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ASYNCHRONOUS OPTIMAL MIXED P2P SATELLITE REFUELING STRATEGIES

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Abstract

In this paper, we study pure peer-to-peer (henceforth abbreviated as P2P) and mixed (combined single-spacecraft and P2P) satellite refueling in circular orbit constellations comprised of multiple satellites. We consider the optimization of two conflicting objectives in the refueling problem and show that the cost function we choose to determine the optimal refueling schedule reflects a reasonable compromise between these two conflicting objectives. In addition, we show that equal time distribution between the forward and return legs for each pair of P2P maneuvers does not necessarily lead to the optimum cost. Based on this idea, we propose a strategy for reducing the cost of P2P maneuvers. This strategy is applied to pure P2P refueling scenarios as well as to mixed refueling scenarios. Furthermore, for the case of a mixed scenario, we propose an asynchronous P2P strategy that also leads to more efficient refueling.

INTRODUCTION

It has long been recognized that servicing and refueling spacecraft in orbit has the potential to revolutionize spacecraft operations by extending the useful lifetime of the spacecraft, by reducing launching and insurance cost, and by increasing operational flexibility and robustness.[1–4] Several studies have been conducted over the past decade investigating the relative merit of satellite refueling when compared to satellite replacement.[1,5,6] Crucial technologies that enable replenishment of satellites with propellant have already been tested or are in the process of being evaluated.[7–12]

Most of the previous studies in the literature have assumed that a single spacecraft alone undertakes the task of refueling the whole constellation. That is, a single service spacecraft plays the role of the sole supplier of fuel.[1,13,14] Recently, an alternative scenario for distributing fuel amongst a large number of satellites has been proposed.[15–17] In this scenario, no single spacecraft is in charge of the complete refueling process. Instead, all satellites share the responsibility of refueling each other on an equal footing. We call this the peer-to-peer (P2P) refueling strategy.[16,17]

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A P2P refueling strategy is, by definition, a *distributed* method for replenishing a constellation of spacecraft with fuel/propellant. Consequently, it offers a great degree of robustness and protection against failures. For instance, with a P2P strategy a failure of a single spacecraft will have almost no impact on the refueling of the rest of the constellation. On the contrary, a failure of the service vehicle in a single-spacecraft scenario will result in the failure of the whole mission.

Although a stand-alone P2P scenario may seem unconventional at first glance, it arises naturally as an essential component of a *mixed refueling strategy*. By mixed refueling strategy we mean a strategy which involves at least two stages. During the first stage a single spacecraft refuels only a certain fraction (perhaps half) of the satellites. During the second stage the satellites that received fuel during the first stage act as go-betweens, and distribute the fuel to the rest of the constellation in a P2P manner. That is, a P2P refueling strategy can be implemented as the final distribution phase of a single-vehicle refueling strategy. In Refs. 18, 19 it has been shown that a mixed refueling strategy is more fuel-efficient than a single-spacecraft strategy, especially for a large number of satellites in the constellation and for short refueling periods. As a matter of fact, it is not difficult to come up with cases for which the single-spacecraft scenario is infeasible (due to the time constraint), while a mixed refueling strategy is still possible.

Pure P2P refueling for circular spacecraft constellations was originally proposed in Ref. 20 as a means to equalize fuel. In that work two P2P cases were analyzed. In the first case the rendezvous costs were negligible when compared to the total amount of fuel exchange taking place. This situation arises only when the satellites are very closely spaced or when the time for refueling is sufficiently large.[16] The optimal matching in this case is very simple, i.e., it is a symmetric matching[§]. For the majority of cases encountered in practice however the cost incurred during the transfers is significant and cannot be neglected in the optimization process. In order to achieve fuel equalization in this case, an optimization problem was formulated in Refs. 17, 19, where the absolute value of the deviation of each satellite's fuel from the *initial* average fuel in the constellation is penalized. Ideally, one would like to minimize the deviation of each satellite's fuel from the *final* average fuel in the constellation. However, without any additional constraints, the later approach may lead to solutions where the satellites perform wasteful maneuvering just to equalize fuel. This undesirable situation does not occur in the formulation used in Ref. 19. By minimizing the deviation from the mean fuel *before* refueling takes place (as opposed to the mean fuel after refueling takes place) we eliminate this possibility. However, this is a rather heuristic way of addressing the objective of a P2P strategy, which is to both equally distribute fuel in the constellation and to ensure as little fuel expenditure as possible in the process.

Fuel equalization and minimum fuel expenditure are two conflicting objectives. Fuel equalization requires transfer of fuel from one satellite to another and hence consumption of fuel because of the required orbital maneuvers. Minimizing total fuel consumption on the other hand, implies few and long transfers. In fact, if the fuel equalization requirement is missing, the optimal solution to the fuel maximization problem is simple: do nothing. No satellites are involved in refueling rendezvous. If, on the other hand, the requirement for fuel minimization

[§]In a symmetric matching the satellite with the most amount of fuel pairs up with the satellite with the least amount of fuel, the satellite with the second most amount of fuel pairs up with the satellite with the second least amount of fuel, etc.

is missing, the opposite occurs: all eligible satellites are involved in refueling rendezvous. In Refs. 16, 17 and 19 the satisfaction of the previous two objectives was addressed via the introduction of a rather artificial cost function that minimizes satellite fuel deviation from the mean fuel in the constellation before refueling takes place. A correct formulation of the problem should involve an explicit incorporation of the two previous conflicting objectives. It is one of the objectives of this paper to fill this gap.

In the first part of the current paper we re-formulate the P2P refueling problem as a minimization problem of a cost function that is a convex combination of the previous two conflicting objectives. The cost function introduced this way is parameterized by a single nonnegative scalar $0 \leq \alpha \leq 1$ that plays the role of the relative weight of the two elementary optimization objectives. The choice of α thus becomes a design parameter to be tuned for best performance. This is a more direct method for formulating the P2P refueling problem than the one used in Refs. 16, 17, 19. Nonetheless, we show that the cost in Refs. 16, 17, 19 corresponds to the cost used herein for a proper choice of the parameter α. This analysis justifies the methodology followed in Refs. 16, 17, 19.

In the second, and major, part of the paper we revisit the P2P refueling problem, with the goal of further improving the transfer costs. Specifically, we relax two of the assumptions made in Refs. 17, 19 while calculating the fuel burnt for the orbital transfers during each fuel transaction. One of the assumptions for the P2P refueling problem studied in Ref. 19 is that when there is a fuel exchange between two satellites in a constellation, the time for the forward journey equals the time for the return journey for all satellite pairs. In the current paper, we will allow for unequal time sharing between the forward and return journeys, and we show that equal time sharing does not lead to optimal fuel consumption. We use this fact to formulate an algorithm that considerably reduces the cost of P2P maneuvers. This algorithm is also applied to a mixed refueling scenario in order to make it a more competitive option to the single-spacecraft refueling scenario. It is also shown that allowing asynchronous P2P maneuvers in such a mixed scenario further brings down the refueling cost. With the help of numerical examples, we demonstrate the improvements over Ref. 19 and we also show how the incorporation of the extensions proposed in this paper make the mixed refueling scenario a far better option than a single spacecraft strategy, particularly when the number of satellites is large.

THE P2P PROBLEM FORMULATION

The Constellation Graph

Given a collection of $n \geq 3$ satellites $\mathcal{C} = \{s_1, \ldots, s_n\}$ with unequal amounts of fuel, the satellites with fuel greater than the average amount of fuel are termed *fuel-sufficient* satellites, whereas the satellites with fuel less than or equal the average amount of fuel in the constellation are termed the *fuel-deficient* satellites. We use $\mathcal{C}_s$ to denote the set of all fuel-sufficient satellites, and $\mathcal{C}_d$ to denote the set of all fuel-deficient satellites. Clearly, $\mathcal{C} = \mathcal{C}_s \cup \mathcal{C}_d$. It is assumed that all satellites are in the same circular orbit, but they do not have to be evenly distributed along the orbit. By a fuel/refuel transaction herein we assume a sequence of events that involves: (i) a satellite firing its thrusters so as to change its orbit and rendezvous with another satellite in the constellation, (ii) exchange of fuel between the two satellites, and (iii) return of the first satellite to its original slot.

It will be assumed that during a refueling transaction, only one satellite, called the *seller*, can give fuel to another satellite. The latter is called the *buyer*. The set of seller satellites will be denoted by $\mathcal{S}$ and the set of buyer satellites will be denoted by $\mathcal{B}$. Depending on the amount of fuel between the two, either of these two satellites can initiate a fuel transaction, i.e., perform a rendezvous with the other satellite, exchange fuel and return to its original orbital slot. The former satellite is said to be the *active* satellite and the latter satellite is said to be the *passive* satellite. The set of active satellites will be denoted by $\mathcal{A}$ and the set of passive satellites will be denoted by $\mathcal{P}$. Note that, in general, $\mathcal{S} \cup \mathcal{B} \subseteq \mathcal{C}$ since not all satellites may be involved in fuel transactions. Similarly, $\mathcal{A} \cup \mathcal{P} \subseteq \mathcal{C}$ for the same reason. Also note that it is not necessarily true that $\mathcal{S} = \mathcal{A}$ or that $\mathcal{B} = \mathcal{P}$, although this typically will be the case. For instance, it may happen that a satellite, say s_i, initiating a fuel transaction receives fuel (i.e., $s_i \in \mathcal{A} \cap \mathcal{B}$) or that a passive satellite is the seller ($s_i \in \mathcal{P} \cap \mathcal{S}$). Furthermore, it is not necessarily true that a fuel sufficient satellite will be active (i.e., $\mathcal{C}_s \not\subseteq \mathcal{A}$). However, a fuel deficient satellite is always a buyer, that is $\mathcal{C}_d \subseteq \mathcal{B}$.

Given now the set $\mathcal{C}$ we may construct a graph $\mathcal{G}$ having as nodes (or vertices) the satellites of $\mathcal{C}$. We call $\mathcal{G}$ the constellation graph. Associated with $\mathcal{G}$ is a set of vertices $\mathcal{V} = \{s_1, s_2, \ldots, s_n\}$ and a set of edges $\mathcal{L} = \{\langle i, j\rangle : s_i, s_j \in \mathcal{V}\}$ connecting the nodes of $\mathcal{G}$. Without loss of generality, we enumerate the vertices such that $i \leftrightarrow s_i$ for all $1 \leq i \leq n$. This allows us in the sequel to refer to "vertex" s_i instead of i without the danger of confusion. We will make no distinction between the edge $\langle j, i\rangle$ and the edge $\langle i, j\rangle$. That is, $\mathcal{G}$ is a undirected graph. This point needs some clarification. Since the propellant required for satellite s_i to rendezvous with satellite s_j is not equal to the propellant required for satellite s_j to rendezvous with satellite s_i, $\mathcal{G}$ is, in principal, a directed graph. By assigning the minimum fuel required between the two transfers $s_i \to s_j$ and $s_j \to s_i$ to the edge $\langle i, j\rangle$ we obtain an undirected graph. This is elaborated upon in the sequel. In the graph $\mathcal{G}$, an edge between two vertices exists if a fuel transaction between the corresponding satellites is permissible. The number of elements of a subset of set $\mathcal{X}$ will be denoted by $|\mathcal{X}|$. Clearly, $|\mathcal{V}| = n$ and for a complete graph $|\mathcal{L}| = n(n-1)/2$.

The set of vertices connected to vertex s_i is called the set of neighbors of s_i, and it is denoted by $\mathcal{N}_i$. The *edge neighborhood* of s_i is defined by $\mathcal{Q}_i = \{\langle i, j\rangle \in \mathcal{L} : s_j \in \mathcal{N}_i\}$. Note that if s_i has no neighbors then no edges are connected to this vertex and $\mathcal{Q}_i = \varnothing$. For example, we may impose that certain satellites are not involved in any fuel transactions due to operational constraints. By removing all satellites which are known a priori that cannot be involved in fuel transactions due to operational restrictions we get the *core constellation graph* $\mathcal{G}^c$. For simplicity, in the sequel we assume that $\mathcal{G} = \mathcal{G}^c$. It should be kept in mind however that the following developments hold verbatim if we replace $\mathcal{G}$ with $\mathcal{G}^c$.

To each edge $\langle i, j\rangle \in \mathcal{L}$ we will assign a (positive) weight that reflects the cost associated with a fuel transaction between the satellites connected by this edge. By an *assignment* or *matching* over the graph $\mathcal{G}$ we mean a partition of $\mathcal{V}$ into two sets $\mathcal{V}_a$ and $\mathcal{V}_b$, such that $|\mathcal{V}_a| = |\mathcal{V}_b|$ along with a subset $\mathcal{M} \subseteq \mathcal{L}$ and a one-to-one mapping $\sigma : \mathcal{V}_a \to \mathcal{V}_b$ such that $\mathcal{M} = \{\langle i, j\rangle : s_i \in \mathcal{V}_a,\ s_j \in \mathcal{V}_b, \text{ and } s_j = \sigma(s_i)\}$. Given the positive weights on each edge, we seek the matching that maximizes the sum of the weights of all edges involved in this matching.

In the next section we show how the problem of finding the optimal pairings of satellites can be reduced to a problem of computing the maximum weighted matching in the constellation graph.

Construction of the Constellation Graph

Let, for convenience, $\mathcal{I}$ denote the index set of the vertices in the (core) constellation graph. That is, $i \in \mathcal{I}$ for $s_i \in \mathcal{G}$. Let f_i^- and f_i^+ denote the fuel contained in each satellite before and after a fuel transaction, respectively. The average amount of fuel in the constellation before and after all fuel transactions will be denoted by $\bar{f}^-$ and $\bar{f}^+$, respectively. That is, $\bar{f}^- = (1/n)\sum_{i\in\mathcal{I}} f_i^-$, and similarly for $\bar{f}^+$. Let p_i^j denote the fuel burnt by satellite $s_i \in \mathcal{A}$ in order to rendezvous with satellite $s_j \in \mathcal{P}$ and return to its original orbital slot. Notice that, in general, $p_i^j \neq p_j^i$. Also note that in a fuel transaction between s_i and s_j either one can be the active satellite, provided that it has enough amount of fuel to rendezvous with the inactive satellite and return to its original orbital slot. Hence, the fuel cost assigned to a single rendezvous between satellites s_i, $s_j \in \mathcal{G}$ is given by

$$p_{ij} = \begin{cases} p_i^j, & \text{if } s_i \text{ can be active, but } s_j \text{ cannot,} \\ p_j^i, & \text{if } s_j \text{ can be active, but } s_i \text{ cannot,} \\ \min\{p_i^j, p_j^i\}, & \text{if either } s_i \text{ or } s_j \text{ can be active,} \\ \infty, & \text{if neither } s_i \text{ nor } s_j \text{ can be active.} \end{cases} \tag{1}$$

The objective is to minimize the square deviation of the fuel distributed among all satellites in the constellation. Therefore, the cost function to be maximized is given by

$$\mathcal{J}_a = -\sum_{i\in\mathcal{I}} |f_i^+ - \bar{f}^-|^2. \tag{2}$$

The contribution of all matched vertices of $\mathcal{G}$ in Eq. (2) is easily computed as

$$-\sum_{i\in\mathcal{I}} \sum_{\langle i,j\rangle\in\mathcal{Q}_i} |f_i^+ - \bar{f}^-|^2 x_{ij}, \tag{3}$$

where x_{ij} is a binary variable associated with each edge as follows

$$x_{ij} = \begin{cases} 1 & \text{if } \langle i,j\rangle \in \mathcal{M}, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

In order to ensure that each satellite is involved in at most one fuel transaction with another satellite we impose the inequality

$$\sum_{\langle i,j\rangle\in\mathcal{Q}_i} x_{ij} \leq 1, \qquad i \in \mathcal{I}. \tag{5}$$

If satellite s_i is not involved in a fuel transaction, then $f_i^+ = f_i^-$. As a result, $x_{ij} = 0$ for all $\langle i,j\rangle \in \mathcal{Q}_i$ and the corresponding edges are not part of the optimal matching. As a matter of fact, we have that $x_{ij} = 0$ for all $\langle i,j\rangle \in \mathcal{L}\backslash\mathcal{M}$.

The contribution to $\mathcal{J}_a$ from all unmatched vertices is

$$-\sum_{i\in\mathcal{I}}\Big(1-\sum_{\langle i,j\rangle\in\mathcal{Q}_i} x_{ij}\Big)|f_i^- - \bar{f}^-|^2 = -\sum_{i\in\mathcal{I}}|f_i^- - \bar{f}^-|^2 + \sum_{i\in\mathcal{I}}\sum_{\langle i,j\rangle\in\mathcal{Q}_i}|f_i^- - \bar{f}^-|^2 x_{ij}. \tag{6}$$

The term $\sum_{i\in\mathcal{I}}|f_i^- - \bar{f}^-|^2$ in the previous expression is constant, and thus it has no effect on the optimization process and it can be neglected. From Eqs. (3) and (7), and summing up the contributions from all satellites, we finally have

$$\mathcal{J}_a' = \sum_{i\in\mathcal{I}}\sum_{\langle i,j\rangle\in\mathcal{Q}_i}\big(|f_i^- - \bar{f}^-|^2 - |f_i^+ - \bar{f}^-|^2\big)x_{ij}. \tag{7}$$

Recalling that each edge $\langle i,j\rangle\in\mathcal{L}$ has contributions from two vertices $i,j\in\mathcal{I}$ of the graph, and rewriting the summation in Eq. (7) as a summation over all edges in the constellation graph, the objective function to be maximized is given by

$$\mathcal{J}_a' = \sum_{\langle i,j\rangle\in\mathcal{L}}\big(|f_i^- - \bar{f}^-|^2 - |f_i^+ - \bar{f}^-|^2 + |f_j^- - \bar{f}^-|^2 - |f_j^+ - \bar{f}^-|^2\big)x_{ij} \tag{8}$$

Letting π_{ij} denote the coefficient of x_{ij} in the previous sum, the problem becomes one of maximizing

$$\mathcal{J}_a' = \sum_{\langle i,j\rangle\in\mathcal{L}}\pi_{ij}x_{ij}. \tag{9}$$

subject to (4) and (5).

Since the objective of the refueling process is to equalize the fuel among all satellites in the constellation, we impose the constraint that after each fuel transaction between any pair of satellites, the two satellites end up with the same amount of fuel. In other words, we impose the condition that $f_i^+ = f_j^+$ for all $i\in\mathcal{I}$ at the end of the refueling process. Noting that the difference between the total fuel in the satellites before and after refueling can be related to the total fuel burnt during the rendezvous,[19] one obtains

$$f_i^+ = f_j^+ = \frac{1}{2}(f_i^- + f_j^- - p_{ij}). \tag{10}$$

Using (10), the weight of each edge in the constellation graph becomes

$$\pi_{ij} = |f_i^- - \bar{f}^-|^2 + |f_j^- - \bar{f}^-|^2 - \frac{1}{2}|f_i^- + f_j^- - 2\bar{f}^- - p_{ij}|^2. \tag{11}$$

Given these weights on the edges of the constellation graph, we seek a matching $\mathcal{M}$ that will maximize the sum of the weights of all edges in $\mathcal{M}$. This is a standard maximum weight matching problem in graph theory.[21] The solution to this problem provides the pairs of satellites involved in the optimal distribution of fuel using a P2P refueling scheme.

AN ALTERNATIVE COST MINIMIZATION FORMULATION

As already mentioned, the two objectives to be satisfied during a P2P refueling scenario are: (i) minimization of the fuel deviation among all satellites in the constellation, and (ii) minimization of the fuel expenditure during the orbital rendezvous transfers. These two objectives are conflicting in nature. For instance, we can fulfil only the first objective by performing continuous orbital transfers until all satellites have the same amount of fuel (perhaps even null). On the other hand, we can satisfy the second objective by not performing any orbital transfers at all. The cost function in Eq. (2) was introduced rather heuristically so that *implicitly* takes into account both of these objectives. In this section we show that this rationale is valid. We do this by introducing an optimization criterion $\mathcal{J}_b$ that incorporates *explicitly* the previous two conflicting objectives, and by unraveling the relationship of the cost $\mathcal{J}_b$ with the cost $\mathcal{J}_a$ in Eq. (2).

Since we seek to minimize the fuel deviation among all satellites in the constellation at the end of the refueling process, we introduce the following cost function to be maximized

$$J_1 = -\sum_{i\in\mathcal{I}} |f_i^+ - \bar{f}^+|^2. \tag{12}$$

Since we also want to minimize the cost incurred during the orbital maneuvers required for the fuel transfers, we also introduce the following cost to be maximized

$$J_2 = -\sum_{\langle \nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu}^2. \tag{13}$$

Given J_1 and J_2, we assign a relative weight between these two costs, and we combine them into a single cost function to be maximized, as follows

$$\mathcal{J}_b = \alpha J_1 + (1-\alpha)\, J_2, \tag{14}$$

where $0 \le \alpha \le 1$ takes care of the relative importance assigned to the two objectives.

The contribution to (12) from the satellites participating in fuel transactions is

$$-\sum_{i\in\mathcal{I}} \sum_{\langle i,j\rangle\in\mathcal{Q}_i} |f_i^+ - \bar{f}^+|^2 x_{ij}. \tag{15}$$

The contribution to J_1 from the satellites not participating in fuel transactions is

$$-\sum_{i\in\mathcal{I}} \Big(1 - \sum_{\langle i,j\rangle\in\mathcal{Q}_i} x_{ij}\Big) |f_i^- - \bar{f}^+|^2. \tag{16}$$

Combining the contributions from the participating (matched) and nonparticipating (unmatched) satellites into (12), one obtains

$$J_1 = -\sum_{i\in\mathcal{I}} \sum_{\langle i,j\rangle\in\mathcal{Q}_i} |f_i^+ - \bar{f}^+|^2 x_{ij} - \sum_{i\in\mathcal{I}} |f_i^- - \bar{f}^+|^2 + \sum_{i\in\mathcal{I}} \sum_{\langle i,j\rangle\in\mathcal{Q}_i} |f_i^- - \bar{f}^+|^2 x_{ij}. \tag{17}$$

The average fuel available in the constellation before and after refueling are related by

$$\bar{f}^+ = \bar{f}^- - \frac{1}{n} \sum_{\langle \nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu}. \tag{18}$$

Using Eq. (18), we may rewrite Eq. (17) as

$$J_1 = \sum_{i\in\mathcal{I}} \sum_{\langle i,j\rangle\in\mathcal{Q}_i} \big(|f_i^- - \bar{f}^-|^2 - |f_i^+ - \bar{f}^-|^2 + \frac{2}{n}(f_i^- - f_i^+) \sum_{\langle \nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu}\big) x_{ij} - \sum_{i\in\mathcal{I}} |f_i^- - \bar{f}^+|^2. \tag{19}$$

A simple calculation yields

$$\sum_{i\in\mathcal{I}} |f_i^- - \bar{f}^+|^2 = \sum_{i\in\mathcal{I}} \Big(|f_i^- - \bar{f}^-|^2 + \frac{2}{n}(f_i^- - \bar{f}^-) \sum_{\langle\nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu} \Big) + \frac{1}{n}\Big(\sum_{\langle\nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu}^2 + \sum_{\langle\nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu} \sum_{\langle m,k\rangle\in\mathcal{M}\setminus\langle\nu,\mu\rangle} p_{mk} \Big).$$

Note also that

$$\sum_{i\in\mathcal{I}} (f_i^- - \bar{f}^-) = 0$$

Moreover, the term $\sum_{i\in\mathcal{I}} |f_i^- - \bar{f}^-|^2$ is constant for a given constellation, and plays no role in the optimization process. Excluding this constant term, we have

$$\sum_{i\in\mathcal{I}} |f_i^- - \bar{f}^+|^2 = \frac{1}{n}\Big(\sum_{\langle\nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu}^2 + \sum_{\langle\nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu} \sum_{\langle m,k\rangle\in\mathcal{M}\setminus\langle\nu,\mu\rangle} p_{mk} \Big).$$

Hence the cost function to be maximized can be written as

$$\begin{aligned} \mathcal{J}_b' &= \alpha \sum_{i\in\mathcal{I}} \sum_{\langle i,j\rangle\in\mathcal{Q}_i} \Big(|f_i^- - \bar{f}^-|^2 - |f_i^+ - \bar{f}^-|^2 + \frac{2}{n}(f_i^- - f_i^+) \sum_{\langle\nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu} \Big) x_{ij} \\ &\quad - \frac{\alpha}{n}\Big(\sum_{\langle\nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu}^2 + \sum_{\langle\nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu} \sum_{\langle m,k\rangle\in\mathcal{M}\setminus\langle\nu,\mu\rangle} p_{mk} \Big) - (1-\alpha) \sum_{\langle\nu,\mu\rangle\in\mathcal{M}} p_{\nu\mu}^2. \end{aligned} \tag{20}$$

Writing the above summation as a summation over the edges and using Eq. (10), it follows that the criterion to be maximized takes the form

$$\begin{aligned} \mathcal{J}_b' &= \alpha \sum_{\langle i,j\rangle\in\mathcal{L}} \Big(|f_i^- - \bar{f}^-|^2 + |f_j^- - \bar{f}^-|^2 - \frac{1}{2}|f_i^- + f_j^- - p_{ij} - 2\bar{f}^-|^2 \Big) x_{ij} \\ &\quad + \frac{\alpha}{n} \sum_{\langle i,j\rangle\in\mathcal{L}} p_{ij} \sum_{\langle m,k\rangle\in\mathcal{L}\setminus\langle i,j\rangle} p_{mk} x_{mk} x_{ij} - (1-\alpha-\frac{\alpha}{n}) \sum_{\langle\nu,\mu\rangle\in\mathcal{L}} p_{\nu\mu}^2 x_{\nu\mu}. \end{aligned} \tag{21}$$

This expression consists of both linear and quadratic terms in the decision variables x_{ij}. This makes the problem a quadratic binary programming problem. One way to solve this problem is by introducing new variables in lieu of the quadratic terms. This also introduces new constraints involving the new and old variables. Formulating these as linear constraints, the problem can be converted to a linear binary programming problem for which efficient algorithms exist.

To this end, consider the quadratic term $x_{ij}x_{mk}$ where x_{ij} and x_{mk} are binary variables. Note that two edges that are part of the matching cannot share the same vertex, that is, if i, j, $m \in \mathcal{I}$, and $x_{im} = 1$, then $x_{ij} = 0$ for all $\langle i,j\rangle \in \mathcal{L}$, $j \neq m$. Thus, we may only consider quadratic terms of the form $x_{ij}x_{mk}$, $\langle i,j\rangle, \langle m,k\rangle \in \mathcal{L}$ and i, j, k, $m \in \mathcal{I}$, all distinct. Let now $\mathcal{I}'$ be a set of indices (of cardinality $|\mathcal{L}|$) generated as follows

$$q = n \times i + j, \quad \text{for all } \langle i,j\rangle \in \mathcal{L},\ i,j \in \mathcal{I}. \tag{22}$$

Conversely, given $q \in \mathcal{I}'$ the corresponding indices i and j are obtained via integer division by n using (22). We can therefore establish a one-to-one correspondence between elements of $\mathcal{I}'$ and $\mathcal{L}$, and we write $q \sim \langle i,j\rangle$ to denote this correspondence.

Considering now distinct indices $i, j, m, k \in \mathcal{I}$, and $p, q \in \mathcal{I}'$ such that $p \sim \langle i, j \rangle$ and $q \sim \langle m, k \rangle$ we introduce new variables defined by

$$x_{pq} = x_{ij} x_{mk}. \tag{23}$$

These new variables are also binary since

$$x_{pq} = \begin{cases} 1, & \text{when } x_{ij} = 1 \text{ and } x_{mk} = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{24}$$

The restrictions in Eq. (24) can be imposed on the new variables by introducing the following three linear constraints

$$x_{pq} \le x_{ij}, \tag{25}$$

$$x_{pq} \le x_{mk}, \tag{26}$$

$$-x_{pq} + x_{ij} + x_{mk} \le 1. \tag{27}$$

The first two constraints ensure that whenever $x_{ij} = 0$ or $x_{mk} = 0$, we have $x_{pq} = 0$. The last of the previous three constraints ensures that $x_{pq} = 1$ when $x_{ij} = 1$ and $x_{mk} = 1$. Hence, the problem of minimizing the two objectives absorbed in Eq. (14) is equivalent to the following *linear* binary integer programming problem

$$\begin{aligned} \mathcal{J}_b' \;=\; & \sum_{\langle i,j \rangle \in \mathcal{L}} \alpha \left(|f_i^- - \bar{f}^-|^2 + |f_j^- - \bar{f}^-|^2 - \frac{1}{2} |f_i^- + f_j^- - 2\bar{f}^- - p_{ij}|^2 \right) x_{ij} \\ & -(1 - \alpha - \frac{\alpha}{n}) \sum_{\langle i,j \rangle \in \mathcal{L}} p_{ij}^2 x_{ij} + \frac{2\alpha}{n} \sum_{\substack{\langle i,j \rangle \in \mathcal{L} \\ \langle m,k \rangle \in \mathcal{L} \setminus \langle i,j \rangle}} p_{ij} p_{mk} x_{pq}, \end{aligned} \tag{28}$$

subject to the constraints given by Eqs. (25), (26), (27), and Eqs. (4)-(5).

The parameter α in Eq. (14) weighs the relative importance for the fulfilment of the two performance objectives we have set for a P2P refueling scenario. If $\alpha = 0$, no fuel equalization is desirable ($\mathcal{J}_b = J_2$), and we only minimize the rendezvous costs. Obviously, in such a case the optimal solution involves no satellite pairings: all satellites remain at their initial orbital slots and the matching set $\mathcal{M}$ is empty. Equivalently, $|\mathcal{M}| = 0$. As we increase the value of α, fuel equalization becomes increasingly important and after a certain value of $\alpha = \bar{\alpha} > 0$ at least one pair of satellites performs a fuel transaction. The matching set $\mathcal{M}$ is non empty, and consequently $|\mathcal{M}| > 0$. For $\alpha = 1$ fuel equalization is the only optimization objective ($\mathcal{J}_b = J_1$), which is achieved with a (perhaps) unacceptably large number of fuel transactions. A compromise between the performance objectives J_1 and J_2 is achieved via an intermediate value of α. To investigate the effect of α in the optimal number of satellite pairings, and compare with the original "two-in-one" cost $\mathcal{J}_a$, several numerical examples have been conducted.

Numerical Example

In this section we investigate numerically the relationship between the solutions obtained via the two costs (2) and (14). Specifically, we show that solutions obtained via (2) correspond to solutions obtained via (14) for a range of values of α that achieve a balanced compromise between the original conflicting optimization objectives J_1 and J_2.

Figure 1 shows a typical variation of $\mathcal{J}_b$ with α for the two constellations C_4 and C_8 in Table 1. The plots are piecewise linear, with each linear portion corresponding to a particular set of pairings of the satellites in the constellation.

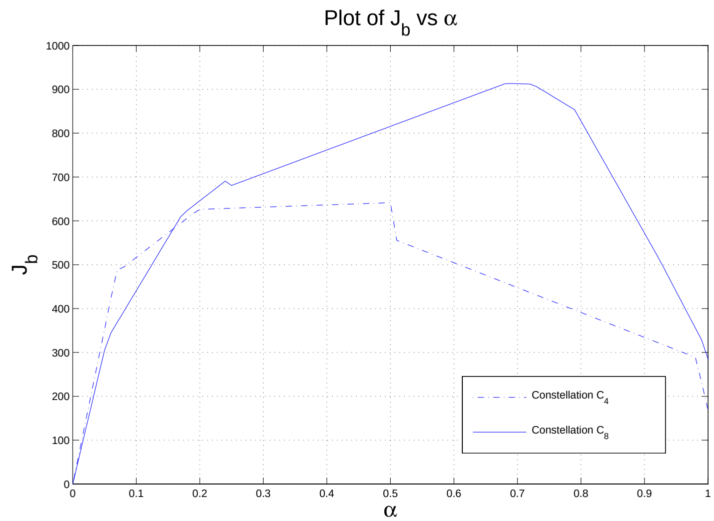

Figure 1: Typical variation of $\mathcal{J}_b$ with respect to α.

Typical variations of the values of the two objective functions J_1 and J_2 are shown in Figures 2 and 3 for the constellations C_4 and C_8, respectively. Each point on the curve in these plots is optimal, corresponding to a particular choice of α. The range of values of α for which the same pairings of satellites occur as with the optimization of $\mathcal{J}_a$ is also shown on these plots. Note that for this range of α the pairings of satellites are the same, hence the values of J_1 and J_2 are also the same.

For this range of values of α we have a reasonable compromise between the two performance specifications J_1 and J_2. Moreover, from these plots it is concluded that the use of the simpler cost $\mathcal{J}_a$ in lieu of $\mathcal{J}_b$ is justified, as the former results in solutions which are identical to those obtained via $\mathcal{J}_b$ for values of α that provide a balance between the objectives J_1 and J_2. The case for using $\mathcal{J}_a$ instead of $\mathcal{J}_b$ is made stronger in light of the fact that the calculation of the optimal matching using the cost $\mathcal{J}_b$ is computationally more intensive than using the cost $\mathcal{J}_a$, owing to the larger number of decision variables and the associated constraints; see (23)-(27). As a result, in practice one can confidently bypass the optimization of $\mathcal{J}_b$ and deal only with the optimization of $\mathcal{J}_a$ when computing the optimal satellite pairings in a P2P scenario. We will make use of this observation in all our subsequent computations from now on.

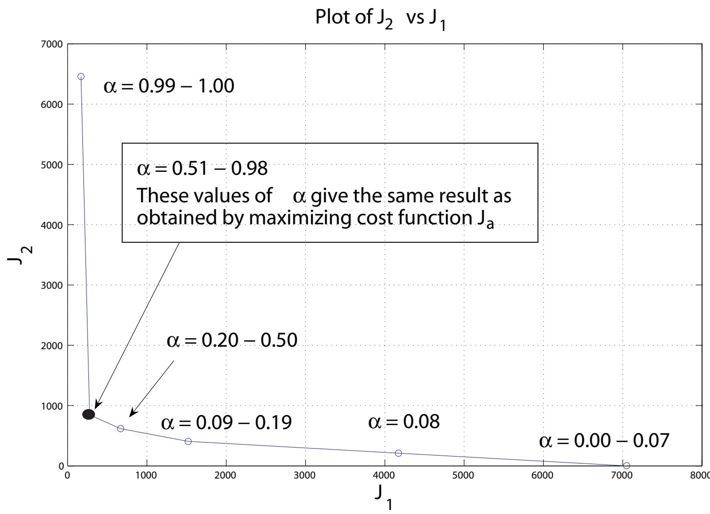

Figure 2: Variation of J_2 with respect to J_1 (constellation C_4).

PURE P2P REFUELING STRATEGIES

It is well known[22,23] that coasting can significantly reduce the fuel expenditure during a rendezvous. Therefore, during each transfer, initial or final coasting intervals play an important role in the overall optimal rendezvous cost. Figure 4 shows a typical variation of the rendezvous cost between two satellites (in terms of non-dimensionalized ΔV) with respect to the transfer time. In this figure the initial separation angle between the satellites is 60 deg and both satellites are in the same circular orbit. The dotted line shows the cost if coasting is not allowed, while the solid line shows the cost when initial coasting is allowed. In the latter case, the active satellite stays for some time in its original orbit and the actual transfer occurs over a smaller time period. Therefore, by allowing a coasting period during an orbital transfer we can reduce the overall cost. The idea of allowing coasting intervals is utilized in this section to propose a strategy for reducing the overall P2P rendezvous cost.

As it is evident from Figure 4 the optimal cost when coasting is included is a non-increasing function of time. That is, the inequality

$$\Delta V(t_{f1}) \leq \Delta V(t_{f2}), \quad \text{for} \;\; t_{f1} \geq t_{f2} \tag{29}$$

holds for any two transfer times t_{f1} and t_{f2}. Note that this monotonicity of ΔV versus the transfer time does not hold if there are no coasting intervals.

In our previous investigation of P2P refueling strategies[20] it was assumed that given the total amount of time to complete each fuel transaction, the time was equally divided between the forward and return orbital transfers for each fuel transaction. Here we relax this restriction. In particular, we show that by allowing unequal transfer times between the forward and return journeys for each fuel transaction, one can reduce the transfer cost.

To see why this is true, let us consider a single refueling maneuver between two satellites s_i and s_j, and let $s_i \in \mathcal{A}$ be the active satellite, and $s_j \in \mathcal{P}$ be the passive satellite. Note

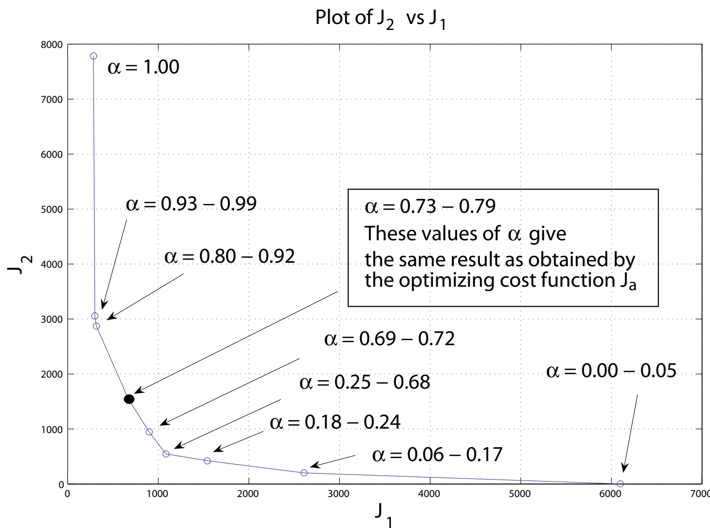

Figure 3: Variation of J_2 with respect to J_1 (constellation C_8).

that either of the two satellites can be the seller or the buyer during the fuel transaction. The amount of fuel spent by s_i to rendezvous with s_j is given by[24]

$$p_{fi} = (m_{si} + f_i^-)(1 - e^{-\Delta V_{ij}/c_{0i}}), \tag{30}$$

where m_{si} is the mass of the permanent structure of satellite s_i, f_i^- is the initial fuel of satellite s_i, f_j^- is the initial fuel of satellite s_j, and ΔV_{ij} is the velocity increase required to transfer from the orbit of satellite s_i to the orbit of satellite s_j. The parameter c_{0i} is defined by $c_{0i} = g_0 I_{\text{spi}}$, where g_0 is the acceleration due to gravity at the Earth's surface, and I_{spi} is the specific thrust of satellite s_i.

The amount of fuel consumed by satellite s_i to return back to its original position after a fuel exchange has taken place[¶] is given by

$$p_{ri} = (2m_{si} + f_i^- + f_j^- - p_{fi})\frac{(1 - e^{-\Delta V_{ji}/c_{0i}})}{(1 + e^{-\Delta V_{ji}/c_{0i}})}, \tag{31}$$

where ΔV_{ji} is the optimum rendezvous cost for the return journey. Note that, in general $\Delta V_{ji} \neq \Delta V_{ij}$. Using the previous equations the total fuel used by satellite s_i during the two transfers is given by

$$p_{ij} = p_{fi} + p_{ri}. \tag{32}$$

Now let us denote by t_{ij} the total time allowed to complete both legs of the fuel transaction between satellites s_i and s_j. Moreover, let t_{ij}^f denote the time for the forward journey and t_{ij}^r denote the time for the return journey, so that

$$t_{ij} = t_{ij}^f + t_{ij}^r. \tag{33}$$

¶It is assumed that during the exchange of fuel the seller satellite gives enough fuel to the buyer satellite so that both have the same amount of fuel at the end of the fuel transaction.[19]

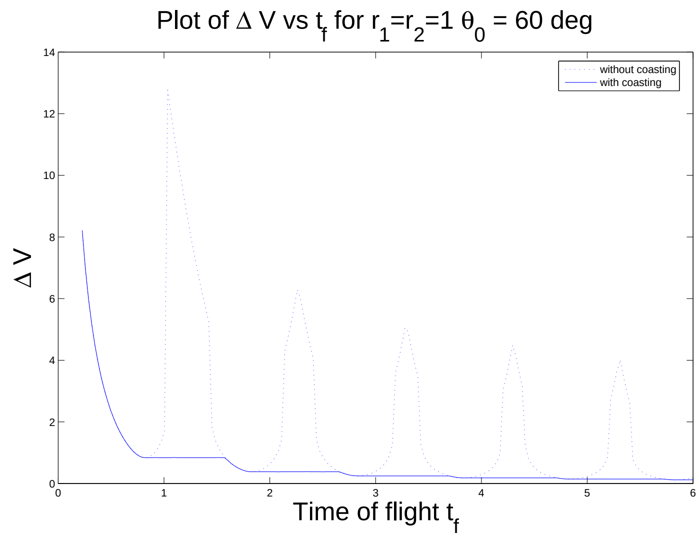

Figure 4: Variation of rendezvous cost with transfer time. Transfer from and to a circular orbit with an initial separation angle of 60 deg.

In case of an equal partition of the total time between the forward and return transfers, we have $t_{ij}^f = t_{ij}^r = t_{ij}/2$. In the sequel we use the superscript I, to denote quantities associated with such an equal time partition transfer. For simplicity, we assume a coasting period for the forward leg, and we will use the superscript II to denote the quantities associated with a transfer with unequal time partition of t_{ij} such that the forward and return legs are completed within the time intervals $t_{ij}^f = t_{ij}/2 - t'_{ij}$ and $t_{ij}^r = t_{ij}/2 + t'_{ij}$, where t'_{ij} denotes the optimal coasting time for the forward leg. Similarly, we will use the superscript III to denote the quantities associated with a transfer with unequal time partition of t_{ij} such that the forward and return legs are completed within the time intervals $t_{ij}^f = t_{ij}/2 + t''_{ij}$ and $t_{ij}^r = t_{ij}/2 - t''_{ij}$, where t''_{ij} denotes the optimal coasting time for the return leg. Let us concentrate on the case where coasting is part of the forward leg.

Note that since coasting periods do not have any effect on the cost, one obtains,

$$\Delta V_{ij}^{\mathrm{I}} = \Delta V_{ij}^{\mathrm{II}},$$

which implies, according to (30) that

$$p_{fi}^{\mathrm{I}} = p_{fi}^{\mathrm{II}}. \tag{34}$$

For the return flight, and since $t_{ij}/2 + t'_{ij} \geq t_{ij}/2$ we have, via (29), that

$$\Delta V_{ji}^{\mathrm{I}} \geq \Delta V_{ji}^{\mathrm{II}},$$

which implies that $e^{-\Delta V_{ji}^{\mathrm{I}}/c_{0i}} \leq e^{-\Delta V_{ji}^{\mathrm{II}}/c_{0i}}$. Using this inequality, it follows that $1 - e^{-\Delta V_{ji}^{\mathrm{I}}/c_{0i}} \geq 1 - e^{-\Delta V_{ji}^{\mathrm{II}}/c_{0i}}$, and also $1 + e^{-\Delta V_{ji}^{\mathrm{I}}/c_{0i}} \leq 1 + e^{-\Delta V_{ji}^{\mathrm{II}}/c_{0i}}$. These two inequalities together yield

$$\frac{1 - e^{-\Delta V_{ji}^{\mathrm{I}}/c_{0i}}}{1 + e^{-\Delta V_{ji}^{\mathrm{I}}/c_{0i}}} \geq \frac{1 - e^{-\Delta V_{ji}^{\mathrm{II}}/c_{0i}}}{1 + e^{-\Delta V_{ji}^{\mathrm{II}}/c_{0i}}} \tag{35}$$

which, via (31), yields

$$p_{ri}^{\mathrm{I}} \geq p_{ri}^{\mathrm{II}}. \tag{36}$$

From equation (34) and inequality (36), the identity (32) yields

$$p_{ij}^{\mathrm{I}} \geq p_{ij}^{\mathrm{II}}. \tag{37}$$

A similar analysis holds when a coasting period of length t'' is part of the return leg, in which case one can show that

$$p_{ij}^{\mathrm{I}} \geq p_{ij}^{\mathrm{III}}. \tag{38}$$

We have therefore shown the following proposition.

Proposition 1. *For each fuel transaction between two satellites in the same circular orbit, and a given total time for the transaction to take place, an equal time allocation between the forward and return legs of the two associated rendezvous transfers is suboptimal.*

We will next utilize this idea to devise a coast time allocation (CTA) algorithm for reducing the fuel coast during each fuel transaction.

Coast Time Allocation Algorithm

The main idea behind the formulation of a fuel-reducing strategy is to allow for unequal time distribution between the forward and the return legs for each fuel transaction. To this end, we consider the following three cases:

- Case-I: $t_{ij}^f = t_{ij}^r = t_{ij}/2$
- Case-II: $t_{ij}^f = t_{ij}/2 - t'_{ij}$ and $t_{ij}^r = t_{ij}/2 + t'_{ij}$
- Case-III: $t_{ij}^f = t_{ij}/2 + t''_{ij}$ and $t_{ij}^r = t_{ij}/2 - t''_{ij}$

Assume a fuel transaction between satellites $s_i \in \mathcal{A}$ and $s_j \in \mathcal{P}$ and let $p_i^{j\mathrm{I}}$, $p_i^{j\mathrm{II}}$ and $p_i^{j\mathrm{III}}$ denote the fuel spent for satellite s_i to rendezvous with s_j and return back to its original position, for each of the previous three cases, respectively. The optimal time sharing is the one that satisfies

$$p_i^{j*} = \min\{p_i^{j\mathrm{I}}, p_i^{j\mathrm{II}}, p_i^{j\mathrm{III}}\}. \tag{39}$$

The corresponding time allocation is then given by

$$(t_{ij}^f, t_{ij}^r) = \begin{cases} (t_{ij}/2, t_{ij}/2), & \text{if } p_i^{j*} = p_i^{j\mathrm{I}}, \\ (t_{ij}/2 - t'_{ij}, t_{ij}/2 + t'_{ij}), & \text{if } p_i^{j*} = p_i^{j\mathrm{II}}, \\ (t_{ij}/2 + t''_{ij}, t_{ij}/2 - t''_{ij}), & \text{if } p_i^{j*} = p_i^{j\mathrm{III}}. \end{cases}$$

We can similarly compute the cost of a single fuel transaction for the case $s_i \in \mathcal{P}$ and $s_j \in \mathcal{A}$. Finally, the optimum fuel consumption between any two satellites s_i, $s_j \in \mathcal{G}$ is given by

$$p_{ij}^* = \begin{cases} p_i^{j*}, & \text{if } s_i \text{ can be active, but } s_j \text{ cannot be active,} \\ p_j^{i*}, & \text{if } s_j \text{ can be active, but } s_i \text{ cannot be active,} \\ \min\{p_i^{j*}, p_j^{i*}\}, & \text{if either } s_i \text{ or } s_j \text{ can be active,} \\ \infty, & \text{if neither } s_i \text{ nor } s_j \text{ can be active.} \end{cases}$$

We demonstrate these results next via a numerical example. Let us consider a single fuel transaction between two identical satellites in the same circular orbit. We assume that the mass of permanent structure for satellites is $m_s = 60$ units, and the characteristic constant of the rocket engine is $c_0 = 2943$ units. The initial fuel of the active satellite is 100 units and of the passive satellite is 10 units. The allowed time to conduct the fuel transaction is chosen to be 12 units.

Figure 5 shows a comparison between the three cases as a function of the separation angle between the two satellites. For all separation angles, an equal time allocation for the forward and return legs of a fuel transaction (Case I) always results in more or equal fuel expenditure than an unequal time allocation (Cases II or III).

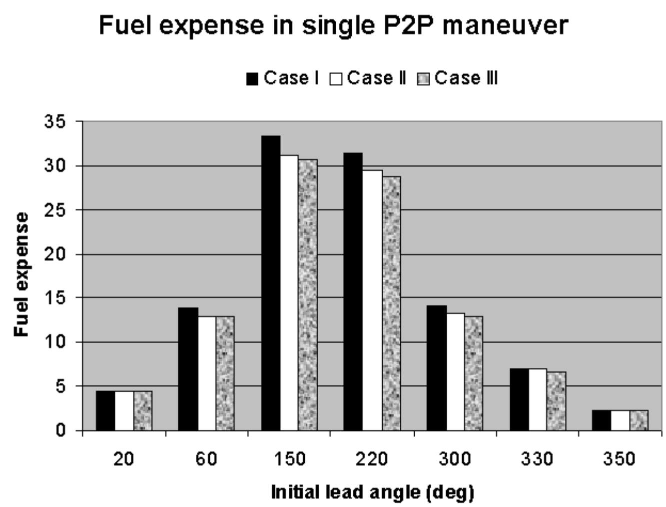

Figure 5: Effect of CTA algorithm to a single P2P maneuver.

The effect of the CTA algorithm when refueling a constellation using a pure P2P strategy is evaluated by the introduction of the following figure of merit

$$G = \frac{\left(\sum_{\langle i,j\rangle \in \mathcal{M}'} p_{ij} - \sum_{\langle i,j\rangle \in \mathcal{M}} p^*_{ij}\right)}{\sum_{\langle i,j\rangle \in \mathcal{M}} p^*_{ij}} \times 100\ \%, \tag{40}$$

where $\mathcal{M}$ is the matching edge set for the optimal time allocation, and $\mathcal{M}'$ is the matching edge set for the refueling strategy under evaluation. We call G the *net percentage gain* of the refueling.

Several circular constellations with a varied number satellites of physical characteristics have been studied, and the final fuel distribution and rendezvous costs associated with both pure P2P and mixed refueling strategies have been computed. The CTA algorithm has been applied to a complete P2P refueling scenario for the constellations given in Table 1. In this table, the initial fuel for each satellite is shown, along with the total allowed time T for the forward and return trips. Note that for all numerical results below one unit of time corresponds to one period of the circular orbit of the constellation.

The corresponding gains are shown in Figure 6. The results in Figure 6 indicate considerable

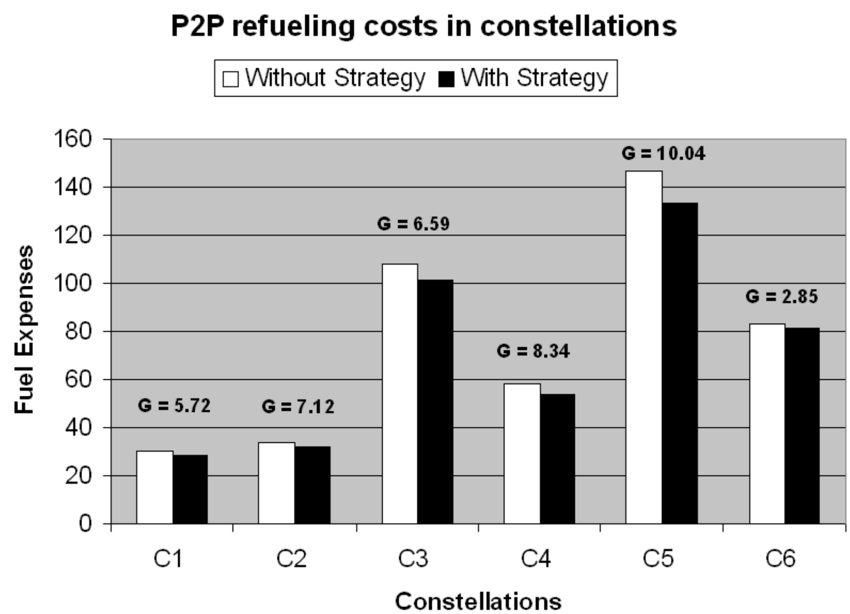

Figure 6: Effect of CTA algorithm to an entire constellation; see also Table 1.

amount of fuel savings if the CTA algorithm is adopted. Note that in most cases, the application of the CTA algorithm has no effect on the satellite pairings. However, for constellation C_6, it was also found that the entire set of optimal pairings of satellites change when the algorithm is applied. This shows that the CTA algorithm can altogether affect the scheduling of the refueling process in order to reduce the cost.

MIXED REFUELING STRATEGIES

So far we have discussed pure P2P refueling strategies for the purpose of equalizing fuel among all satellites in the constellation. Although significant unequal fuel distribution between identical satellites in the same orbit are rather unlikely (except in case of failures), and hence pure P2P strategies seem to be exceptional, nonetheless they arise naturally as a second stage of mixed refueling strategies. This has been demonstrated in Refs. 18, 19, where it was shown that a mixed strategy will typically outperform a single-spacecraft refueling strategy, as the number of satellites in the constellation increases.

Let us consider a constellation in a circular orbit with an even number of satellites $s_i,\ i \in \mathcal{I} = \{1, 2, ..., 2n\}$. For the sake of simplicity, we may assume that all satellites are initially depleted of fuel, that is, $s_i \in \mathcal{C}_d$ for all $i \in \mathcal{I}$. Given a maximum refueling period, say T, we wish to refuel all of the satellites from a service vehicle s_0, such that after time T they all end up with approximately the same amount of fuel. In the process, we also want to minimize the total fuel expenditure during the ensuing orbital maneuvers. Equivalently, we want to

Table 1: Sample Constellations.

Label	Description
C_1	14 satellites, same structure and specific thrust $f_i(0^-)$: 38.8, 36, 35.2, 32.8, 29.6, 27.6, 26.8, 17.6, 14, 8, 6.8, 6.4, 5.6, 0.4 $T = 12$
C_2	6 satellites different structure and specific thrust $f_i(0^-)$: 5, 45, 86, 31, 12, 90 $T = 16$
C_3	18 satellites, same structure and specific thrust $f_i(0^-)$: 62, 50, 40, 98, 70, 25, 88, 20, 72, 30, 82, 54, 42, 66, 35, 10, 90, 45. $T = 8$
C_4	8 satellites, same structure and specific thrust $f_i(0^-)$: 85, 30, 95, 20, 65, 40, 75, 10 $T = 12$
C_5	20 satellites, same structure and specific thrust $f_i(0^-)$: 65, 70, 72, 65, 92, 44, 32, 16, 15, 28, 56, 88, 90, 92, 86, 30, 25, 36, 52, 60. $T = 10$
C_6	7 satellites, different structure and specific thrust $f_i(0^-)$: 25, 40, 70, 82, 12, 95, 42 $T = 8$
C_7	9 satellites, different structure and specific thrust $f_i(0^-)$: 85, 30, 50, 95, 20, 65, 40, 75, 10 $T = 12$
C_8	10 satellites, different structure and specific thrust $f_i(0^-)$: 25, 40, 50, 70, 82, 45, 12, 95, 30, 42 $T = 8$

maximize the total amount of fuel that can be delivered to the constellation. We have two alternatives for solving this problem.

The first alternative is for s_0 to refuel (perhaps sequentially[15]) all other satellites in the constellation. This scenario is shown in Figure 7. The second alternative is a mixed refueling strategy, consisting of two stages. During the first stage, the service vehicle s_0 delivers fuel to half the satellites in the constellation. During the second stage, these satellites share their fuel with the remaining satellites in P2P fashion. This alternative refueling scenario is shown in Figure 8.

Let $\mathcal{I}_1$ denote the index set of the satellites refueled during the first stage by the service vehicle s_0 in a mixed strategy, and let $\mathcal{I}_2 = \mathcal{I} \backslash \mathcal{I}_1$ denote the remaining satellites which are to be refueled during the second stage. Without loss of generality we may assume that $\mathcal{I}_1 = \{1, 2, \ldots, n\}$ and $\mathcal{I}_2 = \{n+1, n+2, \ldots, 2n\}$. Let also $T^{(1)}$ denote the time allotted for the first stage and $T^{(2)} = T - T^{(1)}$ the time allotted for the second (P2P) stage in a mixed strategy.

During $T^{(1)}$ the service vehicle s_0 delivers fuel sequentially to the n satellites s_i ($i \in \mathcal{I}_1$) in an optimal fashion. The optimal time distribution for these transfers, denoted by $t^{(1)}_{i,i+1}$ ($i =$

$1, \ldots, n-1$) then satisfies

$$T^{(1)} = \sum_{i=1}^{n-1} t_{i,i+1}^{(1)}, \tag{41}$$

where the optimal values $t_{i,i+1}^{(1)}$ are calculated by solving a binary integer programming problem.[20] Note that the CTA algorithm can be implemented during this second stage to reduce the cost of the P2P maneuvers as was elaborated in the previous section.

In Ref. 19 we showed that a mixed strategy will, in general, outperform a single-spacecraft strategy, especially as the number of satellites in the constellation increases. In Ref. 19 we assumed only synchronous implementation for the P2P second stage, that is, all P2P maneuvers during the second stage of the mixed refueling scenario, occur simultaneously and they all take time $T^{(2)}$ to be completed. However, we can further improve on the fuel savings incurred during the second stage by allowing asynchronous P2P maneuvers, as described next.

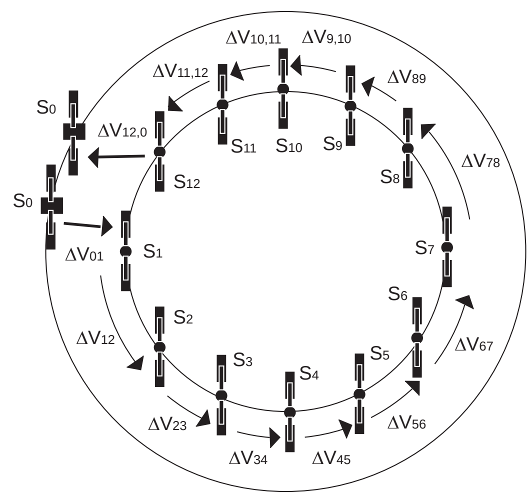

Figure 7: Single-spacecraft refueling scenario.

Asynchronous P2P Refueling

In a synchronous P2P scenario all the satellite rendezvous take place simultaneously. In a mixed refueling strategy, this implies that all fuel deficient satellites (at the end of the first stage) are refueled within the time $T^{(2)}$. Note, however that the time $T^{(2)}$ is binding only for satellite s_n (the last satellite to be visited by s_0 during the first stage of a mixed strategy). All other satellites s_i ($i = 1, \ldots, n-1$) have available $T^{(2)} + \sum_{k=i}^{n-1} t_{k,k+1}^{(1)}$ time units to perform their fuel transactions. Thus, the time available for s_i to complete the P2P maneuver with its matching satellite s_j is given by

$$t_{ij}^{(2)} = \begin{cases} T^{(2)} + \sum_{k=i}^{n-1} t_{k,k+1}^{(1)}, & \text{if } i \in \mathcal{I}_1 \backslash \{n\}, \\ T^{(2)}, & \text{if } i = n. \end{cases} \tag{42}$$

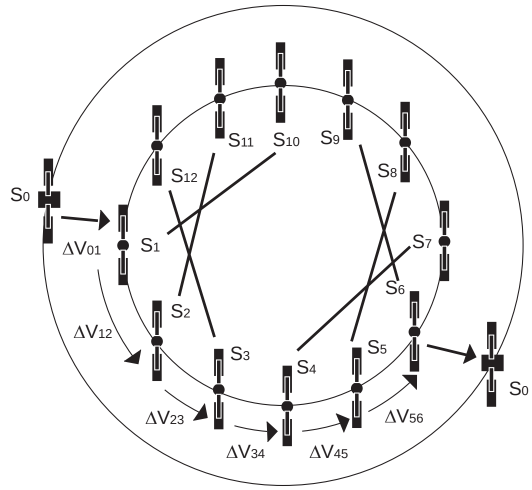

Figure 8: Mixed refueling scenario.

We refer to this strategy as asynchronous P2P refueling, since not all satellite pairs complete their corresponding fuel transactions within the same time period. Since $t_{ij}^{(2)} \geq T^{(2)}$ for all satellite pairs, and referring again to Eq. (29), it is clear that each rendezvous between two satellites will require less fuel than a synchronous implementation. Consequently, the overall fuel consumption for the whole constellation will also be reduced by using an asynchronous P2P implementation. This is demonstrated next via numerical examples.

NUMERICAL EXAMPLES

We next apply the CTA algorithm along with an asynchronous (mixed) P2P refueling strategy to sample constellations. With the help of numerical examples we show how these improvements for a mixed refueling strategy make the latter a competitive alternative to a refueling strategy using a single service vehicle or to mixed synchronous P2P strategies.

To this end, we assume a circular orbit constellation with an even number of satellites. The service spacecraft, denoted by s_0, starts with an initial amount of fuel $f_0(0^-) = 500$ units. We assume that s_0 is initially at a higher circular orbit than the constellation orbit. It is required to return to the same orbit after completing the refueling process with $f_0(T^+) = 10$ units of fuel, where $T = 20$ is the maximum allowed time for completing the whole refueling process. Hence, the total amount of fuel to be delivered to the satellites in the constellation including the fuel to be used during the corresponding orbital transfers is 490 units. The mass of the permanent structure for each satellite is $m_{si} = 60$ units and the characteristic constant of the engine is $c_{0i} = 2943$ units for all satellites.

In the first example, we consider a constellation with six satellites evenly distributed in the circular orbit. The service vehicle s_0 visits all these six satellites and distributes the fuel equally among all satellites in the constellation. There are five rendezvous segments, and the maximum time of transfer allowed for each rendezvous segment is 6 time units. The optimal time distribution for each of these five rendezvous segments, and the corresponding fuel expenditure are given in Table 2.

Table 2: OPTIMAL FUEL CONSUMPTION WITH A SINGLE SERVICE VEHICLE. SIX SATELLITE CONSTELLATION.

Segment	t_{ij}	ΔV_{ij}	Fuel Expense
$i = 1,\ j = 2$	4.1607	0.1676	30.6311
$i = 2,\ j = 3$	4.1607	0.1676	24.8345
$i = 3,\ j = 4$	4.1607	0.1676	19.4244
$i = 4,\ j = 5$	4.1607	0.1676	14.3751
$i = 5,\ j = 6$	3.3570	0.2204	12.5754

At the end of this process, each of the six satellites ends up with an equal amount of fuel $f_i^+ = 56.31$ ($i = 1, 2, ..., 6$). The total amount of fuel used during all these transfers is thus $490 - 6 \times 56.31 = 152.14$ units. Note that these values do not include the fuel consumption for the initial ($\Delta V = 44.2619$) and final ($\Delta V = 6.0094$) transfers of s_0 to and from the constellation orbit, which are constant and thus not part of the optimization process.

Table 3: OPTIMAL FUEL CONSUMPTION DURING THE FIRST STAGE OF A MIXED REFUELING STRATEGY. SIX SATELLITE CONSTELLATION.

Segment	$t_{ij}^{(1)}$	ΔV_{ij}	Fuel Expense
$i = 1,\ j = 2$	4.8279	0.1444	22.0481
$i = 2,\ j = 3$	3.8421	0.1826	16.3783

Table 4: OPTIMAL FUEL CONSUMPTION DURING THE SECOND STAGE OF A MIXED REFUELING STRATEGY. SIX SATELLITE CONSTELLATION.

Pairs	T	$T^{(1)}/T^{(2)}$	Fuel Expense
(s_1, s_6)	20.00	10.17/9.83	9.0299
(s_2, s_4)	15.17	7.85/7.32	23.4042
(s_3, s_6)	11.33	6.00/5.33	31.3522

The optimum solution for a mixed refueling strategy yields that the first step, during which s_0 delivers fuel to satellites s_1, s_2 and s_3 requires two rendezvous segments with total time $T^{(1)} = 8.67$ time units. The optimum time distribution and the corresponding fuel consumption for this step are given in Table 3. The three satellites refueled by s_0 have 133.76 units of fuel each before performing the P2P maneuvers with the remaining satellites s_4, s_5 and s_6. The available time and the corresponding fuel expenditures for the P2P maneuvers are given in Table 4. The final fuel content of each satellite at the end of the refueling process are $f_1(T^+) = f_6(T^+) = 62.37, f_2(T^+) = f_4(T^+) = 55.18$ and $f_3(T^+) = f_5(T^+) = 51.21$. The average amount of fuel in the constellation then is equal to 56.25 units. The total amount of fuel burnt is $490 - 6 \times 56.25 = 152.50$ units, which is 0.24% more than the amount of fuel burnt if the satellites are refueled by a single spacecraft. A single-spacecraft refueling strategy is *marginally* better than a mixed refueling strategy in this case.

For the second example we consider a constellation with twelve satellites evenly distributed in a circular orbit. The total time allowed for refueling is again $T = 20$ time units. There are eleven rendezvous segments with a single-spacecraft refueling strategy. The optimal time distribution for each of the five rendezvous segments and the corresponding fuel consumption are given in Table 5. At the end of this process, each of the six satellites end up with an equal amount of fuel $f_i^+ = 17.31$. The total amount of fuel used during all the transfers is thus $490 - 12 \times 17.31 = 282.28$ units.

For the mixed strategy, there are five rendezvous segments during the first stage, which are all completed within $T^{(1)} = 9.59$ units. The optimal time distribution for each of the five rendezvous segments and the corresponding fuel consumption are given in Table 6. The six satellites refueled by s_0 have fuel 55.53 units each before performing the P2P refueling part. The time available for the P2P maneuvers and the corresponding fuel consumption are given in Table 7. The final fuel content of the satellites are $f_1(T^+) = f_{10}(T^+) = 23.50, f_2(T^+) = f_{11}(T^+) = 23.04, f_3(T^+) = f_{12}(T^+) = 22.45, f_4(T^+) = f_7(T^+) = 21.74, f_5(T^+) = f_8(T^+) = 20.71, f_6(T^+) = f_9(T^+) = 19.35$. The average amount of fuel in the constellation is 21.80 units. The total amount of fuel burnt using the mixed refueling strategy is $490 - 12 \times 21.80 = 228.4$ units, which is about 19% less than the amount of fuel burnt if the satellites are refueled by a single spacecraft. Clearly, the mixed scenario outperforms the single service vehicle option in this case.

Table 5: Optimal Fuel Consumption for Refueling with a Single Service Vehicle. Twelve Satellite Constellation.

Segment	t_{ij}	ΔV_{ij}	Fuel Expense
$i = 1,\ j = 2$	1.9084	0.1821	35.9746
$i = 2,\ j = 3$	1.9084	0.1821	32.1287
$i = 3,\ j = 4$	1.9084	0.1821	28.5604
$i = 4,\ j = 5$	1.9084	0.1821	25.2497
$i = 5,\ j = 6$	1.9084	0.1821	22.1779
$i = 6,\ j = 7$	1.9084	0.1821	19.3278
$i = 7,\ j = 8$	1.9084	0.1821	16.6834
$i = 8,\ j = 9$	1.9084	0.1821	14.2299
$i = 9,\ j = 10$	1.9084	0.1821	11.9535
$i = 10,\ j = 11$	1.9084	0.1821	9.8414
$i = 11,\ j = 12$	0.9163	0.3805	15.8334

Table 6: Optimal Fuel Consumption for First Step of Mixed Refueling Strategy. Twelve Satellite Constellation.

Segment	$t_{ij}^{(1)}$	ΔV_{ij}	Fuel Expense
$i = 1,\ j = 2$	1.9174	0.1822	33.2517
$i = 2,\ j = 3$	1.9174	0.1822	26.8369
$i = 3,\ j = 4$	1.9174	0.1822	20.8556
$i = 4,\ j = 5$	1.9174	0.1822	15.3643
$i = 5,\ j = 6$	1.9174	0.1822	10.2419

Table 7: Optimal Fuel Consumption for Second Step of Mixed Refueling Strategy. Twelve Satellite Constellation.

Pairs	T	$T^{(1)}/T^{(2)}$	Fuel Expense
(s_1, s_{10})	20.00	10.25/9.75	8.5335
(s_2, s_{11})	18.08	9.33/8.75	9.4564
(s_3, s_{12})	16.17	8.43/7.74	10.6270
(s_4, s_7)	14.25	8.00/6.25	12.0585
(s_5, s_8)	12.33	5.75/6.58	14.1137
(s_6, s_9)	10.41	4.74/5.67	16.8236

CONCLUSIONS

In this paper, we have studied peer-to-peer (P2P) satellite refueling scenarios in circular orbit constellations. P2P refueling strategies have been proposed recently as a viable, competitive alternative to single-satellite refueling. Although pure P2P strategies are rather unlikely for constellations with similar satellites, P2P refueling arises naturally as a second stage in mixed refueling strategies. For such mixed strategies we show via numerical examples that an unequal time distribution of the forward and return trips for each satellite pair, along with an asynchronous implementation of the P2P rendezvous sequence, result in more efficient refueling than previous synchronous P2P/mixed or single-spacecraft refueling implementations.

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REFERENCES

[1] E. Lamassoure, J. H. Saleh, and D. E. Hastings. Space systems flexibility provided by on-orbit servicing: Part II. In *AIAA Space Conference and Exhibition*, Albuquerque, NM, 2001. AIAA Paper 2001-4631.

[2] C. M. Reynerson. Spacecraft modular architecture design for on-orbit servicing. In *Proceedings of the IEEE Aerospace Conference*, 2000. Big Sky, MO.

[3] J. Ross, D. Musliner, T. Kreider, J. Jacobs, and M. Fisher. Configurable spacecraft control architectures for on-orbit servicing and upgrading of long life orbital platforms. In *Proceedings of the IEEE Aerospace Conference*, pages 2625–2630, 2004. Big Sky, MO.

[4] D. Manouchehri and A. J. Mauceri. Automated resuply of consumables: Enhancement of space commercialization opportunities. In *The 5th Annual Workshop on Space Operations Applications and Research (SOAR 1991)*, volume 1, pages 407–411, NASA Johnson Space Center, 1991.

[5] S. Smith. Preliminary analysis of the benefits derived to US Air Force spacecraft from on-orbit refueling. In *The 6th Annual Workshop on Space Operations Applications and Research (SOAR 1992)*, volume 2, pages 637–655, 1992. In NASA. Johnson Space Center.

[6] K. Galabova. Architecting a family of space tugs based on orbital transfer mission scenarios. Master's thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA, February 2004.

[7] M. E. Milton and T. R. Tyler. Development and testing of the automated fluid interface system. In *The 27th Aerospace Mechanisms Symposium*, pages 121–135, 1993. In NASA. Ames Research Center.

[8] E. Distefano and C. Noll. Advanced liquid feed experiment. In *AIAA, ASME, SAE, and ASEE, Joint Propulsion Conference and Exhibit, 29th*, June 28-30 1993. Monterey, CA.

[9] S. M. Dominick and J. Tegart. Orbital test results of advanced liquid acquisition device. In *AIAA, ASME, SAE, and ASEE, Joint Propulsion Conference and Exhibit, 30th*, June 27-29 1994. Indianapolis, IN.

[10] http://www.msfc.nasa.gov/news/dart/. (on-line).

[11] W. Hamilton. Automatic refueling coupling for on-orbit spacecraft servicing. In *AIAA, ASME, SAE, and ASEE, Joint Propulsion Conference and Exhibit, 25th. Monterey, CA*, July 10-13, 1989. AIAA Paper 89-2731.

[12] http://www.darpa.mil/tto/programs/astro.html. (on-line).

[13] M. R Johnson. On-orbit spacecraft re-fluiding, 1998. Master's Creative Investigation, US Air Force Institute of Technology.

[14] D. Waltz. *On-orbit Servicing of Space Systems.* Krieger Publishing, Malabar, FL, 1993.

[15] H. Shen and P. Tsiotras. Optimal scheduling for servicing multiple satellites in a circular constellation. In *AIAA/AAS Astrodynamics Specialists Conference*, Monterey, CA, August 5–8, 2002. AIAA Paper 2002-4907.

[16] H. Shen and P. Tsiotras. Peer-to-peer satellite refuelling within a satellite constellation part-I: Zero-cost rendezvous case. In *42nd IEEE Conference on Decision and Control*, pages 4345–4350, Maui, HW, dec 2003.

[17] H. Shen and P. Tsiotras. Peer-to-peer refuelling within a satellite constellation part-II: Nonzero-cost rendezvous case. In *42nd IEEE Conference on Decision and Control*, pages 4363–4368, Maui, HW, dec 2003.

[18] P. Tsiotras and A. de Neilly. Comparison between peer-to-peer and single-spacecraft refueling strategies for spacecraft in circular orbits. In *Infotech at Aerospace*, Sept. 26–29 2005. Crystal City, DC.

[19] H. Shen and P. Tsiotras. Peer-to-peer refueling for circular satellite constellations. *Journal of Guidance, Control, and Dynamics*, 2005. to appear.

[20] H. Shen. *Optimal Scheduling for Satellite Refuelling in Circular Orbits*. PhD thesis, School of Aerospace Engineering, Georgia Institute of Technology, 2003.

[21] B. Bollobás. *Modern Graph Theory*. Springer-Verlag, New York, 1998.

[22] H. Shen and P. Tsiotras. Optimal two-impulse rendezvous between two circular orbits using multiple-revolution lambert's solution. *AIAA Journal of Guidance, Control, and Dynamics*, 26:50–61, 2003.

[23] J. E. Prussing. A class of optimal two-impulse rendezvous using multiple-revolution Lambert solutions. *Advances in Astronautical Sciences*, 106:17–39, 2000.

[24] F. J. Hale. *Introduction to Space Flight*. Prentice Hall, Upper Saddle River, NJ, 1994.