Aggressive Maneuvers on Loose Surfaces: Data Analysis and Input Parametrization

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***Abstract*— In this work we initiate a mathematical analysis of rally racing techniques. An empirical description of Trail-Braking (TB) and Pendulum-Turn (PT) cornering, two of the most common rally racing maneuvers is provided via analysis of data collected during execution of these maneuvers by an expert rally driver. Trail-Braking and Pendulum-Turn maneuvering techniques are reproduced using numerical optimization as special cases of the minimum time cornering problem with specific boundary conditions. We show that a simple parametrization of the control inputs can be used to reproduce these maneuvers using a high fidelity full-car model.**

I. Introduction

The state of the art in autonomous ground vehicles was demonstrated in the 2005 DARPA Grand Challenge, where several teams raced their vehicles autonomously to complete a 131.2 miles unpaved course in the Mojave desert under 10 hours. Technical reports of the teams participating in the final event can be found in [1]. The winning team was from Stanford University, which completed the course at a mean speed of approximately 19 mph. It is envisioned that the next generation of autonomous ground vehicles will be able to travel autonomously such long distances faster than these moderate speeds, and even perhaps as fast as human (expert) car drivers.

The problem of trajectory planning for high-speed ground vehicles is typically dealt with in the literature by means of numerical optimization [2], [3], [4], [5]. These results demonstrate that numerical techniques allow one to incorporate accurate, high order dynamical models, thus producing realistic results. On the other hand, these numerical optimization approaches are computationally intensive, and they cannot be readily applied in cases where the environment changes unpredictably. Analytical approaches have also been introduced in the literature [6], [7], [8], [9], [10], [11]. These analytical methodologies are computationally less intensive than numerical approaches. However, the assumptions used in the formulation of trajectory optimization problems that aim at an analytic solution tend to oversimplify the problem.

A new approach to real-time path planning of autonomous vehicles, which overcomes the limitations of both numerical and analytical optimization techniques has been developed in [12], [13], [14] for aggressive autonomous operation of robotic helicopters. In these references the path optimization is solved by first generating (off-line) a library of maneuvers. By scheduling these maneuvers on the fly using a maneuver automaton, one is able to perform real-time path optimization by numerically pasting together the prerecorded maneuvers from the maneuver library.

The above maneuver automaton scheme is promising for path planning of ground vehicles. The maneuver library can be constructed off-line (perhaps via numerical optimization) thus bypassing the computational bottleneck of on-line computations. For off-road aggressive driving scenarios the maneuver repertoire must be enriched with expert rally racing techniques. Unlike paved road racing or closed-circuit racing of high-performance vehicles (e.g., F1), to date there has been no concrete amount of work correlating driving techniques used by expert rally drivers with mathematical models.

In this work, we use empirical information collected from our interaction with expert rally race drivers in order to develop a mathematical and computationally tractable framework that succinctly formalizes this empirical information. We first present an empirical description of Trail-Braking (TB) and Pendulum-Turn (PT), two of the most common rally racing cornering maneuvers, based on analysis of data collected during execution of these maneuvers by an expert rally driver. Next, we reproduce these techniques using numerical optimization as special cases of the minimum time cornering problem. We introduce a simple input parametrization to reproduce these maneuvers reducing considerably the search space of the optimization problem. We solve the optimization problem numerically using a high fidelity full-car model and derive maneuvers that match the empirical descriptions.

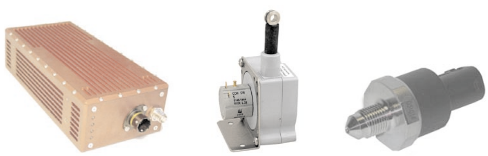
II. Rally Maneuvers Data Collection

Rally racing is a form of motor competition that takes place on public or private roads, typically on loose surfaces, with modified production cars. In fact, often the modifications on the cars may be limited (Group N cars). Hence, rally racing provides an excellent platform for research and development of automotive systems for improving the safety and performance of passenger vehicles. As an example, we mention the development of All-Wheel-Drive (AWD) road vehicles after the successful introduction of the Audi Quattro in rally racing in the 1980's. Despite these technological successes, the techniques and driving style of expert rally race drivers have not yet been fully analyzed using a rigorous mathematical framework, at least in a way such that it can help researchers develop control systems which can operate a vehicle during extreme or abnormal driving conditions. This knowledge remains empirical and exclusive to few expert rally race drivers.

In this section we present a description of Trail-Braking and Pendulum-Turn, two of the most commonly used rally racing maneuvers [15] based on data collected during the execution of these maneuvers by an expert rally driver. Our test driver was Mr. Tim O'neil, five times US and North American Rally Champion and rally driving instructor. The test took place at the facilities of Team O'neil Rally School and Car Control Center in Dalton, New Hampshire. The vehicle used was a Group A 2004 Subaru Impreza WRX STI (Fig. 1) owned by CPD Racing and prepared by ProDrive. The maneuvers were executed on track gravel (typically $\mu = 0.5 - 0.6$).

Fig. 1. The 340 BHP, 510 Nm, All-Wheel-Drive CPD Racing Subaru Impreza.

The vehicle was instrumented with the following sensors (Fig. 2): An Oxford Technical Solutions RT3000 inertial measurement and GPS unit [16] to measure 3 axes accelerations, 3 axes rotational rates, absolute position and heading. The RT3000 was provided by the Research and Advanced Engineering department of Ford Motor Company. Two string extension potentiometers were fitted on the steering column and the throttle cable to measure the steering wheel angle and throttle position respectively. Also, a pressure transducer was placed in the brake line using a T-fitting to measure the brake pressure. More information on these sensors can be found at [17]. A DL2 data logger and GPS system [17] was used to collect the data from the potentiometers and pressure transducer. The data from the RT3000 was directly logged to a portable PC.



(a) (b) (c)

Fig. 2. (a) The RT3000 inertial measurement and GPS unit; (b) String extension potentiometer for steering wheel angle measurement; (c) Pressure transducer for brake pressure measurement.

The goal of this test was to capture the actions of the driver (steering, braking and throttle commands) while performing the Trail-Braking and Pendulum-Turn maneuvers on a loose surface, and the resulting vehicle response as a rigid body, rather than internal dynamics concerning the engine/transmission, brake/suspension/steering systems.

A. Trail-Braking

Trail-Braking is one of the techniques used by rally drivers to negotiate corners at high speeds. Typically, the average driver negotiates a corner by first braking to regulate the speed, then releasing the brakes and steering the vehicle through the corner, and finally accelerating after the exit of the corner. Trail-Braking is used when the approach speed to the corner is high and the braking must continue even after the steering of the vehicle has started. The test driver first executed a Trail-Braking maneuver around a tight, approximately $80 - 90$ deg corner on track gravel. The driver's normalized steering command with time is shown in Fig. 3(a) and the normalized throttle and braking commands in Fig. 3(b). The vehicle's speed with time is shown in Fig. 3(c) and the vehicle slip angle calculated at the rear axle is shown in Fig. 3(d). The vehicle's pitch angle is shown in Fig. 7(a). Figure 4 finally shows the trajectory of the vehicle using the absolute position and heading measurements. Cones where used to define the limits of the road and the apex of the corner and they are denoted by crosses in Fig. 4. The origin has been moved to the apex of the corner.

During $0 \leq t \leq 2$ sec the driver regulated the speed at $75 - 80$ km/h (Fig. 3(c)) travelling straight.At $t = 2$ sec the car reached the 25 m mark (distance from the apex - Fig. 4) and the driver braked hard (Fig. 3(b)). At $2 \leq t \leq 3.5$ sec the driver progressively released the brakes and increased the steering angle towards the direction of the corner (Fig. 3(a)). Deceleration of the vehicle resulted in normal load transferring from the rear to the front wheels, which can be seen from the decrease of the pitch angle in Fig. 7(a). As the rear axle load was reduced, the friction of the rear tires was also reduced and the vehicle started rotating counterclockwise, oversteering with an increasing slip angle (Fig. 3(d)). At $t = 3.5$ sec as the vehicle was reaching the apex of the corner at a high slip angle, the driver took action to stabilize the vehicle and exit the corner. The driver reduced the steering angle, progressively released the brakes and applied throttle. At $4 \leq t \leq 5$ sec the driver was counter-steering and progressively increasing the throttle command. As the vehicle was accelerating the rear axle normal load increased and hence, so did the friction of the rear tires. The counterclockwise rotation was damped, the slip angle was reduced to zero and the vehicle accelerated straight exiting the corner. For safety, and due to limited space, the driver chose to accelerate only until the vehicle exited the corner, cutting-off the throttle just before $t = 5$ sec.

B. Pendulum-Turn

The Pendulum-Turn is another high speed cornering maneuver. It is used when the vehicle approaches the corner at high speed coming from the inner edge of the road,

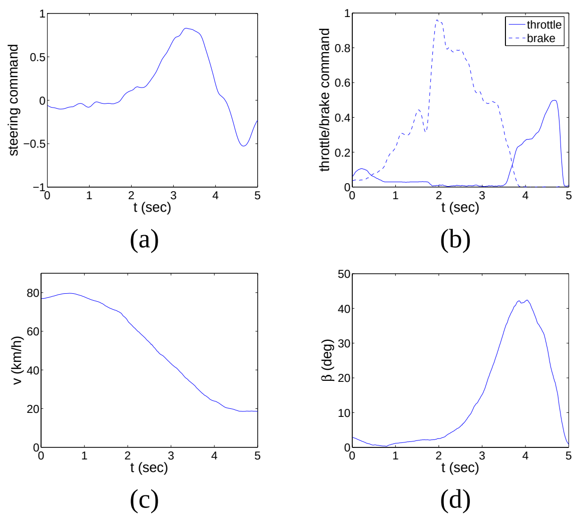


Fig. 3. Trail-Braking maneuver experimental data: (a) Normalized steering command; (b) Normalized throttle and braking commands; (c) Vehicle speed; (d) Vehicle slip angle.

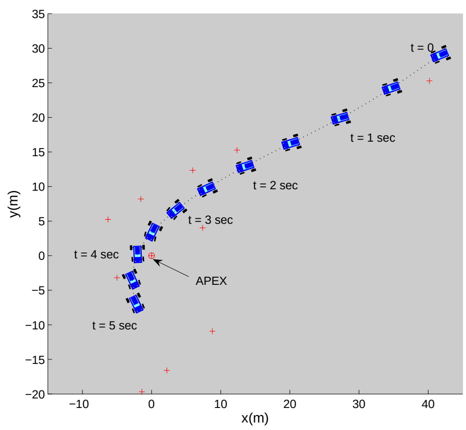


Fig. 4. A Trail-Braking maneuver trajectory reproduced from experimental data.

for S-turns, or for connecting successive sharp corners. If there is not enough time for the driver to place the vehicle at the outer edge of the road and use Trail-Braking, the Pendulum-Turn is then appropriate. The test driver executed a Pendulum-Turn maneuver around the same $80-90$ deg corner on track gravel. The driver's normalized steering command with time is shown in Fig. 5(a) and the normalized throttle and braking commands in Fig. 5(b). The vehicle's speed with time is shown in Fig. 5(c) and the vehicle slip angle calculated at the rear axle is shown in Fig. 5(d). The pitch angle is shown in Fig. 7(b) and the vehicle's trajectory in Fig. 6.

The vehicle approached the corner at high speed, close to the inner (with respect to the corner) limit of the road (left side of the road for an upcoming left turn). As the vehicle was travelling straight at 70 km/h (Fig. 5(c)) the driver increased the steering wheel angle towards the opposite direction of the corner (Fig. 5(a)) while still applying some throttle (Fig. 5(b)). A $t = 0.5$ sec the driver applied a progressively increasing braking command and released the throttle. This resulted in a decrease of the normal load at the rear axle and the friction generated at the rear wheels. This can also be seen by the decrease of the pitch angle (Fig. 7(b)). The decrease of the rear tires load and generated friction resulted in a clockwise oversteering rotation with an increasing magnitude of slip angle (Fig. 5(d)). At approximately $t = 1.5$ sec the driver turned the steering wheel towards the direction of the corner, released the brakes and applied a short throttle command ("throttle blip"). This short acceleration resulted in normal load being transferred to the rear axle and an increase of the rear tires friction to damp the clockwise rotation and initiate a counterclockwise rotation towards the direction of the corner. In order to make the counterclockwise rotation of the vehicle more aggressive and achieve the desired orientation the driver applied the brakes once more at $t = 2$ sec. The vehicle slip angle was increasing in magnitude once again. A few instants before the vehicle reached its desired heading for the exit of the corner, at around $t = 3$ sec the driver counter-steered, released the brakes and applied the throttle. Similarly to the exit part of the Trail-Braking maneuver the normal load transfer to the rear axle and increase of the friction of the rear tires was used to damp the aggressive counterclockwise rotation of the vehicle. Once again the driver cut-off the throttle when the vehicle exited the corner driving straight ($t = 5$ sec).

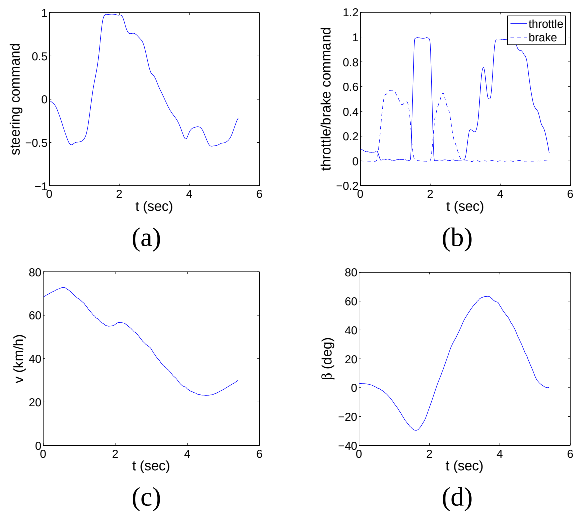


Fig. 5. Pendulum-Turn maneuver experimental data: (a) Normalized steering command; (b) Normalized throttle and braking commands; (c) Vehicle speed; (d) Vehicle slip angle.

III. Numerical Optimization

In the following we propose a simple input parametrizations for each of the TB and PT maneuvers. We reproduce these maneuvers using numerical optimization with a high order vehicle model. We consider the minimum time problem through a 90 deg corner and use CarSim [18] to integrate the full-car vehicle dynamics of an All-Wheel-Drive sedan. The control inputs to be optimized are the steering, braking and throttle commands. We present a parametrization of the control signals in accordance to

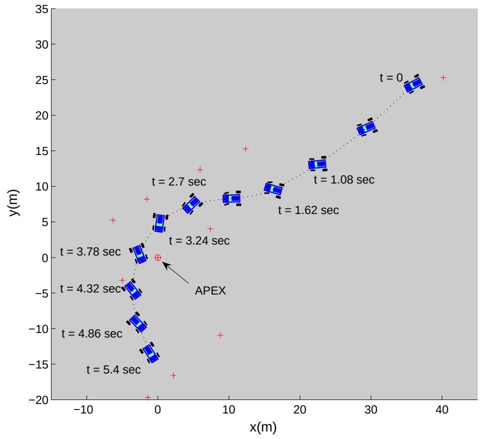


Fig. 6. A Pendulum-Turn maneuver trajectory reproduced from experimental data.

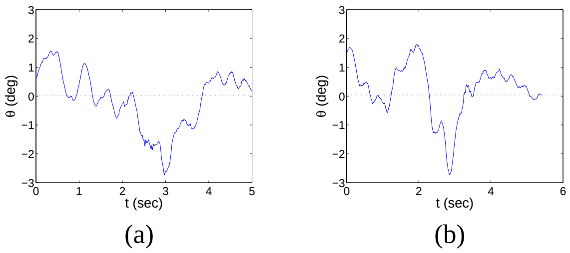


Fig. 7. Vehicle pitch angle: (a) Trail-Braking, (b) Pendulum-Turn.

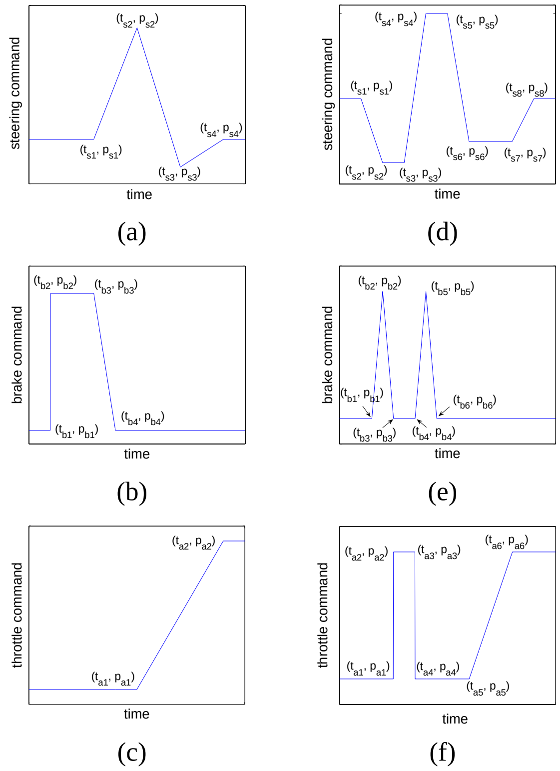


Fig. 8. Input parametrization for the Trail-Braking (a-c) and Pendulum Turn (d-f) maneuvers.

the empirical descriptions and collected data of Section II.

A. *Trail-Braking*

We propose the steering, braking and throttle commands parametrization of Figs 8(a)-(c) for the Trail-Braking maneuver:

According to the empirical description of Section II the TB maneuver starts with the vehicle braking hard while driving straight - see braking command of Fig. 8(b) for $t_{b2} \le t \le t_{b3}$. The driver then applies increasing steering command towards the direction of the corner ($t_{s1} \le t \le t_{s2}$ in Fig. 8(a)) and progressively decreases braking ($t_{b3} \le t \le t_{b4}$ in Fig. 8(b)). Next, the driver decreases the steering command and counter-steers ($t_{s2} \le t \le t_{s3}$ in Fig. 8(a)) and progressively applies the throttle ($t_{a1} \le t \le t_{a2}$ in Fig. 8(c)) to stabilize the vehicle for the exit of the corner. The steering angle is reduced to zero ($t_{s3} \le t \le t_{s4}$ in Fig. 8(a)) and the vehicle exits the corner while accelerating hard ($t \ge t_{a2}$ in Fig. 8(c)).

Next, we find the optimal set of parameters (t_{si}, p_{si}), $i = 1...4$, (t_{bi}, p_{bi}), $i = 1...4$ and (t_{ai}, p_{ai}), $i = 1, 2$, to solve the minimum time cornering problem through the $90\,$deg corner of Fig. 10 subject to the following boundary conditions:

$$\begin{aligned} &x_0 = 18 \text{ m},\ y_0 = -45 \text{ m},\ \dot{x}_0 = 70 \text{ km/h},\ \dot{y}_0 = 0,\\ &\psi_0 = \pi/2,\ \dot{\psi}_0 = 0,\\ &x_f = -10 \text{ m},\ y_f = 12 \text{ m},\ \dot{y}_f = 0,\\ &\psi_f = \pi,\ \dot{\psi}_f = 0. \end{aligned} \tag{1}$$

In accordance to the optimization using the low order vehicle model we enforce that the vehicle is stabilized to straight driving right after it exits the corner ($x_f = -10$ m). In addition, we fix the final position of the optimization at $y_f = 12$ m, which is near the inner limit of the road with respect to the corner, in order to enforce a *late apex* trajectory. Rally drivers typically follow *late apex* trajectories in order to stabilize their vehicle as soon as possible after each corner and react to unexpected road condition changes ahead that are typical in rally racing [15]. The simulation continues up to $x = -45$ m with the vehicle accelerating on a straight line. The optimization was performed in Matlab using a nonlinear minimization algorithm (Nelder-Mead).

The calculated optimal control inputs are shown in Figs. 9(a) and (b). The vehicle speed and vehicle slip angle are shown in Fig. 9(c) and (d) respectively. Figure 15(a) shows the normal load at the front and rear axles demonstrating the load transfer effect during the maneuver: The driver brakes while steering to decrease the rear axle load and generated friction to induce oversteer and aggressively change the vehicle's orientation, or accelerates while steering to increase the rear axle friction, reduce oversteer and stabilize the vehicle. The vehicle's trajectory is shown in Figs 10 and 11. The resulting trajectory is in agreement with the empirical description of Section II regarding the Trail-Braking maneuver.

B. *Pendulum-Turn*

We propose the steering, braking and throttle commands parametrization of Fig. 8(d)-(f) for the Pendulum-

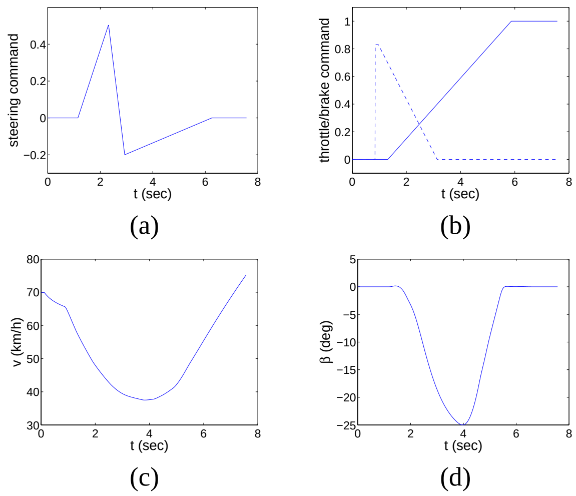


Fig. 9. A Trail-Braking maneuver using a high fidelity full-car model: (a) Steering command; (b) Throttle/Brake command; (c) Vehicle speed; (d) Vehicle slip angle; (e) Normal load on front and rear axles.

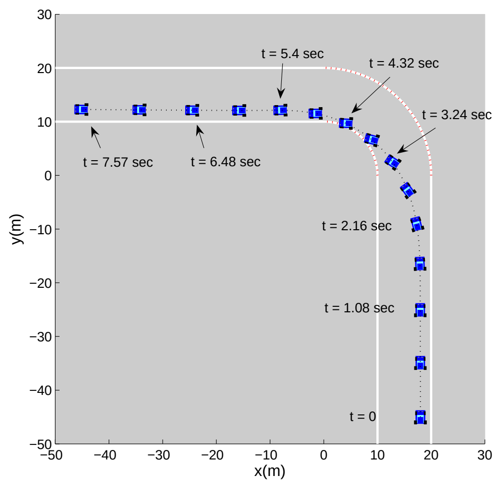


Fig. 10. A Trail-Braking maneuver using a high fidelity full-car model: trajectory

Turn maneuver, according to the empirical description of Section II:

The PT maneuver starts with the driver turning towards the opposite direction of the corner ($t_{s1} \leq t \leq t_{s3}$ in Fig. 8(d)). While the steering command increases the driver applies the brakes ($t_{b1} \leq t \leq t_{b3}$ in Fig. 8(e)) in order to transfer load from the rear to the front axle and induce an oversteering rotation of the vehicle away from the corner. As the vehicle moves away from the inner limit of the road the driver applies a "throttle blip" ($t_{a1} \leq t \leq t_{a3}$ in Fig. 8(f)) to damp the rotation of the vehicle away from the corner. The driver then steers towards the direction of the corner ($t_{s3} \leq t \leq t_{s5}$ in Fig. 8(d)) and applies the brakes once more to make the vehicle rotate fast towards the desired orientation ($t_{b4} \leq t \leq t_{b6}$ in Fig. 8(e)). Finally, the driver decreases the steering command and counter-steers ($t_{s5} \leq t \leq t_{s8}$ in Fig. 8(d)) and progressively applies the throttle ($t_{a5} \leq t \leq t_{a6}$ in Fig. 8(f)) to stabilize the vehicle for the exit of the corner.

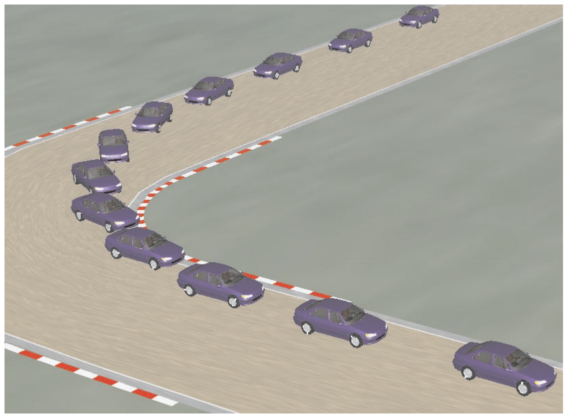


Fig. 11. A Trail-Braking maneuver using a high fidelity full-car model: 3D visualization.

The steering angle is reduced to zero ($t \geq t_{s8}$ in Fig. 8(d)) and the vehicle exits the corner accelerating hard ($t \geq t_{a6}$ in Fig. 8(f)).

Next, we find the optimal set of parameters (t_{si}, p_{si}), $i = 1...8$, (t_{bi}, p_{bi}), $i = 1...6$ and (t_{ai}, p_{ai}), $i = 1...6$, to solve the minimum time cornering problem across the 90 deg corner of Fig. 10 subject to the boundary conditions (1), except for $x_0 = 12$ m and $\dot{y}_0 = 60$ km/h. Once again, we enforce that the vehicle is stabilized to straight driving right after it exits the corner ($x_f = -10$ m) and fix the final position of the optimization at $y_f = 12$ m in order to enforce a late apex trajectory. The simulation continues up to $x = -45$ m with the vehicle accelerating on a straight line.

The calculated optimal control inputs are shown in Figs. 12(a) and (b). The vehicle speed and vehicle slip angle are shown in Fig. 12(c) and (d) respectively. Figure 15(b) shows the normal load at the front and rear axles demonstrating the load transfer effect during the maneuver. The vehicle's trajectory is shown in Figs 13 and 14. The resulting trajectory is in agreement with the empirical description Section II regarding the Pendulum-Turn maneuver.

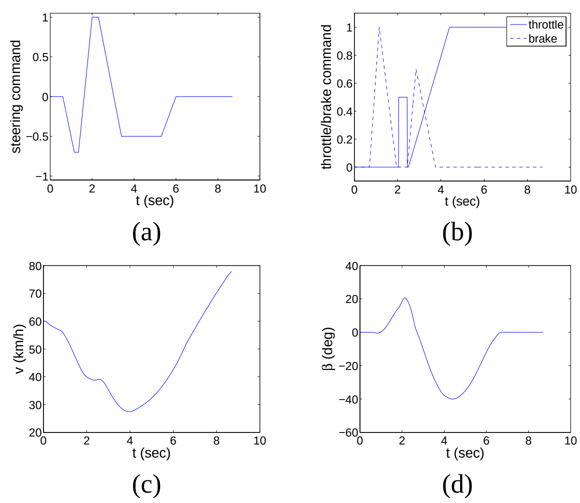


Fig. 12. A Pendulum-Turn maneuver using a high fidelity full-car model: (a) Steering command; (b) Throttle/Brake command; (c) Vehicle speed; (d) Vehicle slip angle.

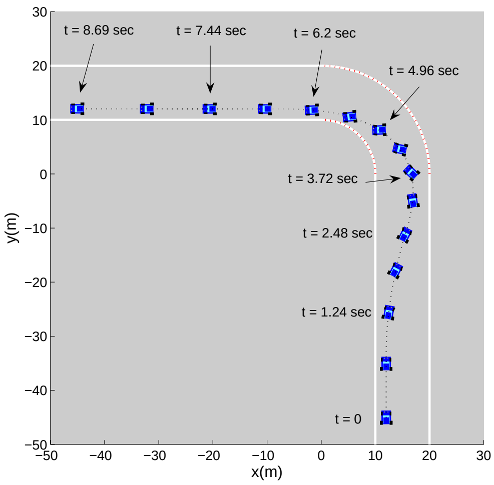


Fig. 13. A Pendulum-Turn maneuver using a high fidelity full-car model: trajectory.

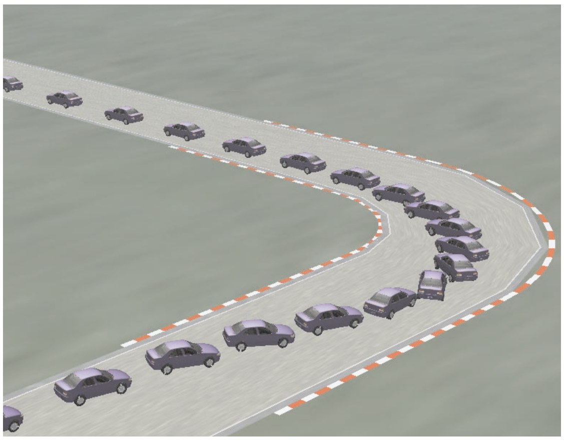


Fig. 14. A Pendulum-Turn maneuver using a high fidelity full-car model: 3D visualization.

IV. Conclusions

In this work we have initiated a mathematical analysis of rally racing techniques. We have concentrated our efforts on two specific techniques for high speed cornering used extensively by rally drivers, namely the Trail-Braking and the Pendulum-Turn. First, we collected experimental data during execution of these maneuvers by an expert driver. Trail-Braking and Pendulum-Turns were then reproduced as special cases of the minimum time cornering problem. We derived a simple input parametrization and incorporated of high fidelity vehicle model to generate TB and PT maneuvers via numerical optimization.

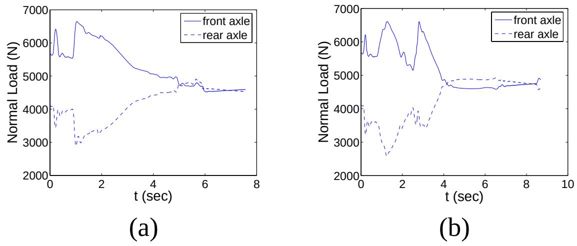


Fig. 15. Front and rear axle normal load: (a) Trail-Braking, (b) Pendulum-Turn.

Acknowledgment

The authors would like to thank Mr. Tim O'Neil and his colleagues at the Team O'Neil Rally School and Car Control Center for the instructional courses and informative discussions on rally driving, as well as for generously offering use of their facilities and vehicles and finally participating in the data collection process. The authors would also like to thank Mr. Christopher Nave and Mr. Steve Hermann from the Research and Advanced Engineering Department of Ford Motor Company for their valuable assistance during the experiments. This work has been supported by Ford Motor Company through the URP program.

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