A COOPERATIVE P2P REFUELING STRATEGY FOR CIRCULAR SATELLITE CONSTELLATIONS

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In this paper, we discuss the problem of peer-to-peer (P2P) refueling of satellites in a circular constellation. In particular, we propose a cooperative P2P (C-P2P) refueling strategy, in which the satellites involved in P2P maneuvers are allowed to engage in cooperative rendezvous. We discuss a formulation of the proposed C-P2P strategy and a methodology to determine the optimal C-P2P assignments. We show that in order to reduce the fuel expenditure in a C-P2P maneuver, the amount of fuel exchanged between the two satellites is such that the satellite performing the larger-ΔV transfer during the return trip, ends up having just enough amount of fuel to be fuel-sufficient. Finally, with the help of numerical examples, we provide a comparison of the P2P and the C-P2P refueling strategies. It is found that a C-P2P strategy is beneficial when the fuel-deficient satellites in the constellation do not have enough fuel to complete a non-cooperative rendezvous.

INTRODUCTION

On-orbit servicing (OOS) of spacecraft has received significant attention in the last decade. Although the current practice in space industry is to replace the spacecraft after their design lifetime, there have also been a few instances of on-orbit servicing. The first instances of OOS can be traced to the servicing missions for the SkyLab Space Station in 1970s. OOS missions were also undertaken for the Solar Maximum Mission (SMM) and the Russian Space Station. The most visible instance of an OOS mission was perhaps the repair of the Hubble Space Telescope (HST).[1–4]

Waltz defines OOS as work done in space by man or machine or by both. He classifies the objectives of OOS into three broad categories: assembly, maintenance, and servicing. Reynerson[5] introduced a notion of cost in describing on-orbit servicing and he defined a serviceable spacecraft as one for which the benefits of OOS outweigh the associated cost. A recent customer-centric approach to studying OOS classifies the objectives of OOS into three functions, namely life extension, upgrade, and modification.[2,3]

Replenishment of consumables (e.g., propellant) is one aspect of OOS. Satellites need a regular fuel-budget for stationkeeping. Providing fuel-deficient satellites with propellant has significant benefits by extending their lifetime. The potential profitability of refueling relatively lightweight geostationary communication satellites with long lifetimes has been emphasized in Ref. 6. Saleh et al.[3] provide numerical examples that point out the promise of refueling in OOS operations. The authors of Ref. 3 also remark that refueling presents little risk, but offers immense gains if it is performed at the end of the spacecraft lifetime. An account of technical and economic feasibility of on-orbit satellite servicing can also be found in Ref. 7. It should be noted that apart from extending the lifetime of satellites, refueling capabilities of a servicing mission enables extraordinary mission flexibility by allowing for orbital maneuvering, which would otherwise considerably shorten the spacecraft lifetime because of high fuel consumption.

The conventional wisdom suggests refueling one or more fuel-deficient satellites in a constellation using a single refueling spacecraft.[8] Recently, an alternative refueling strategy has been investigated by the second

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author and his students. This is the so-called peer-to-peer (P2P) refueling strategy.[9–12] In a P2P strategy satellites distribute fuel amongst themselves in the absence of a single refueling spacecraft. This is achieved by having satellites with excess fuel sharing their resources (propellant) with those depleted of it. Although a stand-alone P2P strategy might seem unconventional at first glance, P2P comes as a natural choice for distributing fuel amongst several satellites in a mixed refueling strategy.[10,11] In such a scenario, an external refueling spacecraft, either launched from Earth or coming from a different orbit, replenishes half of the satellites in a constellation before returning back to its original orbit. The satellites which receive fuel from the external refueling spacecraft distribute the fuel with the rest of the satellites in the constellation via P2P refueling. Numerical studies have shown that such a mixed refueling strategy is a competitive alternative to the single-service vehicle refueling strategy and, in fact, outperforms the latter, as the number of satellites in the constellation increases and/or the time to refuel decreases.[10] Furthermore, the incorporation of additional cost-reducing strategies, such as the coasting time allocation strategy and asynchronous P2P maneuvers,[11] leads to further improvements by reducing the fuel expenditure of the P2P phase of the mixed refueling strategy even more.

The original studies[9,11] perceived P2P refueling as a means of equalizing fuel in the constellation. A subsequent alternative formulation imposed a minimum fuel requirement on each satellite, and perceived P2P refueling as a means of achieving fuel-sufficiency[d] for all satellites in the constellation.[12] An extension of P2P refueling, known as the Egalitarian P2P (E-P2P) strategy, has been shown to further reduce the overall fuel expenditure during the refueling process.[13–15]

In all previous work on P2P refueling all rendezvous between the satellites participating in a fuel exchange have been assumed to be non-cooperative. In other words, one of the satellites is active and performs all the orbital maneuvers, whereas the other satellite is passive and stays in its original orbital slot throughout the whole process. In general, the rendezvous need not be non-cooperative. In fact, cooperative rendezvous (between two single satellites) has been studied in the literature for quite some time. The earliest works on cooperative rendezvous considered a rendezvous between systems with linear or non-linear dynamics and with various performance indices.[16,17] The idea of using differential games to study cooperative rendezvous problems has also been discussed.[18] The optimal terminal maneuver of the active satellites engaged in a cooperative impulsive rendezvous has been studied in Ref. 19. The determination of the optimal terminal maneuver involves the optimization of the common velocity vector after the rendezvous. Methods to determine optimal fixed-time impulsive cooperative rendezvous using primer vector theory were given in Ref. 20. These methods accommodate cases of fuel-constraints on the satellites themselves, and enable the addition of a mid-course impulse to the trajectory. For the case of fixed-time impulsive maneuvers, cooperative rendezvous may be advantageous when the time allotted for the maneuver is relatively short. Examples show that a non-cooperative solution becomes cheaper once the time allotted for the rendezvous is large enough for Hohmann transfers to be feasible. The minimum fuel rendezvous of two power-limited spacecraft has also been studied using non-linear analysis as well as Clohessy-Wiltshire (C-W) equations.[21,22] For spacecraft engaging in a rendezvous maneuver, cooperative rendezvous is always found to be cheaper than a non-cooperative rendezvous. Constrained and unconstrained circular terminal orbits have also been analyzed in Ref. 21, where it has been found that the cooperative solution still remains the cheaper option. Analytical solutions using the C-W equations can be used to predict the nature of the terminal orbit of the rendezvous. For instance, in the case of a cooperative rendezvous between two satellites in a circular orbit, the two satellites meet at an orbital slot that is mid-way between the two original slots, each satellite essentially removing half of the phase angle.[21]

In all previous studies on P2P refueling,[9–14] only non-cooperative rendezvous had been considered. The primary contribution of this paper is the application of cooperative rendezvous to the problem of P2P refueling. The goal is to reduce the fuel expenditure incurred during the refueling process. As in all previous work on the subject, all orbital transfers considered in this work are time-fixed two-impulse orbital transfers.

In the forthcoming sections, we review the P2P refueling strategy, and provide a formulation for the solution of the C-P2P strategy refueling problem. We also determine the amount of fuel shared by satellites engaged in a C-P2P maneuver so that the fuel expenditure incurred during the maneuver is minimum. Finally, with the help of numerical examples, we illustrate the merits of C-P2P refueling strategies.

[d]By fuel-sufficiency, we mean that a satellite has a sufficient amount of fuel to remain operational.

P2P REFUELING STRATEGY

In this section, we will discuss in detail the P2P refueling strategy. This sets the stage for the subsequent developments. We first introduce the basic notation and we then formulate the problem as an optimization problem over a (bipartite) constellation graph.

Notations

Let us consider a circular constellation consisting of n satellites, distributed over n orbital slots in a circular orbit of radius R. Let the set of n satellites be given by $\mathcal{S} = \{s_i : i = 0, 1, 2, \ldots, n\}$, where s_0 represents a fictitious satellite, the purpose of which will become clear shortly. Let the set of n orbital slots be given by $\Phi = \{\phi_i \in [0, 2\pi) : i = 1, 2, \ldots, n,\ \phi_i \neq \phi_j\}$. We introduce a mapping $\sigma_t : \Phi \mapsto \mathcal{S}$ that, at time $t \geq 0$, assigns to each orbital slot a satellite from $\mathcal{S}$. In particular, $\sigma_t(\phi_j) = s_i$ implies that the satellite s_i occupies the orbital slot ϕ_j at time t. If the slot ϕ_j is empty at time t, we write $\sigma_t(\phi_j) = s_0$. Also, let the fuel content of satellite s_i at time t be denoted by $f_{i,t}$. In particular, let the initial fuel content of satellite s_i be denoted by f_i^- and the final fuel content be denoted by f_i^+; that is, $f_i^- = f_{i,0}$ and $f_i^+ = f_{i,T}$, where T is the time allotted for refueling. Also, let $\underline{f}_i$ denote the minimum amount of fuel for the satellite s_i to remain operational, and let $\bar{f}_i$ denote the maximum fuel capacity of the same satellite. *Fuel-sufficient* satellite are those which have at least the required amount of fuel; the remaining satellites are *fuel-deficient*.

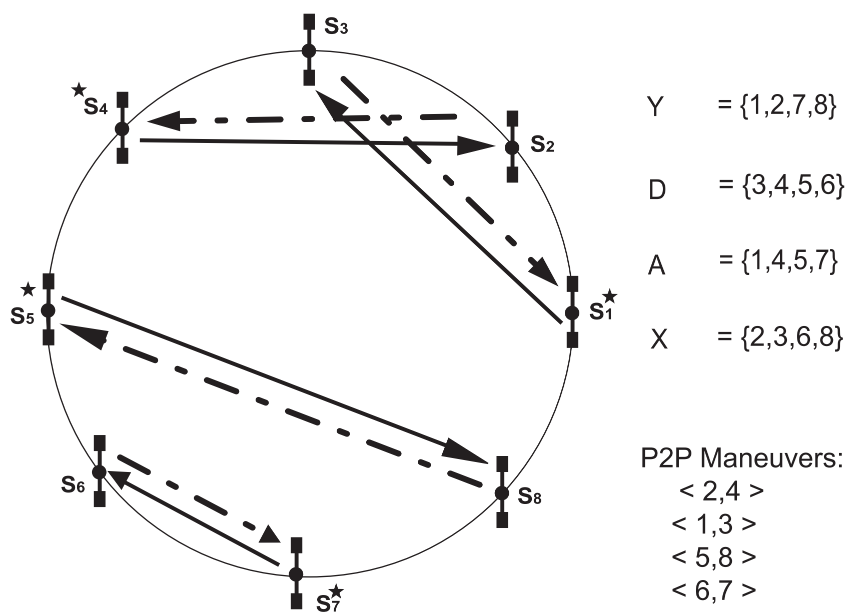


Figure 1. Notation for P2P refueling.

It the following, we will need to keep track of the indices of the satellites participating in the refueling process under different roles. To this end, let $\mathcal{I} = \{1, 2, \ldots, n\}$. The fuel-sufficient satellites have excess fuel and are thereby capable of sharing this fuel with other satellites in the constellation. The fuel-deficient satellites are low on (or depleted of) fuel. Let $\mathcal{I}_{s,0}$ denote the set comprised of the indices of the fuel-sufficient satellites, and let $\mathcal{I}_{d,0}$ denote the set having as elements the indices of the fuel-deficient ones. Clearly, $\mathcal{I}_{s,t} = \{i : f_{i,t} \geq \underline{f}_i\}$, $\mathcal{I}_{d,t} = \{i : f_{i,t} < \underline{f}_i\}$ and $\mathcal{I}_{s,0} \cup \mathcal{I}_{d,0} = \mathcal{I}$. The objective of P2P refueling is therefore to achieve $f_i^+ \geq \underline{f}_i$ for all $i \in \{1, 2, \ldots, n\}$ by expending the minimum amount of fuel during the ensuing orbital transfers.

During a P2P refueling transaction between a fuel-sufficient and a fuel-deficient satellite, one of them (henceforth referred to as the *active* satellite) performs an orbital transfer to rendezvous with the other satellite (henceforth referred to as the *passive* satellite). After the fuel exchange takes place between the two, the active satellite returns to its original orbital slot. We will denote the index set of active satellites by $\mathcal{I}_a \subseteq \mathcal{I}$ and the index set of passive satellites by $\mathcal{I}_p \subset \mathcal{I}$. For convenience, let also $\mathcal{J}_{s,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{s,t}\}$ denote the index set of orbital slots occupied by the fuel-sufficient satellites at time t, and let $\mathcal{J}_{d,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{d,t}\}$ denote the index set of the orbital slots occupied by fuel-deficient satellites at time t. Also, let $\mathcal{J}_a = \{j : \sigma_0(\phi_j) = s_i, i \in \mathcal{I}_a\}$ denote the index set of the orbital slots occupied by the active satellites before any orbital maneuver commences. Finally, let $\mathcal{J}_p = \{j : \sigma_0(\phi_j) = s_i, i \in \mathcal{I}_a\}$ denote the index set of the orbital slots occupied by the passive satellites before any orbital maneuver commences. Figure 1 illustrates these concepts. For the situation depicted in Fig. 1, we assume that $\sigma_0(\phi_i) = s_i$. Also, satellites s_1, s_2, s_7 and s_8 are the fuel-sufficient satellites while the remaining ones are the fuel-deficient satellites. The active satellites are marked with '$\star$', the forward trips are marked by a solid arrow, and the return trips are marked by a dashed arrow. Furthermore, for each satellite s_i, we denote the mass of its permanent structure by m_{spi} and the specific thrust of its engine by I_{spi}. We denote the gravitational acceleration on the surface of the earth by g_0. For each satellite s_i, we therefore define the characteristic constant as $c_{0i} = g_0 I_{\text{spi}}$. Finally, we denote the optimal rendezvous cost required for an orbital transfer from slot ϕ_i to slot ϕ_j by ΔV_{ij}. The fuel expended by satellite s_μ to perform the orbital transfer from slot ϕ_i to slot ϕ_j will be denoted by p_{ij}^μ.

Fuel Expenditure in a P2P Maneuver

Let us consider a P2P maneuver between satellite s_μ, initially occupying the orbital slot ϕ_i, and satellite s_ν, initially occupying the orbital slot ϕ_j. Hence, $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$. Without loss of generality, assume s_μ to be a fuel-sufficient satellite and s_ν to be a fuel-deficient satellite, that is, $\mu \in \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_{d,0}$. Either of the two satellites may be active during a refueling transaction. Accordingly, two different refueling transactions are possible. In the first case, the fuel-sufficient satellite s_μ is active. Therefore, $\mu \in \mathcal{I}_a \cap \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_p \cap \mathcal{I}_{d,0}$. The fuel consumed by the active satellite s_μ to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j is given by:

$$p_{ij}^\mu = \left(m_{\text{s}\mu} + f_\mu^-\right)\left(1 - e^{-\frac{\Delta V_{ij}}{c_{0\mu}}}\right). \tag{1}$$

The fuel content of satellite s_μ after its forward trip (but before fuel exchange takes place) is $f_\mu^- - p_{ij}^\mu$. After the fuel exchange takes place between the two satellites, s_μ performs another orbital transfer and returns to its original orbital slot ϕ_i. Since the fuel consumption during the transfer is minimized when the active satellite returns to its final slot with exactly the required minimum amount of fuel to remain operational, the amount of fuel consumed during the return trip is given by

$$p_{ji}^\mu = \left(m_{\text{s}\mu} + \underline{f}_\mu\right) e^{\frac{\Delta V_{ji}}{c_{0\mu}}}\left(1 - e^{-\frac{\Delta V_{ji}}{c_{0\mu}}}\right). \tag{2}$$

In order for satellite s_ν to become fuel-sufficient after the fuel transaction, we must therefore have,

$$\left(f_\nu^- + f_\mu^-\right) - \left(\underline{f}_\mu + \underline{f}_\nu\right) \geq p_{ij}^\mu + p_{ji}^\mu. \tag{3}$$

If the above condition does not hold, then the P2P refueling transaction is not feasible. Also, if satellite s_μ does not have enough fuel to carry out the orbital transfer during the forward trip, that is, if $p_{ij}^\mu \geq f_\mu^-$, then the P2P refueling transaction is also not feasible.

In the second case, the fuel-deficient satellite s_ν is active. The fuel consumed for the active satellite s_ν to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j is given by

$$p_{ji}^\nu = \left(m_{\text{s}\nu} + f_\nu^-\right)\left(1 - e^{-\frac{\Delta V_{ji}}{c_{0\nu}}}\right). \tag{4}$$

The fuel content of satellite s_ν after its forward trip (but before fuel exchange takes place) is $f_\nu^- - p_{ji}^\nu$. The amount of fuel consumed during the return trip (during which the satellite s_ν travels from the orbital slot

ϕ_i to the orbital slot ϕ_j) is given by

$$p_{ij}^{\nu} = \left(m_{s\nu} + \underline{f}_{\nu}\right) e^{\frac{\Delta V_{ij}}{c_{0\nu}}} \left(1 - e^{-\frac{\Delta V_{ij}}{c_{0\nu}}}\right). \tag{5}$$

Before the return trip (but after the fuel exchange takes place) the fuel on board satellite s_ν is $\underline{f}_\nu + p_{ij}^\nu$. The fuel transferred to satellite s_ν during the fuel exchange is $(\underline{f}_\nu + p_{ij}^\nu) - (f_\nu^- - p_{ji}^\nu)$. The fuel on board satellite s_μ after the fuel transaction is $f_\mu^- - (\underline{f}_\nu + p_{ij}^\nu) + (f_\nu^- - p_{ji}^\nu)$. In order for the satellite s_μ to be fuel-sufficient after the fuel transaction, we must have

$$\left(f_\mu^- + f_\nu^-\right) - \left(\underline{f}_\nu + \underline{f}_\mu\right) \geq p_{ji}^\nu + p_{ij}^\nu. \tag{6}$$

If the above condition does not hold, then a P2P refueling transaction is not feasible. Also, if the satellite s_ν does not have enough fuel to carry out the orbital transfer during the forward trip, that is, if $p_{ji}^\nu \geq f_\nu^-$, then the P2P refueling transaction is also not feasible.

P2P Formulation

Consider an undirected bipartite graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the two partitions being $\mathcal{J}_{s,0}$ and $\mathcal{J}_{d,0}$. There exists an edge $\langle i, j\rangle \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0}$ if the satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ can engage in a P2P refueling transaction such that at the end of the refueling process, both the satellites end up being fuel-sufficient. Let $\mathcal{E} \subseteq \mathcal{J}_{s,0} \times \mathcal{J}_{d,0}$ be the set of all edges in $\mathcal{G}$. To each edge $\langle i, j\rangle \in \mathcal{E}$, we assign a cost c_{ij} that equals the fuel expenditure incurred during the P2P refueling transaction between the two corresponding satellites. Recognizing that either of the two satellites engaged in a P2P refueling transaction can be the active one, we define the cost associated with each edge $\langle i, j\rangle$ as follows:

$$c_{ij} = \begin{cases} p_{ij}^\mu + p_{ji}^\mu, & \text{if } s_\mu \text{ can be active, but } s_\nu \text{ cannot,} \\ p_{ji}^\nu + p_{ij}^\nu, & \text{if } s_\nu \text{ can be active, but } s_\mu \text{ cannot,} \\ \min\{p_{ij}^\mu + p_{ji}^\mu, p_{ji}^\nu + p_{ij}^\nu\}, & \text{if either } s_\mu \text{ or } s_\nu \text{ can be active,} \\ \infty, & \text{if neither } s_\mu \text{ nor } s_\nu \text{ can be active.} \end{cases} \tag{7}$$

We are interested in a set $\mathcal{M} \subseteq \mathcal{E}$ of $|\mathcal{I}_{d,0}|$ edges that has minimum total cost, and such that all fuel-deficient satellites are involved in fuel transactions. Let us also associate with each edge $\langle i, j\rangle \in \mathcal{E}$ a binary variable x_{ij} defined as

$$x_{ij} = \begin{cases} 1, & \text{if } \langle i, j\rangle \in \mathcal{M}, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

We thereby have the following optimization problem:

$$\min_{\mathcal{M} \subseteq \mathcal{E}} \sum_{\langle i,j\rangle \in \mathcal{E}} c_{ij} x_{ij}, \tag{9}$$

such that

$$\sum_{j \in \mathcal{J}_{d,0}} x_{ij} \leq 1, \text{ for all } i \in \mathcal{J}_{s,0}, \tag{10}$$

$$\sum_{i \in \mathcal{J}_{s,0}} x_{ij} = 1, \text{ for all } j \in \mathcal{J}_{d,0}, \tag{11}$$

Constraint (10) implies that a fuel-sufficient satellite can be assigned to at most one refueling transaction, while constraint (11) implies that a fuel-deficient satellite has to be assigned to a refueling transaction.

Next, we illustrate the P2P refueling scenario with a couple of examples. To this end, let us consider some sample constellations given in Table 1. The optimal assignments for each case can be obtained by solving the optimization problem outlined in the previous section. In particular, we discuss in detail the optimal P2P assignments obtained in the case of constellations C_1 and C_1'.

Table 1. Sample Constellations.

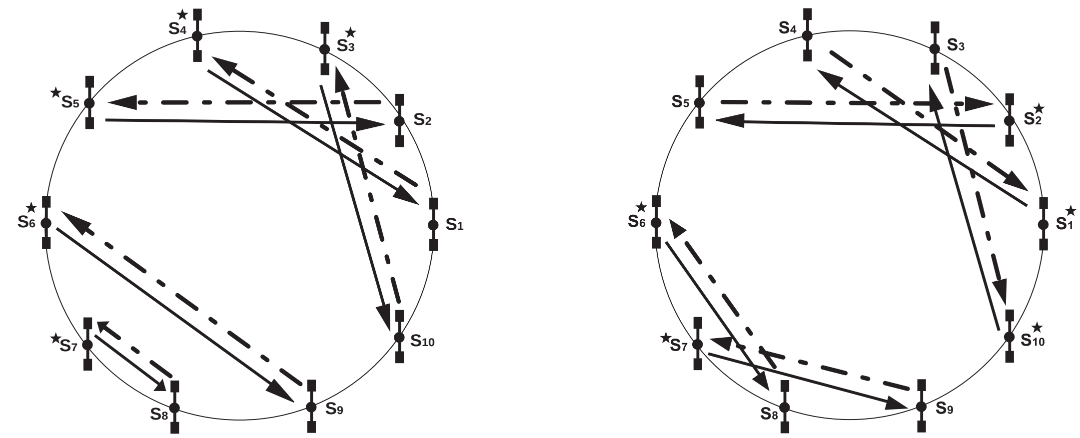
Label	Description
C_1	10 satellites, Altitude $= 35,786$ Km, $T = 12$ f_i^-: $30,30,6,6,6,6,6,30,30,30$ $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{si} = 70$ for all satellites
C_1'	10 satellites, Altitude $= 35,786$ Km, $T = 12$ f_i^-: $30,30,1.5,1.5,1.5,1.5,1.5,30,30,30$ $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{si} = 70$ for all satellites
C_2	16 satellites, Altitude $= 1,200$ Km, $T = 30$ f_i^-: $30,30,30,30,30,30,1.5,1.5,1.5,1.5,1.5,1.5,1.5,1.5,1.5,1.5$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_3	12 satellites, Altitude $= 12,000$ Km, $T = 20$ f_i^-: $25,25,25,25,25,25,2,2,2,2,2,2$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_4	18 satellites, Altitude $= 6,000$ Km, $T = 25$ f_i^-: $25,25,25,25,25,25,25,25,25,6,6,6,6,6,6,6,6,6$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_5	14 satellites, Altitude $= 1,400$ Km, $T = 35$ f_i^-: $25,25,25,25,25,25,25,1.5,1.5,1.5,1.5,1.5,1.5,1.5$ $\bar{f}_i = 25$, $\underline{f}_i = 10$, $m_{si} = 75$ for all satellites

Example 1. *P2P refueling strategy for a constellation of* 10 *satellites.*

Consider the constellation C_1 given in Table 1. This constellation consists of 10 satellites evenly distributed in a circular orbit. The maximum allowed time for refueling is $T = 12$ orbital periods. Each satellite s_i, where $i = 1, \ldots, 10$, has a minimum fuel requirement of $\underline{f}_i = 12$ units, while the maximum amount of fuel each satellite can hold is $\bar{f}_i = 30$ units. Each satellite has a permanent structure of $m_{si} = 70$ units, and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{J}_{s,0} = \{1, 2, 8, 9, 10\}$ and those of the fuel-deficient satellites are $\mathcal{J}_{d,0} = \{3, 4, 5, 6, 7\}$. The optimal P2P assignments obtained after solving the optimization problem (9)-(11) is $s_4 \rightarrow s_1$, $s_5 \rightarrow s_2$, $s_7 \rightarrow s_8$, $s_6 \rightarrow s_9$, $s_3 \rightarrow s_{10}$, and the total fuel consumption for all P2P maneuvers is 26.07 units. The indices of the active satellites in this case are $\mathcal{J}_a = \{3, 4, 5, 6, 7\}$. Note that $\mathcal{J}_a = \mathcal{J}_{d,0}$, that is, the fuel-deficient satellites are the active ones for the P2P refueling strategy. The optimal P2P assignments are shown in Fig. 2(a). The active satellites are marked by '$\star$'. The forward trips are marked by solid arrows, while the return trips are marked by dotted arrows. The fuel-deficient satellites, having the lesser mass than their fuel-sufficient counterparts, incur much lesser fuel expenditure during their forward trips. This results in lower fuel expenditure during the overall refueling process and hence, we all the fuel-deficient satellites are active.

Example 2. *P2P refueling strategy when all satellites do not have enough fuel to be active.*

We consider the constellation C_1' which is the same as the constellation C_1 of Example 1, except that now the fuel-deficient satellites contain only 1.5 units of fuel (not sufficient to carry out some of the large-ΔV transfers). Hence, it is not possible for all the fuel-deficient satellites to be active. In this case, the optimal P2P assignments are: $s_1 \rightarrow s_4$, $s_2 \rightarrow s_5$, $s_6 \rightarrow s_8$, $s_7 \rightarrow s_9$, $s_{10} \rightarrow s_3$. The total fuel expended during all P2P maneuvers is 29.61. Figure 2(b) shows the optimal P2P maneuvers in the constellation. Note that $\mathcal{J}_{d,0} = \{3, 4, 5, 6, 7\}$ and $\mathcal{J}_a = \{1, 2, 6, 7, 10\}$, so that we no longer have $\mathcal{J}_{d,0} = \mathcal{J}_a$. The 1.5 units of fuel for satellites s_4, s_5 and s_3 are not sufficient to complete the forward trip. Instead, it is possible to carry out the P2P maneuvers by having the fuel-sufficient satellites s_1, s_2 and s_{10} to be active.



(a) Constellation C_1 (b) Constellation C_1'

Figure 2. Optimal assignments for P2P refueling.

C-P2P REFUELING STRATEGY

In this section, we formulate the Cooperative P2P (C-P2P) refueling problem as an optimization problem over a suitable bipartite constellation graph. Recall that in the C-P2P strategy, we allow cooperative rendezvous between the satellites engaging in a P2P maneuver. To this end, let us consider a set of slots $\Phi' \supseteq \Phi$ on the constellation orbit. These slots are positions where a cooperative rendezvous can take place between two satellites in the constellation. Let $\mathcal{K}$ denote the set of indices for these slots. Now, let us consider a C-P2P maneuver between two satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ occupying the orbital slots ϕ_i and ϕ_j, where $i, j \in \mathcal{J}$. Let these satellites engage in a cooperative rendezvous at the orbital slot ϕ_k, where $k \in \mathcal{K}$. During the first phase of the cooperative P2P maneuver, the two satellites s_μ and s_ν transfer to the orbital slot ϕ_k. After the rendezvous, the satellites s_μ and s_ν are engaged in a fuel exchange and then, in the second phase of the P2P maneuver, the satellites s_μ and s_ν transfer to their original orbital slots ϕ_i and ϕ_j respectively. Without loss of generality, let us assume that s_μ is the fuel-sufficient satellite and that s_ν is the fuel-deficient satellite, that is, $f_\mu^- \geq \underline{f}_\mu$ and $f_\nu^- < \underline{f}_\nu$.

Note that in a non-cooperative P2P maneuver, the amount of fuel exchanged by the two satellites can be determined by the fact that the active satellite returns with just enough fuel to be fuel-sufficient. Unlike the non-cooperative case, the amount of fuel exchanged between the satellites in the cooperative case affects the return trips of both the active satellites. Hence, a natural question that arises here is how to obtain the amount of fuel that must be shared between the two satellites. Of course, the objective is to spend as little fuel during each C-P2P maneuver as possible.

Fuel Expenditure During a C-P2P Maneuver

In this section, we determine the amount of fuel exchange that leads to minimum fuel expenditure during the maneuver. To this end, let us denote by g_μ^ν the amount of fuel that is transferred from satellite s_μ to satellite s_ν.

The fuel consumed by the active satellite s_μ to transfer from the orbital slot ϕ_i to the orbital slot ϕ_k is given by:

$$p_{ik}^\mu = \left(m_{s\mu} + f_\mu^-\right)\left(1 - e^{-\frac{\Delta V_{ik}}{c_{0\mu}}}\right). \tag{12}$$

Similarly, the fuel expenditure for satellite s_ν to transfer from the orbital slot ϕ_j to the orbital slot ϕ_k is given by:

$$p^{\nu}_{jk} = \left(m_{s\nu} + f^{-}_{\nu}\right)\left(1 - e^{-\frac{\Delta V_{jk}}{c_{0\nu}}}\right). \tag{13}$$

The fuel content of satellite s_{μ} after its forward trip (but before the fuel exchange takes place) is $f^{-}_{\mu} - p^{\mu}_{ik}$, while that of satellite s_{ν} is $f^{-}_{\nu} - p^{\nu}_{jk}$. The amount of fuel that s_{μ} imparts to s_{ν} is g^{ν}_{μ}. Hence, the fuel content of satellite s_{μ} just after the fuel exchange takes place is $f^{-}_{\mu} - p^{\mu}_{ik} - g^{\nu}_{\mu}$, while that of satellite s_{ν} is $f^{-}_{\nu} - p^{\nu}_{jk} + g^{\nu}_{\mu}$. During the return trip, the fuel expenditure of satellite s_{μ} to transfer from slot ϕ_k to slot ϕ_i is given by

$$p^{\mu}_{ki} = \left(m_{s\mu} + f^{-}_{\mu} - p^{\mu}_{ik} - g^{\nu}_{\mu}\right)\left(1 - e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}\right), \tag{14}$$

while that of satellite s_{ν} to transfer from slot ϕ_k to slot ϕ_j is given by

$$p^{\nu}_{kj} = \left(m_{s\nu} + f^{-}_{\nu} - p^{\nu}_{jk} + g^{\nu}_{\mu}\right)\left(1 - e^{-\frac{\Delta V_{kj}}{c_{0\nu}}}\right). \tag{15}$$

The final fuel content of satellite s_{μ} after the cooperative P2P maneuver is given by $f^{+}_{\mu} = f^{-}_{\mu} - p^{\mu}_{ik} - g^{\nu}_{\mu} - p^{\mu}_{ki}$, while that of satellite s_{ν} is given by $f^{+}_{\nu} = f^{-}_{\nu} - p^{\nu}_{jk} + g^{\nu}_{\mu} - p^{\nu}_{kj}$. Using the above equations, we have

$$f^{+}_{\mu} = \left(m_{s\mu} + f^{-}_{\mu} - g^{\nu}_{\mu} - p^{\mu}_{ik}\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - m_{s\mu}, \tag{16}$$

and

$$f^{+}_{\nu} = \left(m_{s\nu} + f^{-}_{\nu} + g^{\nu}_{\mu} - p^{\nu}_{jk}\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - m_{s\nu}. \tag{17}$$

We therefore have

$$\begin{aligned} f^{+}_{\mu} + f^{+}_{\nu} = {} & \left(m_{s\mu} + f^{-}_{\mu} - p^{\mu}_{ik}\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - g^{\nu}_{\mu} e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} + \left(m_{s\nu} + f^{-}_{\nu} - p^{\nu}_{jk}\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} \\ & + g^{\nu}_{\mu} e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - \left(m_{s\mu} + m_{s\nu}\right). \end{aligned} \tag{18}$$

Minimizing the fuel expenditure during a C-P2P maneuver is the same as maximizing the total fuel content $f^{+}_{\mu} + f^{+}_{\nu}$ of the satellites after the maneuver. From the above equation, $f^{+}_{\mu} + f^{+}_{\nu}$ is maximized when

$$g^{\nu}_{\mu} e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - g^{\nu}_{\mu} e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} = g^{\nu}_{\mu}\left(e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}\right)$$

is maximized. Recall that both satellites need to be fuel-sufficient after the P2P maneuver. Satellite s_{μ} will be fuel-sufficient if

$$f^{+}_{\mu} \geq \underline{f}_{\mu},$$

that is,

$$\left(m_{s\mu} + f^{-}_{\mu} - g^{\nu}_{\mu} - p^{\mu}_{ik}\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - m_{s\mu} \geq \underline{f}_{\mu},$$

or

$$g^{\nu}_{\mu} e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} \leq \left(m_{s\mu} + f^{-}_{\mu} - p^{\mu}_{ik}\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - \left(m_{s\mu} + \underline{f}_{\mu}\right),$$

and hence,

$$g^{\nu}_{\mu} \leq \left(m_{s\mu} + f^{-}_{\mu} - p^{\mu}_{ik}\right) - \left(m_{s\mu} + \underline{f}_{\mu}\right) e^{\frac{\Delta V_{ki}}{c_{0\mu}}}.$$

Also, satellite s_{ν} will be fuel-sufficient if

$$f^{+}_{\nu} \geq \underline{f}_{\nu},$$

that is,

$$\left(m_{s\nu} + f^{-}_{\nu} + g^{\nu}_{\mu} - p^{\nu}_{jk}\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - m_{s\nu} \geq \underline{f}_{\nu},$$

or,

$$g^{\nu}_{\mu} e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} \geq \left(m_{s\nu} + \underline{f}_{\nu}\right) - \left(m_{s\nu} + f^{-}_{\nu} - p^{\nu}_{jk}\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}},$$

and hence,

$$g_{\mu}^{\nu}e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} \geq \left(m_{s\nu} + \underline{f}_{\nu}\right) - \left(m_{s\nu} + f_{\nu}^{-} - p_{jk}^{\nu}\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}}.$$

The conditions of fuel-sufficiency on the satellites provide us with a lower bound $g_{\mu}^{\nu}|_{\ell}$ on the amount of fuel exchange, given by

$$g_{\mu}^{\nu}|_{\ell} = \left(m_{s\nu} + \underline{f}_{\nu}\right) e^{\frac{\Delta V_{kj}}{c_{0\nu}}} - \left(m_{s\nu} + f_{\nu}^{-} - p_{jk}^{\nu}\right). \tag{19}$$

It also provides an upper bound $g_{\mu}^{\nu}|_{u}$ on the amount of fuel exchange, given by

$$g_{\mu}^{\nu}|_{u} = \left(m_{s\mu} + f_{\mu}^{-} - p_{ik}^{\mu}\right) - \left(m_{s\mu} + \underline{f}_{\mu}\right) e^{\frac{\Delta V_{ki}}{c_{0\mu}}}. \tag{20}$$

As mentioned already, we need to maximize $g_{\mu}^{\nu}\left(e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}\right)$. This is maximized if

$$g_{\mu}^{\nu} = \begin{cases} g_{\mu}^{\nu}|_{\ell}, & e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} < e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}, \\ g_{\mu}^{\nu}|_{u}, & e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} > e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}. \end{cases} \tag{21}$$

Clearly, if $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} = e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$, then g_{μ}^{ν} can assume any value in the interval $g_{\mu}^{\nu}|_{\ell} \leq g_{\mu}^{\nu} \leq g_{\mu}^{\nu}|_{u}$.

To determine the final fuel content of the satellites when the fuel exchange is optimal, we need to consider two cases. If $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} < e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$, we have

$$\begin{aligned} f_{\nu}^{+} &= \left(m_{s\nu} + f_{\nu}^{-} + g_{\mu}^{\nu}|_{k} - p_{jk}^{\nu}\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} - m_{s\nu} \\ &= \left(m_{s\nu} + f_{\nu}^{-} - \left(m_{s\nu} + f_{\nu}^{-} - p_{jk}^{\nu}\right) - p_{jk}^{\nu}\right) e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} + \left(m_{s\nu} + \underline{f}_{\nu}\right) - m_{s\nu} \\ &= \underline{f}_{\nu}, \end{aligned} \tag{22}$$

which implies that s_{ν} returns with just enough fuel to be fuel-sufficient. On the other hand, if $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} > e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$, we have

$$\begin{aligned} f_{\mu}^{+} &= \left(m_{s\mu} + f_{\mu}^{-} - g_{\mu}^{\nu}|_{u} - p_{ik}^{\mu}\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} - m_{s\mu} \\ &= \left(m_{s\mu} + f_{\mu}^{-} - \left(m_{s\mu} + f_{\mu}^{-} - p_{ik}^{\mu}\right) - p_{ik}^{\mu}\right) e^{-\frac{\Delta V_{ki}}{c_{0\mu}}} + \left(m_{s\mu} + \underline{f}_{\mu}\right) - m_{s\mu} \\ &= \underline{f}_{\mu}, \end{aligned} \tag{23}$$

which implies that s_{μ} returns with just enough fuel to be fuel-sufficient.

If both satellites have the same engine characteristics, then $c_{0\mu} = c_{0\nu}$, and $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} < e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$, equivalently, $\frac{\Delta V_{kj}}{c_{0\nu}} > \frac{\Delta V_{ki}}{c_{0\mu}}$, and hence, $\Delta V_{kj} > \Delta V_{ki}$. Similarly, $e^{-\frac{\Delta V_{kj}}{c_{0\nu}}} > e^{-\frac{\Delta V_{ki}}{c_{0\mu}}}$ implies that $\Delta V_{kj} > \Delta V_{ki}$. We can summarize our findings with the following proposition.

Proposition 1. *If two satellites engaging in a cooperative P2P maneuver have engines with the same specific thrust, the optimal fuel exchange takes place when the satellite making the costlier ΔV transfer returns with just enough fuel to be fuel-sufficient.*

C-P2P Formulation

Similar to solving the P2P refueling problem, let us consider the undirected bipartite graph $\mathcal{G}$ with the two graph partitions being the orbital slots of the fuel-sufficient satellites $\mathcal{J}_{s,0}$ and those of the fuel-deficient satellites $\mathcal{J}_{d,0}$. There exists an edge $\langle i, j\rangle \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0}$ if the satellites $s_{\mu} = \sigma_0(\phi_i)$ and $s_{\nu} = \sigma_0(\phi_j)$ can engage in a cooperative or non-cooperative P2P refueling transaction, such that, at the end of the refueling process, both satellites end up being fuel-sufficient. Let $\mathcal{E} \subseteq \mathcal{J}_{s,0} \times \mathcal{J}_{d,0}$ be the set of all edges in $\mathcal{G}$. To each edge $\langle i, j \rangle \in \mathcal{E}$, we assign a cost c_{ij} that equals the fuel expenditure incurred during the cheapest (among all non-cooperative and cooperative) P2P maneuver between the two. Let the satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ be involved in a cooperative rendezvous at the orbital slot $\phi_k \in \Phi'$, where Φ' is the set of all possible orbital slots on the orbit. Note that $\Phi \subseteq \Phi'$. The fuel expenditure incurred during the cooperative maneuver is given by

$$c_{ij}|_{\phi_k} = \left(p^{\mu}_{ik} + p^{\nu}_{jk}\right) + \left(p^{\mu}_{ki} + p^{\nu}_{kj}\right) \tag{24}$$

Note that $\phi_k = \phi_i$ corresponds to a non-cooperative maneuver, in which the satellite s_ν is active, while $\phi_k = \phi_j$ corresponds to a non-cooperative maneuver, in which the satellite s_ν is active. The minimum over all cooperative and non-cooperative fuel expenditures is assigned to be the weight of the edge $\langle i, j \rangle$. Therefore, we have

$$c_{ij} = \min_{\phi_k \in \Phi'} c_{ij}|_{\phi_k} \tag{25}$$

For convenience, let us also define a function $\mathsf{Coop} : \mathcal{E} \mapsto \Phi'$ such that

$$\mathsf{Coop}\,(i, j) = \arg \min_{\phi_k \in \Phi'} c_{ij}|_{\phi_k} \tag{26}$$

Note that if for edge $\langle i, j \rangle$, the cheapest maneuver is non-cooperative, then $\mathsf{Coop}\,(i, j)$ gives the orbital slot of the passive satellite. We are interested in a set $\mathcal{M}_e \subseteq \mathcal{E}$ of $|\mathcal{I}_{d,0}|$ edges that has minimum total cost and such that all fuel-deficient satellites are involved in fuel transactions. Similarly to what we did for the P2P refueling problem, let us also associate with each edge $\langle i, j \rangle \in \mathcal{E}$ a binary variable x_{ij}, defined as

$$x_{ij} = \begin{cases} 1, & \text{if } \langle i, j \rangle \in \mathcal{M}_e, \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

We can therefore consider the following optimization problem:

$$(\mathrm{CP2P-IP}) : \min_{\mathcal{M}_e \subseteq \mathcal{E}} \sum_{\langle i,j \rangle \in \mathcal{E}} c_{ij} x_{ij}, \tag{28}$$

such that

$$\sum_{j \in \mathcal{J}_{d,0}} x_{ij} \leq 1, \text{ for all } i \in \mathcal{J}_{s,0}, \tag{29}$$

$$\sum_{i \in \mathcal{J}_{s,0}} x_{ij} = 1, \text{ for all } j \in \mathcal{J}_{d,0}. \tag{30}$$

As before, constraint (29) implies that a fuel-sufficient satellite must be assigned to at most one refueling transaction, while constraint (30) implies that a fuel-deficient satellite has to be assigned to a refueling transaction. However, for the C-P2P problem, we require additional constraints to be imposed. For instance, consider two edges $\langle i, j \rangle, \langle q, r \rangle \in \mathcal{M}_e$. Note that if $\mathsf{Coop}\,(i, j) = \mathsf{Coop}\,(q, r)$, then this implies either one of the following:

i) A cooperative rendezvous corresponding to the two edges occur at the same orbital slot, or

ii) A cooperative rendezvous corresponding to one edge occurs at the slot of the passive satellite corresponding to another edge.

Either case is impractical and cannot occur physically. Hence, we have to ensure that the following additional constraint also holds:

$$\mathsf{Coop}\,(i, j) \neq \mathsf{Coop}\,(q, r) \text{ for all } \langle i, j \rangle, \langle q, r \rangle \in \mathcal{M}_e. \tag{31}$$

The determination of the optimal C-P2P solution requires the minimization of the objective given in (28), subject to the constraints (29)-(31).

Methodology

We can solve the optimization problem given by (28)-(30) to find the set of edges $\mathcal{M}_e$. The set $\mathcal{M}_e$ may or may not be a feasible C-P2P solution, because it may or may not satisfy constraint (31). If it does, then we have the optimal C-P2P solution and we are done. If the constraint (31) is not satisfied, then another bipartite matching problem can be set up in order to yield the optimal (and feasible) C-P2P solution for the same set of satellite pairs (or refueling transactions) given by $\mathcal{M}_e$. We discuss below how this can be achieved.

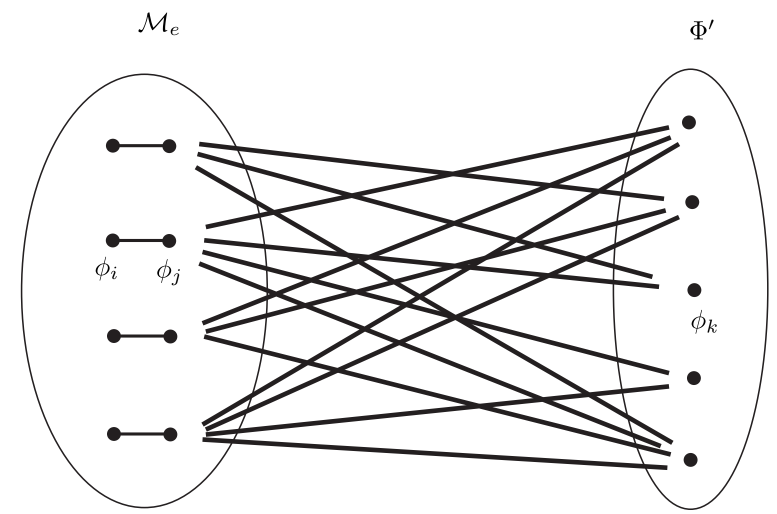


Figure 3. Bipartite graph for determining the C-P2P solution given the fuel transactions $\mathcal{M}_e$.

Let us construct a bipartite graph, with one of the partitions representing the orbital slots given by Φ', and the other partition comprised of nodes representing the edges given by $\mathcal{M}_e$. Figure 3 depicts such a graph. We say that there exists an edge $< \langle i,j \rangle, \phi_k >$ between $\langle i,j \rangle \in \mathcal{M}_e$ and $\phi_k \in \Phi$, if satellites $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$ can engage in a feasible cooperative P2P maneuver at the orbital slot $\phi_k \in \Phi'$, such that at the end of the overall maneuver the satellites return to their original slots with enough amount of fuel to be fuel-sufficient. Let $\mathcal{E}_c$ denote the set of all such edges. We are interested in a set $\mathcal{M}_c \subseteq \mathcal{E}_c$ of edges that assigns to each fuel transaction a slot for cooperative rendezvous and which leads to a feasible C-P2P solution. To this end, let us assign to each edge the binary variable

$$y_{ijk} = \begin{cases} 1, & \text{if } < \langle i,j \rangle, k > \in \mathcal{M}_c, \\ 0, & \text{otherwise.} \end{cases} \tag{32}$$

The following optimization problem yields the optimal C-P2P solution, given the fuel transactions depicted by the infeasible solution $\mathcal{M}_e$:

$$\min_{\mathcal{M}_c \subseteq \mathcal{E}_c} \sum_{< \langle i,j \rangle, \phi_k > \in \mathcal{E}_c} c_{ij}|_{\phi_k}\, y_{ijk}, \tag{33}$$

subject to

$$\sum_{\phi_k \in \Phi'} y_{ijk} = 1, \text{ for all } \langle i,j \rangle \in \mathcal{M}_e, \tag{34}$$

$$\sum_{\langle i,j \rangle \in \mathcal{M}_e} y_{ijk} \leq 1, \text{ for all } \phi_k \in \Phi', \tag{35}$$

Constraint (34) signifies that all fuel transactions need to be assigned a slot for rendezvous, while constraint (35) signifies that an orbital slot can be assigned to at most one refueling transaction. The solution to this optimization problem yields the cheapest feasible C-P2P solution corresponding to the fuel transactions determined by $\mathcal{M}_e$.

NUMERICAL EXAMPLES

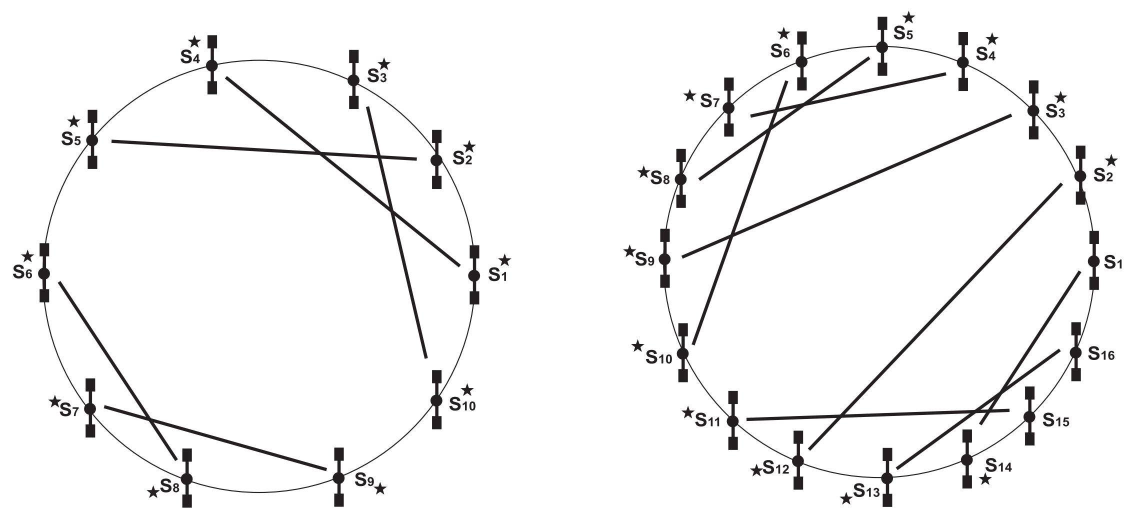
In this section, we will consider sample constellations and will determine the optimal C-P2P refueling strategy for each one of them. We will also compare the total fuel expenditure incurred using C-P2P and P2P refueling of the satellites for the constellations given in Table 1. These numerical examples demonstrate the usefulness of a C-P2P refueling strategy.

Example 3. *C-P2P refueling strategy for constellation* C_1*.*

For this example the orbital slots for cooperative rendezvous to take place have been assumed to be equally spaced at intervals of 9 deg along the orbit. Hence, there are 40 available slots for the cooperative rendezvous to take place, including the 10 orbital slots occupied originally by the satellites. The optimal assignments obtained from the solution of the optimization problem (CP2P-IP) were found to be non-cooperative. Note that since $\Phi \subseteq \Phi'$, the optimal solution of (CP2P-IP) will be the optimal P2P solution if there exists no cooperative solution that is cheaper than the optimal P2P case. In other words, cooperative maneuvers in cases such as in this example do not help in reducing the fuel expenditure of the overall refueling process.

Example 4. *C-P2P refueling strategy for constellation* C_1'*.*

As in the previous example, the orbital slots for cooperative rendezvous to take place are equally spaced at intervals of 9 deg along the orbit. The assignments are determined by solving the optimization problem (CP2P-IP), and are given by: $s_1 \leftrightarrow s_4$, $s_2 \leftrightarrow s_5$, $s_8 \leftrightarrow s_6$, $s_9 \leftrightarrow s_7$ and $s_{10} \leftrightarrow s_3$. All of these maneuvers are cooperative. For instance, satellites s_1 and s_4 rendezvous at the orbital slot with a lead angle of 54 deg with respect to satellite s_1. Similarly, satellites s_8 and s_6 engage in a cooperative maneuver in which both satellites cooperatively rendezvous at the orbital slot with a lead angle of 27 deg. The solution to the C-P2P integer program yields no conflict that violates the additional constraint. Hence, the above solution corresponds to the optimal C-P2P assignments. The fuel expenditure corresponding to this set of C-P2P assignments is 27.19 units, a reduction of about 8% over the optimal P2P fuel expenditure. This example demonstrates the benefit of allowing satellites to engage in cooperative rendezvous when the fuel-deficient satellites do not have enough fuel to complete the non-cooperative rendezvous. Figure 4(a) shows the optimal C-P2P assignments obtained for this example. An important observation for this example is that for each of the C-P2P maneuvers, the cooperative rendezvous takes place in a slot at which the fuel-deficient satellite arrives



(a) Constellation C_1' (b) Constellation C_2

Figure 4. Optimal assignments for C-P2P refueling.

by having exhausted almost all of its fuel. In other words, the fuel-deficient satellite moves as close to the fuel-sufficient satellite as it is permitted by its onboard fuel. The final fuel contents of the satellites after the C-P2P maneuvers have taken place are 12.0, 12.0, 13.1, 13.1, 13.1, 12.0, 12.0, 15.5, 15.5 and 12.0 respectively.

Example 5. *C-P2P refueling strategy for constellation* C_2*.*

Let us now consider the constellation C_2 given in Table 1. The fuel expenditure incurred in the P2P refueling of the satellites in the constellation is 39.67 units. The optimal C-P2P assignments, as determined by solving the (CP2P-IP), are given as follows: $s_1 \leftrightarrow s_{14}$, $s_2 \leftrightarrow s_{12}$, $s_3 \leftrightarrow s_9$, $s_4 \leftrightarrow s_7$, $s_5 \leftrightarrow s_8$, $s_6 \leftrightarrow s_{10}$, $s_{15} \leftrightarrow s_{11}$ and $s_{16} \leftrightarrow s_{13}$. Of these maneuvers, two are non-cooperative, namely the assignments $s_1 \leftrightarrow s_{14}$ and $s_{16} \leftrightarrow s_{13}$. For these, the fuel-deficient satellites have enough fuel to be active. The remaining maneuvers are cooperative. Allowing for cooperative maneuvers reduces the overall fuel expenditure to 36.98 units, which is about 6.8% less than the optimal P2P fuel expenditure. Similarly to the previous example, we have that for the cooperative maneuvers, the fuel-deficient satellites move as close to the fuel-sufficient satellites as permitted by their onboard fuel. Figure 4(b) shows the C-P2P assignments. The final fuel contents of the satellites in the constellation are given by 16.2, 12.3, 12.0, 16.1, 16.1, 14.8, 12.0, 12.0, 12.2, 12.0, 12.0, 12.0, 12.0, 12.0, 14.9 and 16.2 units. The solution generated by the optimization problem (CP2P-IP) does not violate the additional constraint (31). Hence, this is the optimal C-P2P solution.

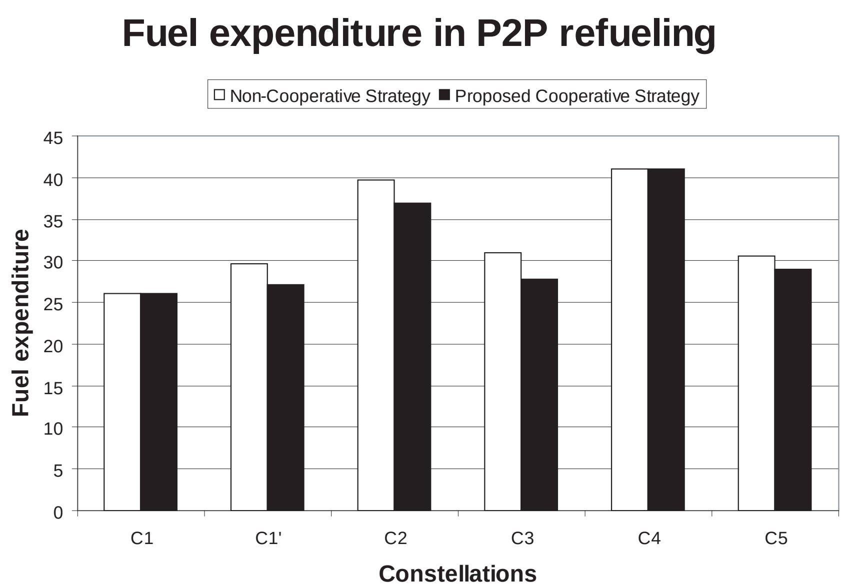


Figure 5. Comparison of P2P and C-P2P refueling strategies for the sample constellations of Table 1.

Figure 5 summarizes the results for the sample constellations of Table 1. The optimal P2P and C-P2P fuel expenditure for these constellations are shown. For the constellations C_1 and C_4, the optimal non-cooperative P2P solution is the cheapest way to redistribute fuel in the constellation. For these, the fuel-deficient satellites have enough fuel to complete a non-cooperative rendezvous. Whenever this is not possible, as in case of the remaining constellations, cooperative maneuvers turn out to be beneficial.

CONCLUSIONS

In this paper we have studied a cooperative P2P (C-P2P) strategy for refueling satellites in a circular constellation. This strategy allows for cooperative rendezvous between the satellites engaging in a fuel exchange via P2P maneuvers. We have proposed a formulation of the C-P2P refueling problem, and have shown that the satellites exchange fuel in such a way that the one making the costlier ΔV transfer returns with just enough amount of fuel to be fuel-sufficient. Finally, with the help of numerical examples, we point out the benefits of a C-P2P strategy, particularly when the fuel-deficient satellites in the constellation do not have enough fuel to complete non-cooperative rendezvous.

Acknowledgement: This work has been supported by AFOSR award no. FA9550-04-1-0135.

References

[1]Waltz, D. M., *On-orbit Servicing of Space Systems*, Krieger, Malabar, FL, 1st ed., 1993.

[2]Lamassoure, E., *A Framework to Account for Flexibility in Modeling the Value of On-Orbit Servicing for Space Systems*, Master's thesis, Dept. of Aeronautics and Astronatics, Massachusetts Institute of Technology, Cambridge, MA, June 2001.

[3]Saleh, J., Lamassoure, E., Hastings, D., and Newman, D., "Flexibility and the Value of On-orbit Servicing: New Customer-centric Perspective," *Journal of Spacecrafts and Rockets*, Vol. 40, 2003, pp. 279–291.

[4]Tatsch, A., Fitz-Coy, N., and Gladun, S., "On-orbit servicing: A Brief Survey," *Performance Metrics for Intelligent Systems Conference*, Gaithersburg, MD, Aug. 2006.

[5]Reynerson, C. M., "Spacecraft Modular Architecture for On-orbit Servicing," *AIAA Space Technology Conference and Exposition*, No. AIAA-99-4473, Albuquerque, NM, Sept. 1999.

[6]Vandenkerckhove, J. A., "Satellite Refueling in Orbit," *Communications Satellite Systems Conference*, No. AIAA Paper 82-0514, San Diego, CA, Mar. 1982.

[7]Sullivan, B. R., *Technical and Economic Feasibility of Telerobotic On-orbit Satellite Refueling*, Ph.D. thesis, University of Maryland, 2005.

[8]Shen, H. and Tsiotras, P., "Optimal Scheduling for Servicing Multiple Satellites in a Circular Constellation," *AIAA/AAS Astrodynamics Specialists Conference*, No. AIAA Paper 02-4907, Monterey, CA, Aug. 2002.

[9]Shen, H. and Tsiotras, P., "Peer-to-Peer Refueling for Circular Satellite Constellations," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 28, 2005, pp. 1220–1230.

[10]Tsiotras, P. and Nailly, A., "Comparison Between Peer-to-Peer and Single Spacecraft Refueling Strategies for Spacecraft in Circular Orbits," *Infotech at Aerospace Conference*, No. AIAA Paper 05-7115, Crystal City, DC, Sept. 2005.

[11]Dutta, A. and Tsiotras, P., "Asynchronous Optimal Mixed P2P Satellite Refueling Strategies," *The Journal of the Astronautical Sciences*, Vol. 54, No. 3-4, Jul-Dec 2006, pp. 543–565.

[12]Salazar, A. and Tsiotras, P., "An Auction Algorithm for Optimal Satellite Refueling," *Georgia Tech Space Systems Engineering Conference*, Atlanta, GA, Nov. 2005.

[13]Dutta, A. and Tsiotras, P., "A Greedy Random Adaptive Search Procedure for Optimal Scheduling of P2P Satellite Refueling," *AAS/AIAA Space Flight Mechanics Meeting*, No. AAS Paper 07-150, Sedona, AZ, Jan. 2007.

[14]Dutta, A. and Tsiotras, P., "A Network Flow Formulation for an Egalitarian P2P Refueling Strategy," *AAS/AIAA Space Flight Mechanics Meeting*, No. AAS Paper 07-151, Sedona, AZ, Jan. 2007.

[15]Dutta, A. and Tsiotras, P., "An Egalitarian Peer-to-Peer Satellite Refueling Strategy," *AIAA Journal of Spacecraft and Rockets*, Vol. 45, No. 3, 2008, pp. 608–618.

[16]Meschler, P., "Time Optimal Rendezvous Strategies," *IEEE Transactions on Automatic Control*, Vol. 8, 1963, pp. 279–282.

[17]Eller, D., Mueller, T., and Aggarwal, J., "On Optimal Cooperative Rendezvous," *IEEE Transactions on Automatic Control*, Vol. 12, April 1967, pp. 202–202.

[18]Wong, R., "Some Aerospace Differential Games," *Journal of Spacecraft and Rockets*, Vol. 4, No. 11, 1967, pp. 1460–1465.

[19]Prussing, J. and Conway, B., "Optimal Terminal Maneuver for a Cooperative Impulsive Rendezvous," *Journal of Guidance, Control and Dynamics*, Vol. 12, 1989, pp. 433–435.

[20]Mirfakhraie, K. and Conway, B., "Optimal Cooperative Time-fixed Impulsive Rendezvous," *Journal of Guidance, Control and Dynamics*, Vol. 17, 1994, pp. 607–613.

[21]Coverstone, V. and Prussing, V., "Optimal Cooperative Power-limited Rendezvous Between Neighboring Circular Orbits," *Journal of Guidance, Control and Dynamics*, Vol. 16, 1993, pp. 1045–1054.

[22]Coverstone, V. and Prussing, J., "Optimal Cooperative Power-limited Rendezvous Between Coplanar Circular Orbits," *Journal of Guidance, Control and Dynamics*, Vol. 17, 1994, pp. 1096–1102.