Reducing Conservatism for Gain-Scheduled $\mathcal{H}_\infty$ Controllers for AMB's

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ABSTRACT

We address the problem of $\mathcal{H}_\infty$ gain-scheduling of a magnetic bearing/rotor system with the speed of the rotor being the scheduling parameter. The rotor exhibits significant gyroscopics. As a result, the plant dynamics change with the rotor speed. Recent results in the theory of LPV systems can be used to design robust, gain-scheduled controllers for such a system. As opposed to traditional gain-scheduling, LPV controllers offer guarantees of stability and performance over the whole range of the rotor speed. However, these control laws can be overly conservative since they ensure stability for arbitrarily fast variations in the scheduling parameter. In this paper we refine these LPV controllers in order to obtain better bounds on the achievable performance in the presence of a priori known bounds of the variation of the rotor speed.

INTRODUCTION

Magnetic bearings are becoming increasingly popular for use in high-speed, high-temperature, low-friction applications. Typical cases where the use of magnetic bearings offer significant advantages over traditional bearings are turbomachinery, machining spindles, fuel pumps, etc. More recently, magnetic bearings have been proposed for use in flywheel systems, either for energy or momentum storage devices (or both). Flywheels typically involve a (heavy) rotor operating at very high speeds. These applications differ from more traditional ones mentioned earlier, because significant gyroscopics change the system dynamics. In addition, the speed of the rotor may vary as demand for energy (or angular momentum) is increased or decreased. This fact makes the design of magnetic bearing controllers for these systems a demanding task. One way to address this problem is to design a series of controllers for each operating speed and then interpolate between these controllers (Matsumura *et al.* 1996). It is well known, however, that this approach does not provide any guarantees for stability and performance (Shamma and Athans 1991).

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Recently, the methodology of gain-scheduled (or self-scheduled) $\mathcal{H}_\infty$ control theory for Linear, Parameter Varying (LPV) systems has been proposed to address this problem (Becker *et al.* 1993, Apkarian *et al.* 1995, Apkarian and Gahinet 1995). This framework is natural for rotor systems, where the plant dynamics is a linear function of the rotor speed. One can therefore use standard results from LPV theory in order to design a controller scheduled on the rotor speed. Reference (Tsiotras and Mason 1996) has used the LPV approach to obtain controllers that achieve disturbance rejection and automatic balancing over the whole operating interval of the rotor speeds. Similar controllers were also proposed in (Sivrioglu and Nonami 1996) and were compared to classical PID controllers. The gain-scheduled control design in these references can be, however, potentially very conservative since it guards against arbitrarily fast variations of the rotor speed.

In order to reduce conservatism, we propose a modification of the previous control design, which takes explicitly into consideration the (a priori known) bounds on the rate of variation of the rotor speed for a magnetic bearing/rotor system. The theory used in this paper has been previously developed in (Wu *et al.* 1995) and (Apkarian and Adams 1997). One of the appealing characteristics of the proposed methodology is that the control design can be reduced to a finite number of Linear Matrix Inequalities (LMIs). Numerical methods, based on interior point algorithms, can then be used to solve these inequalities very efficiently.

MAGNETIC BEARING/ROTOR SYSTEM

We consider the rotor/magnetic bearing system shown in Fig. 1. Two pairs of active

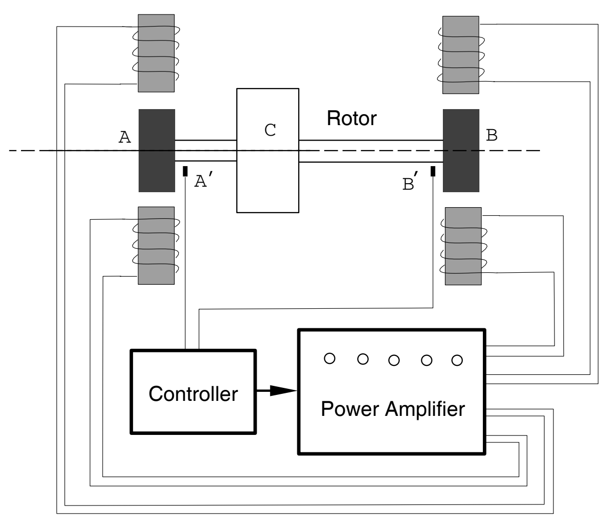

Figure 1: Magnetic bearing/rotor system.

magnetic bearings support the rotor at points A and B. A power amplifier provides the currents at the coils to control the bearings in the x and y directions independently. Eddy current sensors located at points A' and B' measure the gaps at these locations. The signals from these sensors are fed to the controller. (Figure 1 shows only the bearings and the sensors in the y-direction.) The controller generates the necessary voltage signals for driving the power amplifier. The performance objective of the controller is to asymptotically stabilize the system over the whole range of speeds and to minimize an error signal representing a (weighted) sum of the forces at the bearings, the gap displacement at the bearing locations and the control input used. The rotor, which is assumed to operate between 0 and 48,000 rpm, exhibits significant gyroscopics. The first three critical speeds occur at 8,000, 14,000 and 26,000 rpm. In this work, only the first bending mode will be retained in the model. More accurate models should include additional modes. For illustrating the proposed approach, however, keeping only one mode is sufficient.

Under the previous assumptions, the rotor/magnetic bearing system in Fig. 1 can be appropriately described by a linear, parameter-varying system in the form

$$\dot{x} = A(p)x + B_1 w + B_2 u \tag{1a}$$
$$z = C_1 x + D_{11} w + D_{12} u \tag{1b}$$
$$y = C_2 x + D_{21} w \tag{1c}$$

where $x \in \mathbb{R}^n$ is the state of the system, $u \in \mathbb{R}^{m_2}$ is the input (voltage to the power amplifiers) and $y \in \mathbb{R}^{p_2}$ is the measured output (voltage from the sensors). The input $w \in \mathbb{R}^{m_1}$ is an exogenous disturbance. In this work it is assumed that w represents finite energy signals acting at the rotor location (point C in Fig. 1) plus sensor noise. The performance output $z \in \mathbb{R}^{p_1}$ includes all the signals we want to retain small (here forces at the bearings, gap displacements at the bearings and control effort). Note that the state matrix $A(p)$ depends linearly on the rotor speed p by $A = A_0 + pA_1$. All other system matrices are assumed to be constant. Also, without loss of generality it is assumed that the matrices D_{12} and D_{21} have full column and full row rank, respectively.

SELF-SCHEDULED $\mathcal{H}_\infty$ CONTROL OF LPV SYSTEMS

Consider a general LPV system of the form

$$\dot{x} = A(p(t))x + B(p(t))u \tag{2a}$$
$$y = C(p(t))x + D(p(t))u \tag{2b}$$

whose system matrices are fixed *affine* functions of some time-varying parameter vector $p(t) \in \mathcal{P}$, a given polytope. A useful indication of performance for LPV systems is the notion of quadratic $\mathcal{H}_\infty$ performance (Apkarian *et al.* 1995).

Definition 1 (Quadratic $\mathcal{H}_\infty$ performance.) The LPV system in Eqs. (2) is said to have *quadratic $\mathcal{H}_\infty$ performance* γ if there exists a positive definite matrix $X > 0$ which satisfies the following linear matrix inequality (LMI)

$$\begin{pmatrix} A^T(p)X + XA(p) & XB(p) & C^T(p) \\ B^T(p)X & -\gamma I & D^T(p) \\ C(p) & D(p) & -\gamma I \end{pmatrix} < 0 \tag{3}$$

for all values of the parameter vector $p \in \mathcal{P}$.

Quadratic $\mathcal{H}_\infty$ performance guarantees global asymptotic stability and $\mathcal{L}_2$-gain of the map from u to y less than γ

$$\|y\|_2 < \gamma \|u\|_2 \tag{4}$$

for all possible parameter trajectories $p(t) \in \mathcal{P}$. Therefore, quadratic $\mathcal{H}_\infty$ performance establishes internal stability and robust performance in the sense of inequality (4).

It is important to note at this point that the condition in Eq. (3) is independent of the rate of change of the parameter p. In fact, condition (3) guarantees stability and performance in the presence of arbitrarily fast changes of the parameter p. That is, although quadratic performance is a very strong property, it can be at the same time very conservative since, due to physical considerations (e.g., maximum torque output of the motor), it is known that the rotor speed cannot vary arbitrarily fast. One is therefore naturally led to the investigation of controllers which will take into account any *a priori* bounds on the rate of variation of the rotor speed in order to reduce conservatism.

To this end, consider the output feedback *synthesis* problem for the system (1). That is, we seek an output feedback controller with state-space realization

$$\dot{x}_k = A_k(p,\dot{p})x_k + B_k(p,\dot{p})y \tag{5a}$$
$$u = C_k(p,\dot{p})x_k + D_k(p,\dot{p})y \tag{5b}$$

which ensures internal stability and minimal $\mathcal{L}_2$-gain bound γ for the closed-loop system (1)-(5) from the disturbance input w to the performance output z. Note that we allow for the controller to depend not only on the rotor speed but also on its derivative. Both of them are assumed to be measurable on-line (using, say, a tachometer). Moreover, it is assumed that both the bounds of p, as well as of $\dot{p}$ are known

$$p_{min} \le p \le p_{max}, \qquad r_{min} \le \dot{p} \le r_{max} \tag{6}$$

Pairs $(p,\dot{p})$ satisfying these inequalities will be henceforth called *admissible pairs*.

The following result, due to (Apkarian and Adams 1997), gives computable conditions for a solution to the previous problem. Suppose there exist parameter-dependent matrices $X(p)$ and $Y(p)$ such that for all admissible pairs $(p,\dot{p})$ the following Linear Matrix Inequalities (LMI's) are satisfied

$$\left(\begin{array}{c|c} \mathcal{N}_X & 0 \\ \hline 0 & I \end{array}\right)^T \left(\begin{array}{cc|c} \dot{X} + XA + A^TX & XB_1 & C_1^T \\ B_1^TX & -\gamma I & D_{11}^T \\ \hline C_1 & D_{11} & -\gamma I \end{array}\right) \left(\begin{array}{c|c} \mathcal{N}_X & 0 \\ \hline 0 & I \end{array}\right) < 0 \tag{7}$$

$$\left(\begin{array}{c|c} \mathcal{N}_Y & 0 \\ \hline 0 & I \end{array}\right)^T \left(\begin{array}{cc|c} -\dot{Y} + A^TY + AY & YC_1^T & B_1 \\ C_1Y & -\gamma I & D_{11} \\ \hline B_1^T & D_{11}^T & -\gamma I \end{array}\right) \left(\begin{array}{c|c} \mathcal{N}_Y & 0 \\ \hline 0 & I \end{array}\right) < 0 \tag{8}$$

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \ge 0 \tag{9}$$

where $\mathcal{N}_X$ and $\mathcal{N}_Y$ are any bases of the null spaces of $[C_2 \;\; D_{21}]$ and $[B_2^T \;\; D_{12}^T]$, respectively. Then, there exists a gain-scheduled output feedback controller (5) such that the corresponding closed-loop system is internally stable and the map from w to z is bounded above by γ for all admissible pairs $(p, \dot{p})$.

Two observations can be made at this point concerning the system of inequalities (7)-(9). First, for fixed $(p, \dot{p})$ this is a convex optimization problem in terms of $X(p)$ and $Y(p)$, and γ. This is a highly desirable property because it ensures a unique solution, if one exists. On the other hand (7)-(9) constitute an *infinite dimensional* system of inequalities in the $(p, \dot{p})$ space. Hence, it is not at all obvious how one would go about solving this problem in practice. We will return to this issue later on. The second observation has to do with the fact that the previous inequalities merely guarantee the existence of a controller, but they do not actually give any information how to compute one. Moreover, if one such controller exists, it will necessarily be of the same order as the order of the plant.

CONTROLLER CONSTRUCTION

Assume that the inequalities (7)-(9) are satisfied for two parameter-dependent matrices $X(p)$ and $Y(p)$ and some performance level γ. Then a gain-scheduled controller can be constructed for any admissible pair $(p, \dot{p})$ by the following scheme:

- compute a matrix D_k which solves

$$\sigma_{max}(D_{11} + D_{12} D_k D_{21}) < \gamma \tag{10}$$

  and set $D_{cl} = D_{11} + D_{12} D_k D_{21}$. (For instance, in case $\sigma_{max}(D_{11}) < \gamma$, simply let $D_k = 0$ and $D_{cl} = D_{11}$.)

- compute matrices $\hat{B}_k$ and $\hat{C}_k$ by solving the following linear matrix equations

$$\begin{bmatrix} 0 & D_{21} & 0 \\ D_{21}^T & -\gamma I & D_{cl}^T \\ 0 & D_{cl} & -\gamma I \end{bmatrix} \begin{bmatrix} \hat{B}_k^T \\ \star \end{bmatrix} = - \begin{bmatrix} C_2 \\ B_1^T X \\ C_1 + D_{12} D_k C_2 \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} 0 & D_{12}^T & 0 \\ D_{12} & -\gamma I & D_{cl} \\ 0 & D_{cl}^T & -\gamma I \end{bmatrix} \begin{bmatrix} \hat{C}_k \\ \star \end{bmatrix} = - \begin{bmatrix} B_2^T \\ C_1 Y \\ B_1^T + D_{21}^T D_k^T B_2^T \end{bmatrix} \tag{12}$$

- compute the matrix

$$\hat{A}_k = \begin{bmatrix} X B_1 + \hat{B}_k D_{21} & C_1^T + C_2^T D_k^T D_{12}^T \end{bmatrix} \begin{bmatrix} -\gamma I & D_{cl}^T \\ D_{cl} & -\gamma I \end{bmatrix}^{-1} \begin{bmatrix} B_1^T + D_{21}^T D_k^T B_2^T \\ C_1 Y + D_{12} \hat{C}_k \end{bmatrix} \tag{13}$$

- solve for full rank matrices N and M from

$$I - XY = N M^T \tag{14}$$

- compute the controller matrices A_k, B_k, C_k from the equations

$$\begin{aligned} A_k &= N^{-1}(X\dot{Y} + N\dot{M}^T + \hat{A}_k - (A + B_2 D_k C_2)^T - X(A - B_2 D_k C_2)Y \\ &\quad -\hat{B}_k C_2 Y - X B_2 \hat{C}_k)M^{-T} \quad (15) \\ B_k &= N^{-1}(\hat{B}_k - X B_2 D_k) \quad (16) \\ C_k &= (\hat{C}_k - D_k C_2 Y)M^{-T} \quad (17) \end{aligned}$$

Equation (13) shows clearly that, in general, the controller depends on the rate of change $\dot{p}$ through the term $X\dot{Y} + N\dot{M}^T$. Thus, its implementation requires on-line measurements not only of p but also of $\dot{p}$. It has to be pointed out here that the previous algorithm is not the only possible one. An alternative controller construction (based on a fixed controller structure assumption, thus more conservative) is reported in (Apkarian and Adams 1997).

CONTROLLER COMPUTATION

As mentioned previously, inequalities (7)-(9) consist of a convex, but infinite-dimensional optimization problem in terms of the unknowns $X(p)$, $Y(p)$ and γ and the parameters p and $\dot{p}$. One obvious (but computationally expensive) way to solve this problem is to simply grid the parameter space in terms of p and solve the simultaneously system of inequalities. The inequalities (7)-(9) then reduce to the solution of a finite-dimensional optimization problem, parameterized by p and $\dot{p}$. For dense enough grid, this procedure will provide a close to optimal, non-conservative solution to these inequalities. This approach has the drawback, however, that it does not provide a natural way of scheduling for X and Y. The second approach, used here, is to *postulate* a fixed parameter structure for the unknown matrices X and Y. Letting, for example,

$$X(p) = X_0 + pX_1, \qquad Y(p) = Y_0 + pY_1 \quad (18)$$

equations (7)-(9) become a series of LMIs with linear dependence on $\dot{p}$ and linear and quadratic dependence on p.

If p and $\dot{p}$ appeared in these inequalities in an affine way (actually $\dot{p}$ does) then one only need to check these matrix inequalities at the vertices of the polytope defined by $\hat{\mathcal{P}} = [p_{min}, p_{max}] \times [r_{min}, r_{max}]$ in the $(p, \dot{p})$ space. In order to address the non-affine (quadratic) dependence on the parameter p, one can impose a multiconvexity requirement on (7)-(8) as in (Apkarian and Adams 1997). This approach amounts essentially to overbounding the inequalities (7) and (8) by inequalities affine in p by imposing one additional constraint. The details of this idea can be found in (Gahinet *et al.* 1996). The advantage of this approach lies in the fact that one needs to solve the inequalities only at the vertices of the polytope $\hat{\mathcal{P}}$.

Under the previous assumptions, the optimization problem (7)-(9) can be replaced (indeed with some conservatism) with the following one: Find *fixed* matrices X_0, X_1, Y_0 and Y_1 and positive scalars e_x and e_y such that the following inequalities are satisfied *at* *the vertices* of the polytope $\hat{\mathcal{P}}$.

$$\hat{\mathcal{N}}_X^T \left(\begin{array}{cc|c} \left[\begin{array}{c} \dot{p}X_1 + X_0A_0 + A_0^TX_0 \\ p(X_1A_0 + A_0^TX_1 + X_0A_1 + A_1^TX_0) + p^2e_xI \end{array} \right] & (X_0 + pX_1)B_1 & C_1^T \\ B_1^T(X_0 + pX_1) & -\gamma I & D_{11}^T \\ \hline C_1 & D_{11} & -\gamma I \end{array} \right) \hat{\mathcal{N}}_X < 0 \tag{19}$$

$$\hat{\mathcal{N}}_Y^T \left(\begin{array}{cc|c} \left[\begin{array}{c} -\dot{p}\dot{Y}_1 + A_0^TY_0 + A_0Y_0 \\ p(A_0^TY_1 + Y_1A_0 + A_1^TY_0 + Y_0A_1) + p^2e_yI \end{array} \right] & YC_1^T & B_1 \\ C_1Y & -\gamma I & D_{11} \\ \hline B_1^T & D_{11}^T & -\gamma I \end{array} \right) \hat{\mathcal{N}}_Y < 0 \tag{20}$$

$$\left(\begin{array}{cc} X_0 + pX_1 & I \\ I & Y_0 + pY_1 \end{array} \right) \geq 0 \tag{21}$$

$$X_1A_1 + A_1^TX_1 + e_xI > 0, \qquad A_1Y_1 + Y_1A_1^T + e_yI > 0 \tag{22}$$

where the matrices $\hat{\mathcal{N}}_X$ and $\hat{\mathcal{N}}_Y$ are as in (7) and (8).

This formulation, based on the parameterization (18), also provides one with an *ad hoc* procedure to remedy the problem of the controller dependence on the parameter rate $\dot{p}$. Referring to (15), one can, for example, remove the dependence of the controller on $\dot{p}$ by choosing either $X_1 = 0$ or $Y_1 = 0$ in (19)-(22). Note also that by setting both $X_1 = Y_1 = 0$ one recovers the result of quadratic $\mathcal{H}_\infty$ performance of Definition 1.

MAGNETIC BEARING/ROTOR SIMULATIONS

In this section we apply the previous results to the magnetic bearing/rotor system in Fig 1. For simplicity, only the first bending mode will be retained in the model, in which case (1) represents an 12-dimensional realization of the system. Since the performance output z does not depend on the exogenous input w, we have that $D_{11} = 0$, thus we chose $D_{cl} = D_k = 0$. In this case, the equations (11)-(13) simplify to (recall that D_{21} and D_{12} are assumed to be full row and full column matrices, respectively)

$$\hat{B}_k^T = -(D_{21}D_{21}^T)^{-1}(\gamma C_2 + D_{21}B_1^TX) \tag{23a}$$
$$\hat{C}_k = -(D_{12}^TD_{12})^{-1}(\gamma B_2^T + D_{12}^TC_1Y) \tag{23b}$$
$$\hat{A}_k = -(XB_1 + \hat{B}_kD_{21})\frac{B_1^T}{\gamma} - \frac{C_1^T}{\gamma}(C_1Y + D_{12}\hat{C}_k) \tag{23c}$$

and the controller can be computed by

$$A_k = N^{-1}(X\dot{Y} + N\dot{M}^T + \hat{A}_k - A^T - XAY - \hat{B}_kC_2Y - XB_2\hat{C}_k)M^{-T} \tag{24a}$$
$$B_k = N^{-1}\hat{B}_k \tag{24b}$$
$$C_k = \hat{C}_kM^{-T} \tag{24c}$$

where the matrices N and M are as in (14). We point out that *any* factorization of (14) will do for this purpose, and the obvious solution is to choose $N = I$ and $M = (I - XY)^T$ or $M = I$ and $N = I - XY$. In general, a numerically stable (but computationally expensive) approach is to calculate N and M using a singular value decomposition of the matrix $(I - XY)^{-1}$. An efficient algorithm (also used in this work) to avoid the inversion of the matrix $I - XY$ on line is to compute this inverse using the formula

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix}^{-1} = \begin{bmatrix} \star & \star \\ (I - XY)^{-1} & \star \end{bmatrix} \tag{25}$$

where the previous inverse is computed by

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix}^{-1} = T(I + p\Lambda)^{-1} T^T \tag{26}$$

where Λ is diagonal and T is a (parameter *independent*) transformation which satisfies

$$T^T \begin{bmatrix} X_0 & I \\ I & Y_0 \end{bmatrix} T = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \qquad \text{and} \qquad T^T \begin{bmatrix} X_1 & 0 \\ 0 & Y_1 \end{bmatrix} T = \Lambda \tag{27}$$

Since $p \in [-1, 1]$, such a transformation always exists (Apkarian and Adams 1997).

Four different cases were investigated and compared. The calculations were performed using the LMI toolbox of MATLAB (Gahinet *et al.* 1995). The results are tabulated in Table 1. In the first case, a parameter-varying controller was designed ignoring any bounds on the rate of variation of p. This controller corresponds to setting $X_1 = Y_1 = 0$ in equations (19)-(22) and guarantees arbitrarily fast changes of p (it can actually handle the completely unrealistic case $\dot{p} = \infty$). The second and third cases correspond to a parameter varying controller using only speed measurements. A known bound of $2,000$ *rpm* per 30 *sec* was assumed for $\dot{p}$. This controller was obtained by setting either $X_1 = 0$ (second case) or $Y_1 = 0$ (third case). Finally, a controller with both $X_1 \neq 0$ and $Y_1 \neq 0$ was designed.

The resulting values of γ show the improvement in the performance using the parameter-dependent controller based on parameter-dependent solutions of the associated LMI's.

Table 1: Controller performance.

$X(p)$	$Y(p)$	γ
$X_1 = 0$	$Y_1 = 0$	2.4353
$X_1 = 0$	$Y_1 \neq 0$	2.4090
$X_1 \neq 0$	$Y_1 = 0$	2.1857
$X_1 \neq 0$	$Y_1 \neq 0$	2.1698

As expected, the case with $X_1 \neq 0$ and $Y_1 \neq 0$ gives the best performance with an overall improvement of about 11% over the standard quadratic $\mathcal{H}_\infty$ gain-scheduled controller (case #1). The practical controller (i.e., one without measurement of $\dot{p}$) of case #3 has comparable performance to the one in case #4. For this particular example it appears that the case $Y_1 \neq 0$ is better than the case $X_1 \neq 0$. However, this is depends on the problem at hand. In general, both cases need to be examined for the one which gives the best answer.

The values of γ in Table 1 indicate only guaranteed *upper bounds* of the actual performance level. In order to compare the actual achieved performance at each speed, we calculated the corresponding controllers from (23)-(24) and computed the $\mathcal{H}_\infty$ norm of the corresponding closed-loop systems at several speeds. For comparison, we also computed the standard (fixed-speed) $\mathcal{H}_\infty$ controllers at these same speeds. The corresponding values of γ fo theses fixed-speed $\mathcal{H}_\infty$ controllers indicate the best achievable performance at that particular speed and thus provide a measure of the degree of conservatism introduced by the parameter-varying gain-scheduled controllers. The results for all the cases are shown in Fig. 2.

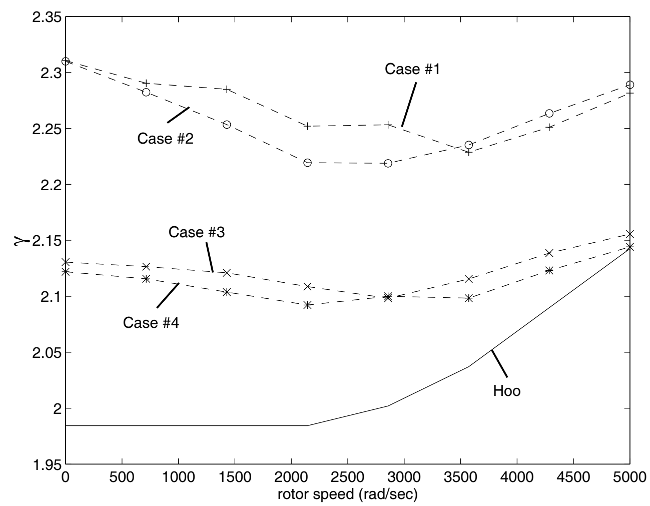

Figure 2: Performance vs. operating speed.

Figure 2 shows that the performance of the fixed-speed $\mathcal{H}_\infty$ controller varies significantly with the rotor speed. The $\mathcal{H}_\infty$ controller performance degrades as the speed increases, i.e., as the gyroscopic effects become dominant. We postulate that his degradation in performance will be more profound for the real system, as higher flexible modes will be excited as the speed is increased. In fact, the $\mathcal{H}_\infty$ controller offers some advantage only at low speeds (less than 3,500 rad/sec). On the other hand, gain-scheduled controllers show a relatively uniform performance over all speeds. The performance at high speeds in essentially the same as the $\mathcal{H}_\infty$ controller (the "best" achievable at that speed). This observation is very encouraging for the use of gain-scheduled controllers for systems with strong gyroscopic effects. Moreover, Fig. 2 shows that allowing X to be dependent on p offers minimal improvement in performance.

CONCLUSIONS

We have presented a methodology to refine existing LPV-based gain-scheduled controller synthesis results for a magnetic bearing/rotor system. The proposed approach is based on parameter-dependent solutions of the linear matrix inequalities of the associated optimization problem along the lines of (Apkarian and Adams 1997). Numerical simulations indicate that significant improvement in the achievable performance can be achieved using parameter-dependent solutions of the associated Linear Matrix Inequalities (LMI's). In fact, he performance at high speeds is essentially the same as the corresponding fixed-speed $\mathcal{H}_\infty$ controller. Future research will concentrate on experimentally validating these results at the Center of Magnetic Bearings at the University of Virginia.

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