg}	diag{0.1, 0.1, 0.1, 0.1}	kg m^2

time. Figure 3 shows that the actual power profile also tracks the required power command exactly. These figures show that the goal of IPACS is achieved successfully. Figure 4 shows the wheel speed histories when each of the two wheel equalization methods is applied. The corresponding gimbal angles and control signals for both methods are shown in Fig. 5 and 6. The attitude histories are similar for both cases.

As seen from Fig. 4(a) the first method achieves exact speed equalization, whereas the second method equalizes the wheels only approximately; see Fig. 4(b). In fact, after the condition number of the matrix C becomes large – lower right plot in Fig. 6 – the second method switches to a MW mode and thus the wheel speeds deviate from each other. The first method still keeps the wheel speeds equalized after the sudden change of the required power profile, whereas the second method shows a tendency of divergence. As expected, in both cases, the wheels spin-up (charge) during sunlight and despin (discharge) during the eclipse.

It is worth pointing out that in all simulations the moment of inertia matrix has been assumed to be completely unknown. Despite this fact, the adaptive control law achieves both attitude and power tracking while equalizing the wheel speeds, as desired.

CONCLUSIONS

In this article, we have developed algorithms for controlling the spacecraft attitude while simultaneously tracking a desired power profile by using a cluster of VSCMGs. For attitude tracking, both a model-based control which assumes exact knowledge of the spacecraft inertia matrix, and an adaptive control that deals with the uncertainty of the inertia matrix have been proposed. These control laws have been augmented with a power tracking algorithm, to solve for a velocity steering law for an IPACS. The scheme is identical with the previous results that decompose the torque into two perpendicular spaces one for attitude control and the other for power tracking.[5,6] Though the VSCMG system does not exhibit gimbal lock in the attitude tracking mode, singularities may still occur if a power profile must also be followed. This problem can be solved using any of the singularity avoidance methods used for a conventional CMG system. A wheel speed equalization method has also been devised to reduce the possibility of singularity and actuator saturation problems. Numerical examples based on a realistic scenario demonstrate the efficacy of the proposed

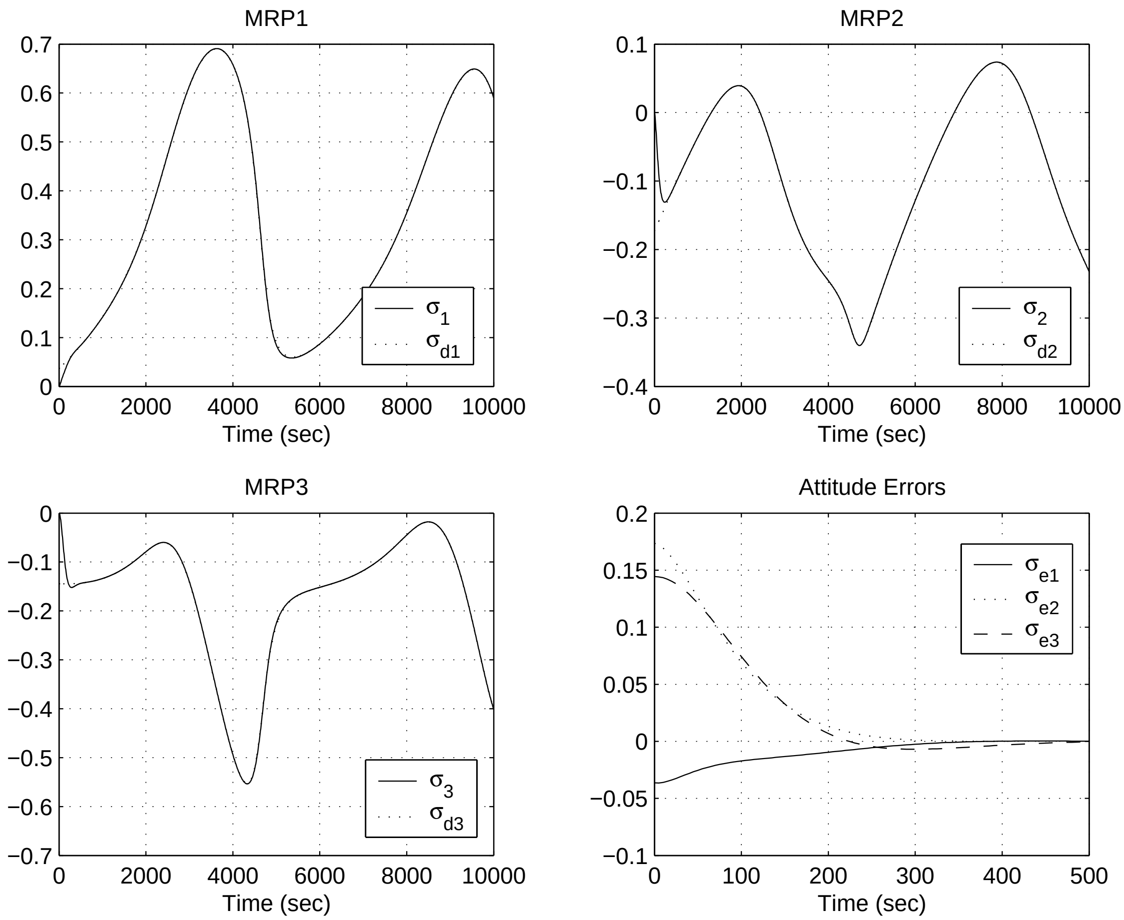


Figure 2: Desired/Actual Attitude and Error Trajectories

control methods.

Acknowledgment: Support for this work has been provided through the AFOSR award F49620-00-1-0374.

References

[1] Roes, J. B., "An Electro-Mechanical Energy Storage System for Space Application," *Progress in Astronauticas and Rocketry*, Vol. 3, Academic Press, 1961, pp. 613–622.

[2] Anderson, W. and Keckler, C., "An Integrated Power/Attitude Control System (IPACS) for Space Application," *Proceedings of the 5th IFAC Symposium on Automatic Control in Space*, 1973.

[3] Cormack III, A., "Three Axis Flywheel Energy and Control Systems," Tech. Rep., NASA TN-73-G&C-8, North American Rockwell Corp., 1973.

[4] Notti, J., Cormack III, A., Schmill, W., and Klein, W., "Integrated Power/Attitude Control System (IPACS) Study : Volume II - Conceptual Designs," Tech. Rep., NASA CR-2384, Rockwell International Space Division, Downey, CA, 1974.

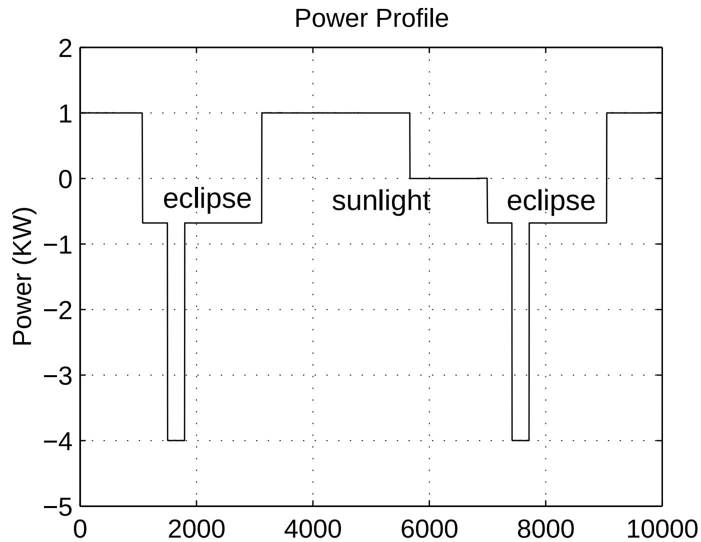


Figure 3: Power Profile

[5] Tsiotras, P., Shen, H., and Hall, C., "Satellite Attitude Control and Power Tracking with Energy/Momentum Wheels," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 1, 2001, pp. 23–34.

[6] Hall, C. D., "High-Speed Flywheels for Integrated Energy Storage and Attitude Control," *American Control Conference*, 1997, pp. 1894–1898, Albuquerque, NM.

[7] Richie, D. J., Tsiotras, P., and Fausz, J. L., "Simultaneous Attitude Control and Energy Storage using VSCMGs : Theory and Simulation," *Proceeding of American Control conference*, 2001, pp. 3973–3979.

[8] Schaub, H., Vadali, S. R., and Junkins, J. L., "Feedback Control Law for Variable Speed Control Moment Gyroscopes," *Journal of the Astronautical Sciences*, Vol. 46, No. 3, 1998, pp. 307–328.

[9] Ford, K. A. and Hall, C. D., "Singular Direction Avoidance Steering for Control-Moment Gyros," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 4, 2000, pp. 648–656.

[10] Oh, H. and Vadali, S., "Feedback Control and Steering Laws for Spacecraft Using Single Gimbal Control Moment Gyro," *The Journal of the Astronautical Science*, Vol. 39, No. 2, 1991, pp. 183–203.

[11] Ahmed, J., Coppola, V., and Bernstein, D., "Adaptive Asymptotic Tracking of Spacecraft Attitude Motion with Inertia Matrix Indentification," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 5, 1998, pp. 684–691.

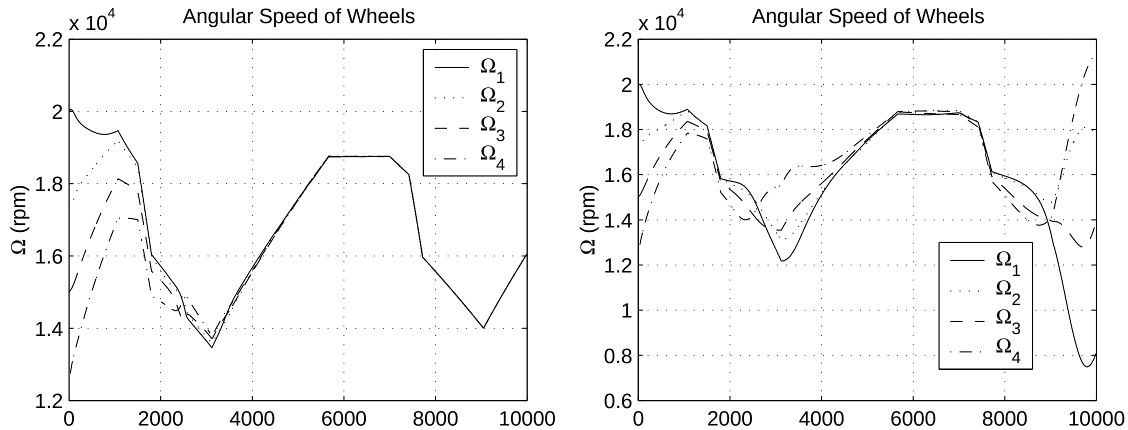
[12] Bishop, R., Paynter, S., and Sunkel, J., "Adaptive Control of Space Station with Control Moment Gyros," *IEEE Control Systems Magazine*, Vol. 12, Oct.1992, pp. 23–28.

[13] Sheen, J. and Bishop, R., "Adaptive Nonlinear Control of Spacecraft," *American Control Conference*, Baltimore, Maryland, 1994, pp. 2867–2871.

[14] Ahmed, J. and Bernstein, D., "Adaptive Control of a Dual-Axis CMG with an Unbalanced Rotor," *Proceedings of the 37th IEEE Conference on Decision and Control*, Tampa, FL, Dec.1992, pp. 4531–4536.

[15] Slotine, J. and Li, W., *Applied Nonlinear Control*, Prentice Hall, Englewood Cliff, NJ, 1991.

[16] Zaremba, A., "An Adaptive Scheme with Parameter Identification for Spacecraft Attitude Control," *Proceedings of American Control Conference*, 1997, pp. 552–556.



(a) Method 1

(b) Method 2

Figure 4: Angular Wheel Speeds with Speed Equalization

[17] Tsiotras, P., "Stabilization and Optimality Results for the Attitude Control Problem," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 4, 1996, pp. 772–777.

[18] Schaub, H. and Junkins, J. L., "Stereographic Orientation Parameters for Attitude Dynamics: A Generalization of the Rodrigues Parameters," *Journal of the Astronautical Sciences*, Vol. 44, No. 1, 1996, pp. 1–19.

[19] Shuster, M. D., "A Survey of Attitude Representations," *Journal of the Astonautical Sciences*, Vol. 41, No. 4, 1993, pp. 439–517.

[20] Tsiotras, P., Junkins, J. L., and Schaub, H., "Higher Order Cayley-Transforms with Applications to Attitude Representations," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 3, 1997, pp. 528–534.

[21] Schaub, H. and Junkins, J. L., "Singularity Avoidance Using Null Motion and Variable-Speed Control Moment Gyros," *Journal of Guidance Control, and Dynamics*, Vol. 23, No. 1, 2000, pp. 11–16.

[22] Khalil, H. K., *Nonlinear Systems*, Prentice Hall, New Jersey, 2nd ed., 1996.

[23] Margulies, G. and Auburn, J. N., "Geometric Theory of Signle-Gimbal Control Moment Gyro Systems," *Journal of the Astronautical Science*, Vol. 26, No. 2, 1978, pp. 159–191.

[24] Cornick, D. E., "Singularity Avoidance Control Laws for Single Gimbal Control Moment Gyros," *Proceedings of the AIAA Guidance and Control Conference*, AIAA, New York, 1979, pp. 20–33.

[25] Heiberg, C. J., Bailey, D., and Wie, B., "Precision Spacecraft Pointing Using Single-Gimbal Control Moment Gyroscopes with Disturbance," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 1, 2000, pp. 77–85.

[26] Horn, R. and Johnson, C. R., *Matrix Analysis*, Cambridge University Press, Cambridge, United Kingdom, 1985.

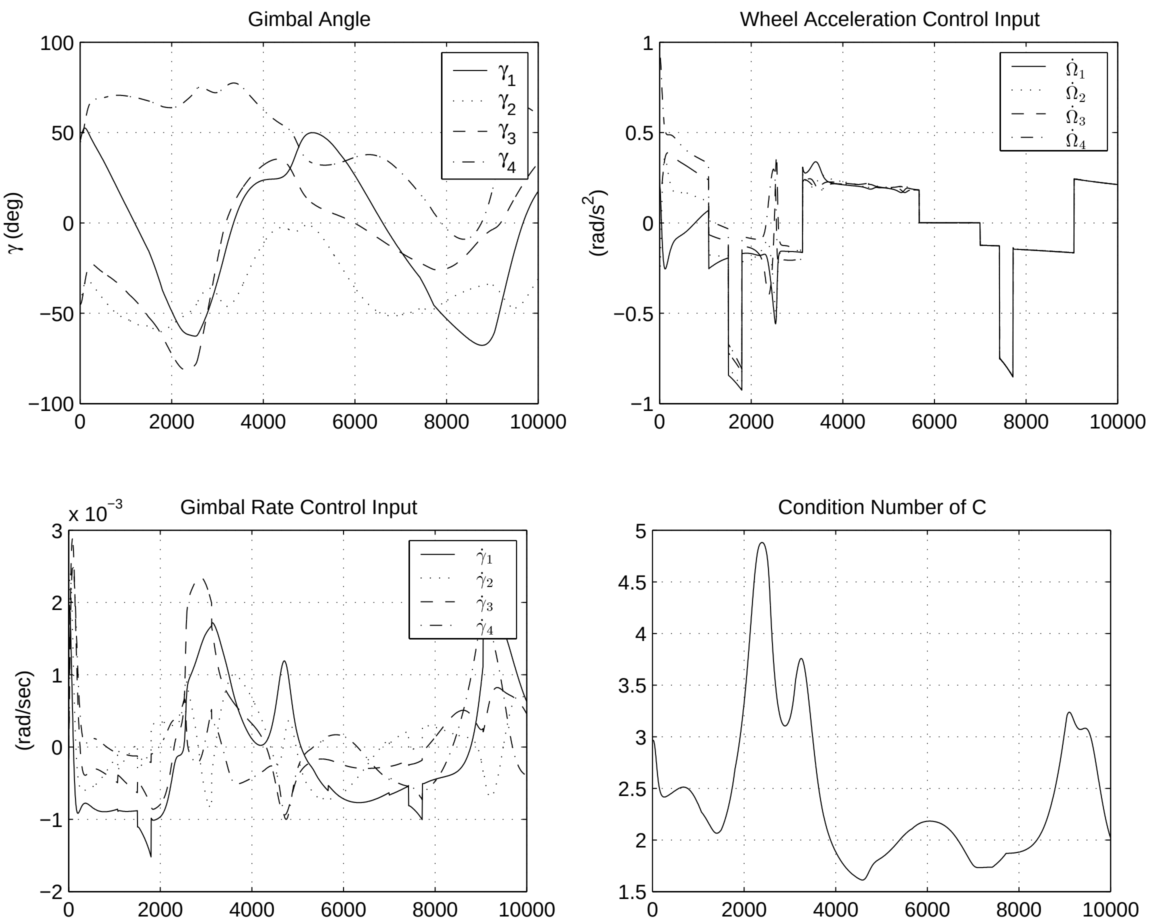


Figure 5: Gimbal Angles, Control Inputs and Condition Number of Matrix C (Method 1)

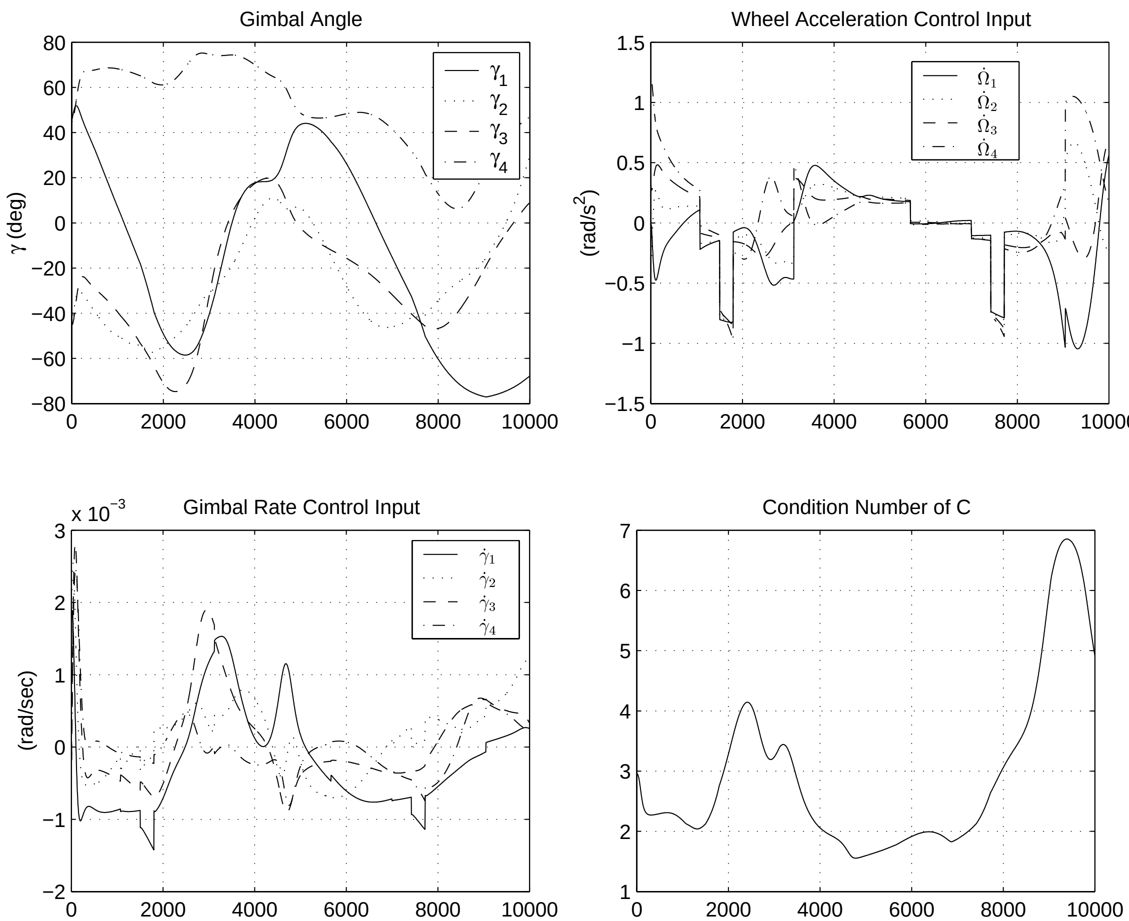


Figure 6: Gimbal Angles, Control Inputs and Condition Number of Matrix C (Method 2)