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Egalitarian Peer-to-Peer Satellite Refueling Strategy

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In this paper we revisit the problem of peer-to-peer refueling of a satellite constellation in orbit with propellant. In particular, we propose the egalitarian peer-to-peer refueling strategy that relaxes the restriction on the active satellites to return to their original orbital slots after all fuel exchanges have been completed. We formulate the problem as a minimum cost flow problem in the so-called constellation network, and minimize the total ΔV subject to flow balance constraints, along with certain additional constraints introduced to avoid conflicts between active and passive satellites. Recognizing that the actual objective is to minimize the total fuel expenditure, instead of ΔV, we also propose a method to improve the results by performing a local search around the minimum-ΔV solution. We also provide explicit upper and lower bounds on the suboptimality of the obtained results. With the help of numerical examples, it is shown that the proposed egalitarian peer-to-peer refueling strategy leads to considerable reduction in terms of the total fuel expenditure over the baseline peer-to-peer strategy.

Nomenclature

b_i	=	supply/demand corresponding to a node in $\mathcal{V}_n$
$\mathcal{C}(\mathcal{M})$	=	cost of an egalitarian peer-to-peer solution $\mathcal{M}$
$\mathcal{C}_{\mathrm{LB}}$	=	optimal value of objective function of the optimization problem
c_{ij}	=	cost of an edge in $\mathcal{G}_n$
c^{ℓ}_{ij}	=	cost of an edge in $\mathcal{G}_\ell$
$c(i,j,k)$	=	cost of a triplet (i,j,k) representing an egalitarian peer-to-peer maneuver
d	=	distance between two triplets t_p and t_q
$\mathcal{E}_\ell$	=	set of edges in $\mathcal{G}_\ell$
$\mathcal{E}_f$	=	set of edges in $\mathcal{G}_n$ representing forward trips of active satellites
$\mathcal{E}_n$	=	set of edges in $\mathcal{G}_n$
$\mathcal{E}_r$	=	set of edges in $\mathcal{G}_n$ representing return trips of active satellites
$\mathcal{E}_s$	=	set of source arcs in $\mathcal{G}_n$
$\mathcal{E}_t$	=	set of sink arcs in $\mathcal{G}_n$
$\bar{f}_i$	=	maximum fuel capacity of satellite s_i
$\underline{f}_i$	=	minimum fuel requirement by satellite s_i to remain operational
$f_{i,t}$	=	fuel content of satellite s_i at time t
$\mathcal{G}$	=	constellation digraph
$\mathcal{G}_\ell$	=	bipartite graph used for calculating lower bound on cost of optimal e-p2p solution
$\mathcal{G}_n$	=	constellation network
g_0	=	acceleration due to gravity at surface of the earth
$I_{\mathrm{sp}\mu}$	=	specific thrust of satellite s_μ
$\mathcal{J}$	=	index set for satellites/orbital slots
$\mathcal{J}_a$	=	index set for orbital slots of active satellites
$\mathcal{J}_p$	=	index set for orbital slots of passive satellites
$\mathcal{J}_r$	=	index set of orbital slots available for active satellites to return
$\mathcal{J}_{d,t}$	=	index set for orbital slots of fuel-deficient satellites at time t
$\mathcal{J}_{s,t}$	=	index set for orbital slots of fuel-sufficient satellites at time t
$\mathcal{M}$	=	egalitarian peer-to-peer solution composed of a set of triplets
$\mathcal{M}^*$	=	optimal egalitarian peer-to-peer solution
$\mathcal{M}_H$	=	egalitarian peer-to-peer solution obtained after local search on $\mathcal{M}_{\mathrm{IP}}$
$\mathcal{M}_{\mathrm{IP}}$	=	Egalitarian peer-to-peer solution yield by the optimization problem
$\mathcal{M}_{\mathrm{P2P}}$	=	optimal egalitarian peer-to-peer solution
$m_{s\mu}$	=	mass of permanent structure of satellite s_μ
$N_2(t_p,t_q)$	=	two-exchange neighborhood of a triplet pair comprising t_p and t_q
$\mathcal{N}(\mathcal{M})$	=	neighborhood of an egalitarian peer-to-peer solution $\mathcal{M}$
p^{μ}_{ij}	=	fuel expenditure required for an orbital transfer by satellite s_μ from slot ϕ_i to slot ϕ_j
$\mathcal{Q}(i,j,k)$	=	edge in $\mathcal{E}_\ell$ corresponding to a triplet (i,j,k)
s_i	=	satellite with index $i \in \mathcal{J}$
T	=	total time allotted for refueling
$\mathcal{T}$	=	set of feasible triplets in the constellation graph
t_p	=	triplet (i_p,j_p,k_p) with index p
$\mathcal{V}_n$	=	set of vertices in $\mathcal{G}_n$
x_{ij}	=	binary variable corresponding to an arc $(i,j) \in \mathcal{E}_n$ or an edge $\langle i,j\rangle$ in $\mathcal{E}_\ell$
ΔV_{ij}	=	velocity change required for a transfer from slot ϕ_i to slot ϕ_j
η	=	suboptimality measure
ϕ_i	=	orbital slot with index i

I. Introduction

SERVICING and refueling spacecraft in orbit has the potential to revolutionize spacecraft operations by extending the useful lifetime of the spacecraft, by reducing launching and insurance cost, and by increasing operational flexibility and robustness. The traditional approach toward satellite refueling is to consider a single refueling spacecraft that supplies fuel to all satellites depleted of fuel [1]. Recently, an alternative scenario for distributing fuel amongst a large number of satellites has been proposed [2]. In this scenario, no single spacecraft is in charge of the complete refueling process. Instead, satellites exchange fuel amongst themselves in pairs, with the fuel-sufficient satellites providing fuel to the fuel-deficient satellites. We call the latter the peer-to-peer (P2P) refueling strategy [2].

A P2P refueling strategy offers a great degree of robustness and protection against failures by providing a *distributed* method to replenish a constellation with fuel/propellant. For instance, with a P2P strategy, a failure of a single spacecraft will have almost no

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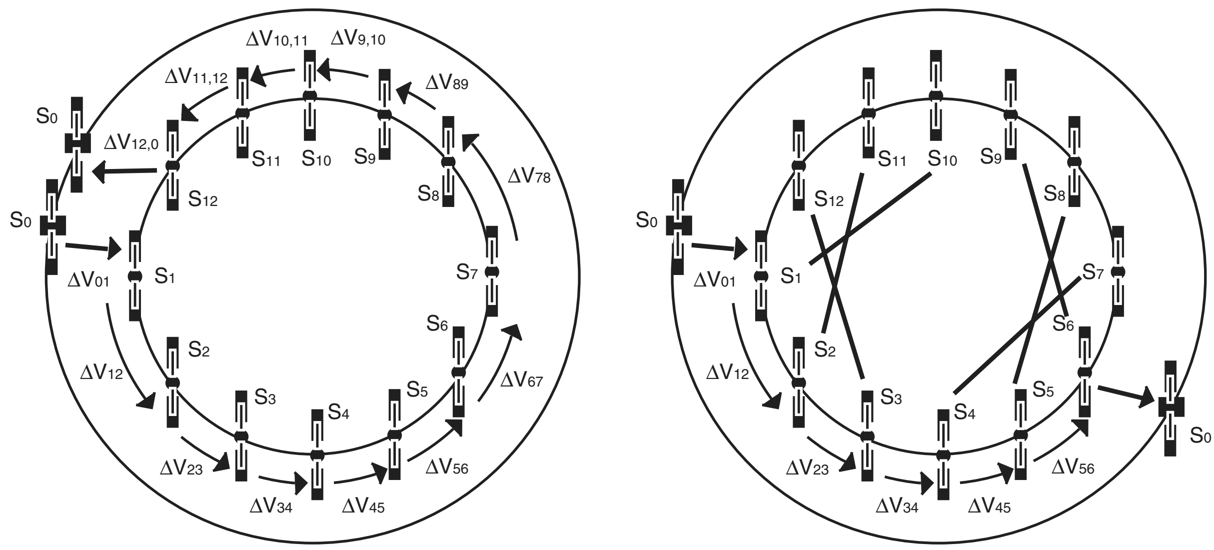


a) Single vehicle refueling **b) Mixed refueling strategy**

Fig. 1 Single-service vehicle and mixed refueling strategies.

impact on the refueling of the remaining satellites in the constellation. On the contrary, a failure of the service vehicle in a single-spacecraft strategy will result in the failure of the whole mission. Also, a P2P refueling strategy arises naturally as an essential component of a *mixed refueling strategy* [3]. Such a strategy involves two stages. During the first stage, a single spacecraft refuels some of the satellites (typically half) in the constellation. During the second stage, the satellites that received fuel during the first stage distribute the fuel to the remaining satellites via P2P maneuvers. See Fig. 1b.

It has been shown in [3] that a mixed refueling strategy may be more fuel-efficient than a single-spacecraft strategy (Fig. 1a), especially for a large number of satellites and for short refueling periods [4]. It has also been shown that the incorporation of a coasting time allocation strategy or of asynchronous P2P maneuvers make the mixed refueling strategy an even more competitive alternative than single-service vehicle refueling strategies in terms of fuel expenditure [3].

The original studies [2–5] on P2P refueling used this strategy as a means to equalize fuel in a satellite constellation. To achieve fuel equalization in the constellation, these studies considered an optimization problem in which the deviation of each satellite's fuel from the initial average fuel in the constellation was minimized. Under such a formulation, the problem of finding the optimal assignment between fuel-sufficient and fuel-deficient satellites reduces to a problem of finding the maximum weighted matching in the so-called constellation graph. This maximum matching problem can be solved using standard methods like Edmond's algorithm [6].

The P2P refueling problem can alternatively be formulated by imposing a minimum fuel requirement on each satellite to remain operational. Satellites having at least the required amount of fuel are called *fuel-sufficient*, whereas those which do not have the required amount of fuel are called *fuel-deficient*. We therefore seek to determine the optimal assignment of satellites so that all satellites end up being fuel-sufficient after the refueling process is over. The objective is to achieve fuel-sufficiency for all satellites by expending as little fuel as possible during the ensuing orbital transfers. A decentralized approach that uses auctions to determine the optimal assignments has also been proposed [7].

In all of the aforementioned studies [2–4,7], it has been assumed that all active satellites return to their original orbital slots after the refueling process is over. Recent studies [8,9] aimed at relaxing this constraint by allowing the active satellites to return to any of the available orbital slots left vacant by other (active) satellites. The underlying assumption behind such a consideration is that all satellites are similar and perform the same functions, so that any satellite can be used in lieu of any other satellite in the constellation. We call this the egalitarian P2P (E-P2P) refueling strategy. The study of optimal E-P2P refueling strategies is the main focus of the current paper.

The E-P2P refueling problem can be formulated as a three-index assignment problem [8]. It is known that three-index assignment problems are NP-complete (nondeterministic polynomial time) [10]. The general multi-index assignment problem was first stated by Pierskalla [11] as an extension of the two-index assignment problem. The three-index assignment problem, which is a special case of the multi-index assignment problem, can be viewed as a matching problem on a complete tripartite graph. Several suboptimal algorithms have been proposed for the three-index assignment problem. A branch-and-bound algorithm was proposed by Balas and Saltzman [12]. Approximation algorithms for three-index assignment problems with triangle inequalities were addressed by Crama and Spieksma [13]. For multi-index assignment problems in k-partite graphs with decomposable costs,[‡] Bandelt et al. [14] introduced two approximate algorithms, each one of which solves a sequence of two-index assignment problems. Another class of algorithms that has been developed for solving the three-index assignment problem includes the Greedy Random Adaptive Search Procedure (GRASP) [15–17], which generates good quality solutions for the three-index assignment problem by constructing low-cost feasible solutions. These feasible solutions can then be improved by performing a local search about these solutions. In [8], we have used the GRASP method to determine the optimal assignments for E-P2P refueling. Alternatively, the E-P2P refueling problem can be modeled as a minimum cost flow problem in an appropriately constructed constellation network [9].

In this paper, we consider a network flow formulation, which gives a good quality solution much faster than the GRASP algorithm. We formulate the E-P2P refueling problem as a minimum cost flow problem in an appropriately constructed network. An optimal flow in this network provides us with a set of E-P2P maneuvers that has the minimum ΔV cost. Recognizing that our real objective is to minimize the total fuel expenditure (as opposed to minimizing total ΔV), we propose a local search method to improve the obtained solution. Local search methods have been found to be very useful in improving the cost of solutions for the three-index assignment

[‡]By decomposable costs, we mean that the cost of a clique in the k-partite graph is a function of the cost of the edges induced by the clique. Note that a clique is a subgraph in which all vertices are pairwise adjacent. For a k-partite graph, a clique comprises exactly one node from each partition of the k-partite graph.

problem [8,16,17]. In this paper, we perform a local search based on a two-exchange neighborhood [12,16,17]. We also derive upper and lower bounds on the fuel expenditure corresponding to the optimal assignments for E-P2P refueling. The lower bound can be calculated by solving a separate two-index assignment problem, and is useful for providing a measure of suboptimality of the E-P2P solution. A P2P solution, in which the active satellites are constrained to return to their original orbital slots, provides an upper bound for the E-P2P solution.

The network flow formulation minimizes the total ΔV rather than the total fuel expenditure, and hence the results obtained using this method are suboptimal when compared with those obtained by GRASP. However, from a computational point of view, the network flow formulation generates the optimal E-P2P assignments several times faster than the (nonparallelized) GRASP method.

II. Problem Formulation

We consider a constellation consisting of n satellites distributed over n orbital slots in a circular orbit. Let the set of n satellites be given by $\mathcal{S} = \{s_i: i = 1, 2, \ldots, n\}$ and let the set of n orbital slots be given by $\Phi = \{\phi_i \in [0, 2\pi): i = 1, 2, \ldots, n, \phi_i \neq \phi_j\}$. For the satellite s_i, let $\underline{f}_i$ denote the minimum amount of fuel to remain operational, and let $\bar{f}_i$ denote the maximum fuel capacity of the satellite. Also, let the fuel content of satellite s_i at time t be denoted by $f_{i,t}$. The initial fuel content of satellite s_i will therefore be $f_{i,0}$.

Satellite s_i is said to be fuel-sufficient at time t if $f_{i,t} \geq \underline{f}_i$. Otherwise, it is said to be fuel-deficient. In an E-P2P refueling strategy, the fuel-sufficient and the fuel-deficient satellites undergo fuel transactions amongst themselves, such that, at the end of refueling, each satellite has at least the required amount of fuel to remain operational. In other words, the objective of refueling is to ensure that $f_{i,T} \geq \underline{f}_i$ for all $i = 1, 2, \ldots, n$.

It will be convenient to keep track of the indices of the orbital slots of the satellites participating in the refueling process under different roles. To this end, let $\mathcal{J} = \{1, 2, \ldots, n\}$. Let $\mathcal{J}_{s,t}$ denote the index set of orbital slots occupied by fuel-sufficient satellites at time t, and let $\mathcal{J}_{d,t}$ denote the index set of orbital slots occupied by fuel-deficient satellites at time t. During an E-P2P refueling transaction between a fuel-sufficient and a fuel-deficient satellite, one of them (henceforth referred to as the *active* satellite) performs an orbital transfer to rendezvous with the other satellite (henceforth referred to as the *passive* satellite). After the fuel exchange takes place between the two, the active satellite returns to any of the available orbital slots. Let $\mathcal{J}_a$ denote the index set of orbital slots occupied by the active satellites before any orbital maneuver commences, let $\mathcal{J}_p$ denote the index set of orbital slots occupied by the passive satellites before any orbital maneuver commences, and let $\mathcal{J}_r$ denote the index set of orbital slots available for the active satellites to return to after they have undergone fuel transactions with the passive satellites. Clearly, the available slots are the same as the ones initially occupied by the active satellites. Hence, $\mathcal{J}_r = \mathcal{J}_a$.

A. Constellation Graph

The E-P2P refueling problem can be formulated on an undirected tripartite graph [8], denoted by $\mathcal{G}$, consisting of three partitions, namely, $\mathcal{J}_{s,0}$, $\mathcal{J}_{d,0}$, and $\mathcal{J}_a$. We will denote an E-P2P maneuver in $\mathcal{G}$ by the triplet $(i, j, k) \in \mathcal{J}_a \times \mathcal{J}_p \times \mathcal{J}_r$; by this, we mean that the active satellite s_μ, which initially occupies the orbital slot ϕ_i, carries out an orbital transfer to rendezvous with the passive satellite s_ν, which occupies the orbital slot ϕ_j, undergoes a fuel transaction with s_ν, and finally performs another orbital transfer to return to the orbital slot ϕ_k left vacant by another active satellite. Note that a fuel transaction can only be between a fuel-sufficient and a fuel-deficient satellite. Nonetheless, either the fuel-deficient or the fuel-sufficient satellite can be active (that is, the one that initiates the refueling transaction).

The situation is depicted in Fig. 2. In this figure, the active satellites are marked by a star, the forward trips are shown by solid arrows, and return trips are shown by dotted arrows. The corresponding index sets indicate the fuel-sufficient satellites and the fuel-deficient satellites before refueling commences, along with the indices of active and passive satellites. For this example, and for the sake of simplicity, we assume that each satellite index coincides with the index of the orbital slot occupied initially by this satellite in the orbit.

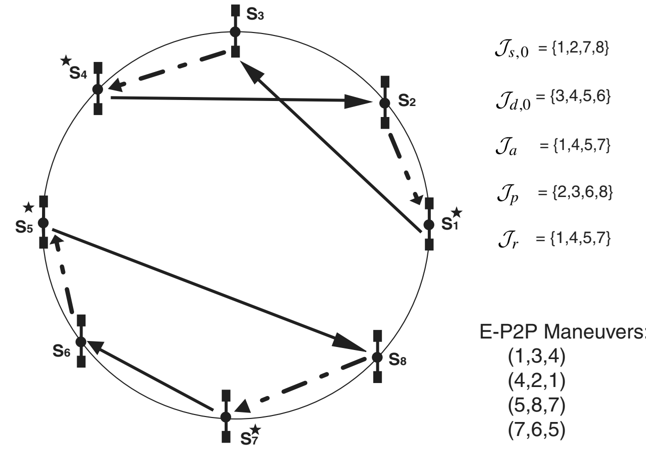


Fig. 2 Typical E-P2P refueling strategy.

We say that an E-P2P maneuver is *feasible* if and only if both satellites s_μ and s_ν involved in the maneuver end up being fuel-sufficient after the E-P2P maneuver. Accordingly, we say that the triplet (i, j, k) is feasible. Let $\mathcal{T} \subseteq \mathcal{J}_a \times \mathcal{J}_p \times \mathcal{J}_r$ denote the set of feasible triplets in the constellation graph. To each feasible triplet $(i, j, k) \in \mathcal{T}$, we assign a cost $c(i, j, k)$ that equals the fuel expenditure incurred during the E-P2P maneuver. We therefore have

$$c(i, j, k) = p_{ij}^{\mu} + p_{jk}^{\mu} \tag{1}$$

where p_{ij}^{μ} is the fuel expenditure incurred by satellite s_μ during the forward trip (when it moves from the orbital slot ϕ_i to the orbital slot ϕ_j) and p_{jk}^{μ} is the fuel expenditure incurred by s_μ during its return trip (when it moves from the orbital slot ϕ_j to the orbital slot ϕ_k). We define as a *feasible E-P2P solution* a set $\mathcal{M}$ of $|\mathcal{J}_{d,0}|$ feasible triplets that has the following properties:

1) An active and a passive satellite should feature in a single E-P2P maneuver, that is, $i \neq i'$, $j \neq j'$ for all triplets (i, j, k), $(i', j', k') \in \mathcal{M}$.

2) The returning positions for all active satellites are distinct, that is, $k \neq k'$ for all triplets (i, j, k), $(i', j', k') \in \mathcal{M}$.

3) The return positions are the slots left vacant by the active satellites.

4) The orbital slots of the passive satellites cannot be the return positions for any of the active satellites.

The cost $\mathcal{C}(\mathcal{M})$ of a feasible E-P2P solution $\mathcal{M}$ is defined to be the sum of the costs of all triplets in $\mathcal{M}$, that is,

$$\mathcal{C}(\mathcal{M}) = \sum_{(i,j,k)\in\mathcal{M}} c(i, j, k) \tag{2}$$

The *optimal E-P2P solution* is a feasible E-P2P solution $\mathcal{M}^*$ that achieves the minimum value of Eq. (2) among all feasible sets of triplets. That is, $\mathcal{C}(\mathcal{M}^*) \leq \mathcal{C}(\mathcal{M})$ for all feasible $\mathcal{M} \subset \mathcal{T}$.

The determination of the assignments for E-P2P refueling can be posed as an optimization problem, in which we are required to find a set $\mathcal{M} \subset \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$ of $|\mathcal{J}_{d,0}|$ triplets such that none of the triplets in $\mathcal{M}$ share a vertex, and such that the total cost of the triplets in $\mathcal{M}$ is a minimum. The GRASP method has been used to solve this problem [8] and yields a good quality solution, but is computationally expensive. This motivates the network flow formulation in this paper.

Note that the P2P refueling problem is a special case of E-P2P refueling, in which the active satellites return to their respective original slots. The P2P refueling problem can be solved as a two-index assignment problem in an undirected bipartite constellation graph having as node partitions the orbital slots of the fuel-sufficient and the fuel-deficient satellites. Each (undirected) edge $\langle i, j\rangle$ in this bipartite graph has an associated active satellite, a passive satellite, and a cost for the maneuver. The optimal P2P solution is a set of edges (equal to the number of the fuel-deficient satellites) that do not share a node, and the sum of the cost of the edges is a minimum. We can represent this set of edges by a set of triplets $\mathcal{M}_{\text{P2P}}$ by noting that for each edge, the return slot is the original slot of the active satellite. The fuel expenditure for optimal P2P refueling in the constellation is therefore given by $\mathcal{C}(\mathcal{M}_{\text{P2P}})$.

B. Feasible Conditions and Maneuver Cost

Let us consider a triplet $(i, j, k) \in \mathcal{M}$, where $\mathcal{M}$ is a feasible E-P2P solution. Also, let s_μ be the satellite that is initially at orbital slot ϕ_i, and let s_ν be the satellite that is originally at orbital slot ϕ_j. The fuel consumed by satellite s_μ to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j is given by:

$$p_{ij}^{\mu} = (m_{s\mu} + f_{\mu,0})(1 - e^{-\frac{\Delta V_{ij}}{c_{0\mu}}}) \tag{3}$$

where $m_{s\mu}$ is the mass of the permanent structure of satellite s_μ, and ΔV_{ij} is the velocity change required for a two-impulse transfer from the orbital slot ϕ_i to the orbital slot ϕ_j. The parameter $c_{0\mu}$ is defined by $c_{0\mu} = g_0 I_{\text{sp}\mu}$, where g_0 is the acceleration due to gravity at the Earth's surface and $I_{\text{sp}\mu}$ is the specific thrust of satellite s_μ. The fuel content of satellite s_μ after its forward trip (but before the fuel exchange takes place) is $f_{\mu,0} - p_{ij}^{\mu}$. Because the fuel consumption during a maneuver is minimized if the active satellite returns to its assigned slot with exactly the required minimum amount of fuel to remain operational, the amount of fuel consumed during the return trip, is given by

$$p_{jk}^{\mu} = (m_{s\mu} + \underline{f}_{\mu})e^{\frac{\Delta V_{jk}}{c_{0\mu}}}(1 - e^{-\frac{\Delta V_{jk}}{c_{0\mu}}}) \tag{4}$$

where ΔV_{jk} is the velocity change required for the transfer from the orbital slot ϕ_j to the orbital slot ϕ_k. Before the return trip (but after the fuel exchange takes place), the fuel onboard satellite s_μ is $\underline{f}_{\mu} + p_{jk}^{\mu}$. The fuel transferred to satellite s_ν during the fuel exchange is $(f_{\mu,0} - p_{ij}^{\mu}) - (\underline{f}_{\mu} + p_{jk}^{\mu})$, assuming that the satellite s_ν has enough fuel capacity to accommodate this amount of fuel. The fuel onboard satellite s_ν after it is refueled is $f_{\nu,0} + (f_{\mu,0} - p_{ij}^{\mu}) - (\underline{f}_{\mu} + p_{jk}^{\mu})$. For satellite s_ν to become fuel-sufficient after the fuel transaction, we must therefore have

$$(f_{\nu,0} + f_{\mu,0}) - (\underline{f}_{\mu} + \underline{f}_{\nu}) \geq p_{ij}^{\mu} + p_{jk}^{\mu} \tag{5}$$

If the preceding condition does not hold, then the E-P2P refueling transaction between s_μ and s_ν is not feasible. Also, if satellite s_μ does not have enough fuel to carry out the orbital transfer during the forward trip, that is, if $p_{ij}^{\mu} \geq f_{\mu,0}$, then the E-P2P refueling transaction is also not feasible.

III. Methodology for Solving the E-P2P Refueling Problem

In this section, we propose a methodology for solving the E-P2P problem. We use the directed constellation graph $\mathcal{G}$ to set up a constellation network flow. We then pose the problem as a minimum cost flow problem. The solution to the minimum cost flow problem will correspond to a set of maneuvers for which the total ΔV is minimized. We also describe a local search method for improving this solution by performing a local search in its N_2 neighborhood [17]. This is similar to what is typically done in the case of the GRASP method.

A. Constellation Digraph

Recall that the constellation graph $\mathcal{G}$ is a tripartite graph with partitions $\mathcal{J}_a$, $\mathcal{J}_p$, $\mathcal{J}_r$. Because we do not know a priori which satellites are active, we take $\mathcal{J}_a = \mathcal{J}_p = \mathcal{J}_r = \mathcal{J}$. On the constellation digraph $\mathcal{G}$ we represent an E-P2P maneuver (i, j, k) by the directed edges (i, j) and (j, k), where $i \in \mathcal{J}_a$, $j \in \mathcal{J}_p$, and $k \in \mathcal{J}_r$. Because a fuel transaction can only be between a fuel-sufficient and a fuel-deficient satellite, we also have that either $i \in \mathcal{J}_{s,0}$ and $j \in \mathcal{J}_{d,0}$, or $i \in \mathcal{J}_{d,0}$ and $j \in \mathcal{J}_{s,0}$. Let the set of edges representing all possible forward trips be denoted by $\mathcal{E}_f$. The return maneuver from the orbital slot ϕ_j to the orbital slot ϕ_k, where $k \neq j$, can be represented by a directed edge (j, k) where $j \in \mathcal{J}_p$ and $k \in \mathcal{J}_r$, where $j \neq k$. Let $\mathcal{E}_r$ denote the set of all possible return trips.

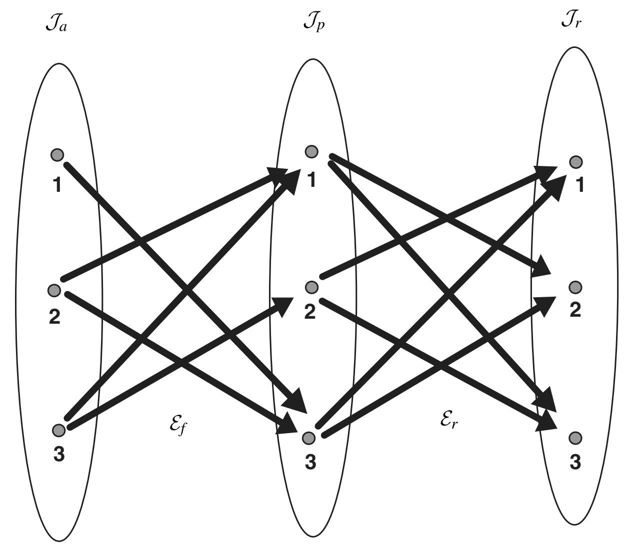


Fig. 3 Directed constellation graph.

Figure 3 shows the digraph for a constellation, with vertices representing orbital slots of satellites and edges representing orbital maneuvers. Note that a pair of directed edges $(i, j) \in \mathcal{E}_f$ and $(\ell, k) \in \mathcal{E}_r$ represents an E-P2P maneuver if and only if $\ell = j$.

B. Cost Assignment

With each orbital transfer represented by a directed edge $(i, j) \in \mathcal{E}_f \cup \mathcal{E}_r$, we associate a cost c_{ij} as follows

$$c_{ij} = \Delta V_{ij} \tag{6}$$

where ΔV_{ij} is the required velocity change for a satellite to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j. Note that the calculation of ΔV_{ij} requires, in general, the solution of a two-impulse multirevolution Lambert problem [18].

We should point out here that, ideally, the cost c_{ij} should be the fuel consumption [refer to Eqs. (1), (3), and (4)] during the transfer. However, the amount of fuel depends on the mass of the satellite performing the transfer, which may not be known a priori. For instance, recall that the edge $(j, k) \in \mathcal{E}_r$ represents a valid return trip for any of the E-P2P maneuvers in which an edge $(i, j) \in \mathcal{E}_f$ represents a forward trip. The active satellites that can carry out the orbital transfer from orbital slot ϕ_j to orbital slot ϕ_k are all those that initially occupy the orbital slots ϕ_i such that $(i, j) \in \mathcal{E}_f$. For each of these active satellites, the fuel expenditure for the return trip represented by the edge $(j, k) \in \mathcal{E}_r$ is different. Therefore, if fuel expenditure is used to define the cost of edges, no unique value can be assigned to an edge $(j, k) \in \mathcal{E}_r$. This is the reason we use Eq. (6), tacitly recognizing the fact that the results will necessarily be suboptimal in terms of actual fuel consumption.

C. Constellation Network Flow

Given the constellation digraph $\mathcal{G}$, we now set up the constellation network $\mathcal{G}_n$, and show that the E-P2P problem can be formulated as a minimum cost flow problem over the constellation network $\mathcal{G}_n$. To this end, we add a source node s and a sink node t to the constellation digraph $\mathcal{G}$. For all $i \in \mathcal{J}_a$, we also add an arc (s, i) with associated cost $c_{si} = 0$. We denote the set of these arcs by $\mathcal{E}_s$. Similarly, for all $k \in \mathcal{J}_r$, we add an arc (k, t) with associated cost $c_{kt} = 0$. We denote the set of these arcs by $\mathcal{E}_t$. Let $\mathcal{V}_n$ denote the set of notes of $\mathcal{G}_n$, and let $\mathcal{E}_n$ denote the set of arcs (directed edges) of $\mathcal{G}_n$. A depiction of $\mathcal{G}_n$ is given in Fig. 4. Let us now consider a $s \to t$ flow in the network $\mathcal{G}_n$. By an $s \to t$ flow, we mean a flow from the source s to the sink t passing through the nodes $i \in \mathcal{J}_a$, $j \in \mathcal{J}_p$, and $k \in \mathcal{J}_r$ in that order, that is, a flow along the directed path $\{s \to i \to j \to k \to t\}$. Note that the flow $s \to t$ passes through the arcs $(s, i) \in \mathcal{E}_s$, $(i, j) \in \mathcal{E}_f$, $(j, k) \in \mathcal{E}_r$, and $(k, t) \in \mathcal{E}_t$. Of these, the arcs (i, j) and (j, k) constitute an E-P2P maneuver (i, j, k), and the sum of the costs of all these edges is the total cost of the E-P2P maneuver (i, j, k). The remaining arcs (s, i) and (k, t) have zero cost and therefore the cost of a unit flow along the path $\{s \to i \to j \to k \to t\}$ is the total cost of the corresponding E-P2P maneuver. We can therefore associate an E-P2P maneuver with a unique $s \to t$ flow.

D. Network Flow Minimization Problem

We will now formulate the E-P2P refueling problem as a minimum cost flow problem. It is to be noted here that the integrality property [19] states that if all arc capacities and supplies/demands of the nodes are integers, the minimum cost flow problem has an integral optimal solution. In other words, we can arrive at the optimal solution by considering only integer values of the flow variables. In the formulation of our problem, we will include integer arc capacity and integer supply/demand for each node, and will show that a feasible integral flow in the constellation corresponds to a feasible E-P2P solution $\mathcal{M}$. Let us introduce a flow variable x_{ij} for each edge $(i, j) \in \mathcal{E}_n$. The flow variable x_{ij} equals the amount of flow through the edge (i, j). We consider $x_{ij} \in \{0, 1\}$. Clearly, for each edge (i, j), the capacity which is the maximum amount of flow that is permissible through that edge equals one. In addition, let b_i denote the amount of supply at node i, such that $b_i < 0$ denotes demand at the node. For all nodes except the source and sink node we have $b_i = 0$. For the source and sink nodes, we have $b_s = |\mathcal{J}_{d,0}|$ and $b_t = -|\mathcal{J}_{d,0}|$, respectively. This implies that we wish to send a flow equal to $|\mathcal{J}_{d,0}|$ through the network from the source to the sink, given that no edge allows more than one unit of flow through it.

All nodes in the constellation network $\mathcal{G}_n$ are required to satisfy the usual flow balance equations

$$\sum_{j:\ (i,j)\in\mathcal{E}_n} x_{ij} - \sum_{j:\ (j,i)\in\mathcal{E}_n} x_{ji} = b_i \quad \text{for all } i \in \mathcal{V}_n \tag{7}$$

However, our initial consideration $\mathcal{J}_a = \mathcal{J}_p = \mathcal{J}_r = \mathcal{J}$ requires the introduction of additional constraints. First, note that the orbital slots of the active satellites are the same as the available return orbital slots. Hence, if the flow passes through a node $i \in \mathcal{J}_a$, then the flow has to pass through the node $i \in \mathcal{J}_r$. Moreover, if the flow does not pass through the node $i \in \mathcal{J}_a$, no flow should then pass through $i \in \mathcal{J}_r$. This constraint can be written as

$$x_{si} = x_{it} \quad \text{for all } i \in \mathcal{J}_a = \mathcal{J}_r \tag{8}$$

Second, note that the satellite originally occupying the orbital slot ϕ_i cannot be simultaneously the active satellite and the passive satellite with respect to two different E-P2P maneuvers. Hence, the network should not allow two $s \to t$ flows, one that passes through node $i \in \mathcal{J}_a$ and the other that passes through $i \in \mathcal{J}_p$. This implies the following constraint

$$x_{sj} + \sum_{i:\ (i,j)\in\mathcal{E}_f} x_{ij} \le 1 \quad \text{for all } j \in \mathcal{J}_p \tag{9}$$

Finally, given the constellation network $\mathcal{G}_n$, we seek to find the minimum cost flow in this network by solving the following egalitarian peer-to-peer integer problem (EP2P-IP):

$$\text{(EP2P-IP):}\ \min \sum_{(i,j)\in\mathcal{E}_n} c_{ij} x_{ij} \tag{10}$$

subject to the constraints (7–9) and $x_{ij} \in \{0, 1\}$.

Note that in Fig. 4, the source sends a total flow equal to $|\mathcal{J}_{d,0}|$ to the sink via the network. Because the capacity of each edge is unity, an integral flow in the network will be composed of $|\mathcal{J}_{d,0}|$ flows from s to t. The flow from the source reaches $|\mathcal{J}_{d,0}|$ nodes in $\mathcal{J}_a$. Some of these nodes correspond to active fuel-sufficient satellites, and the remaining to active fuel-deficient satellites. Because of the definition of edges $\mathcal{E}_f$, the flow emanating from nodes corresponding to active fuel-sufficient satellites reach an equal number of nodes (in $\mathcal{J}_p$) corresponding to the passive and fuel-deficient satellites. Therefore, the flow originating from the source reaches $|\mathcal{J}_{d,0}|$ fuel-deficient satellites. However, the constraint (9) ensures that all these nodes are distinct. Hence, the network ensures that the flow passes through the nodes corresponding to all $|\mathcal{J}_{d,0}|$ fuel-deficient satellites. In other words, all fuel-deficient satellites are involved in a fuel transaction during the E-P2P maneuvers represented by the optimal flow in the network as required by the problem statement.

Remark 1: Note that in the network flow formulation for the E-P2P problem, the supply or demand at each node representing an orbital slot of a satellite is zero. If we let $b_s = b_t = 0$, but add an arc (t, s) in the network $\mathcal{G}_n$ and impose a flow $|\mathcal{J}_{d,0}|$ through this arc from the sink to the source, then the problem remains unaltered. All nodes in the augmented network (see Fig. 5) now have zero demand/supply and the flow in the network has to be a circulation. We know that a

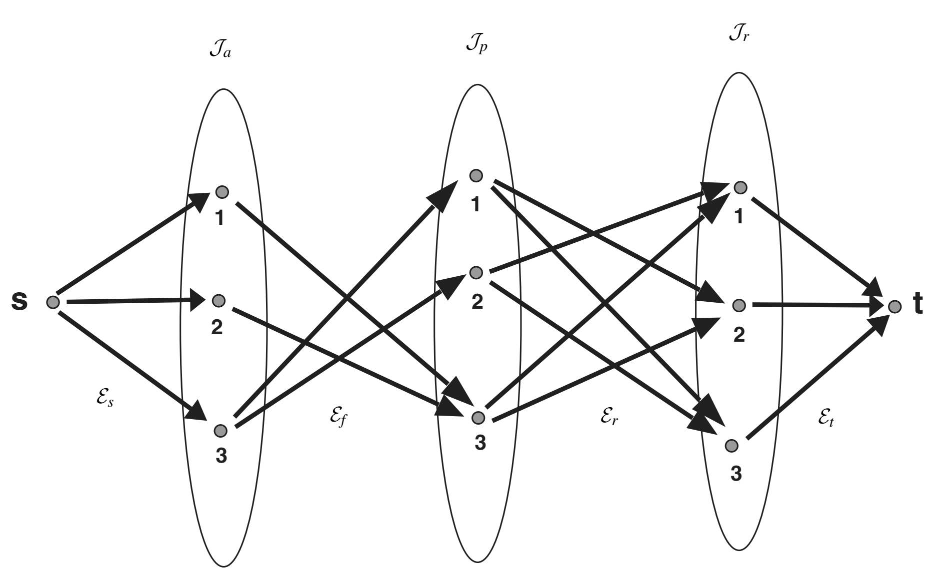


Fig. 4 Constellation flow network.

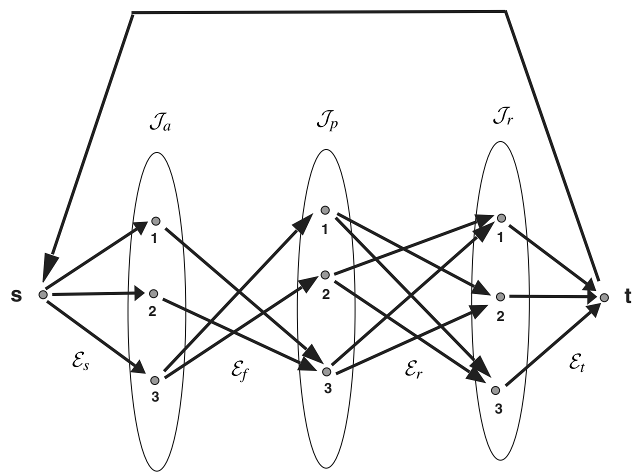


Fig. 5 Constellation flow network with an additional (t, s) arc.

circulation can always be decomposed into cycles [19]. Hence, the optimal cost flow should be in the form of cycles.

Let the solution obtained by solving the constrained optimization problem (EP2P-IP) be denoted by $\mathcal{M}_{\text{IP}}$. The corresponding fuel expenditure, calculated using Eqs. (1), (3), and (4), is therefore given by $\mathcal{C}(\mathcal{M}_{\text{IP}})$. Note that $\mathcal{M}_{\text{IP}}$ is, in general, suboptimal because, instead of minimizing the total fuel expenditure, we minimize the total ΔV. Next, we discuss a local search method to improve the solution $\mathcal{M}_{\text{IP}}$. To do this, we first define a neighborhood of the solution $\mathcal{M}_{\text{IP}}$ and then look for a cheaper solution (in terms of lower fuel expenditure) in that neighborhood.

E. Local Search About $\mathcal{M}_{\text{IP}}$

During a local search method, the feasible E-P2P solution $\mathcal{M}_{\text{IP}}$ is improved upon by searching its neighborhood for a better solution with lesser fuel expenditure than $\mathcal{C}(\mathcal{M}_{\text{IP}})$. If an improvement is detected, the solution is updated and a new neighborhood search is initialized about the new solution. The definition of the neighborhood $\mathcal{N}(\mathcal{M})$ of the feasible E-P2P solution $\mathcal{M}_{\text{IP}}$ is crucial for the performance of the local search. Here, we use the two-exchange neighborhood suggested in [17]. Note that $\mathcal{M}_{\text{IP}}$ consists of $|\mathcal{J}_{d,0}|$ triplets. For convenience, let us denote the triplet (i_ℓ, j_ℓ, k_ℓ) by t_ℓ. We can therefore represent $\mathcal{M}_{\text{IP}} = \{t_\ell\}$, where $\ell \in \{1, 2, \ldots, |\mathcal{J}_{d,0}|\}$. The difference between two triplets t_p and t_q is given by the set $\{r\colon t_{p,r} \neq t_{q,r}, r = 1, 2, 3\}$ and the distance between the triplets is defined by the cardinality of the difference. In other words,

$$d(t_p, t_q) = |\{r\colon t_{p,r} \neq t_{q,r}, r = 1, 2, 3\}| \tag{11}$$

Using Eq. (11), we can define the two-exchange neighborhood of the triplet pair $(t_p, t_q) \in \mathcal{M}_{\text{IP}} \times \mathcal{M}_{\text{IP}}$ as

$$N_2(t_p, t_q) = \{(t'_p, t'_q) \in \mathcal{T} \times \mathcal{T}\colon d(t_p, t'_p) + d(t_q, t'_q) = 2\} \tag{12}$$

The neighborhood of the solution $\mathcal{M}_{\text{IP}}$ consists of the union of all two-exchange neighborhoods of all possible triplet pairs $(t_p, t_q) \in \mathcal{M}_{\text{IP}}$, that is,

$$\mathcal{N}(\mathcal{M}_{\text{IP}}) = \bigcup_{(t_p,t_q)\in\mathcal{M}_{\text{IP}}} N_2(t_p, t_q) \tag{13}$$

During the local search phase, the cost of each $\mathcal{M}' \in \mathcal{N}(\mathcal{M}_{\text{IP}})$ is compared with the cost of $\mathcal{M}_{\text{IP}}$. If none of its neighbors have a lower cost, then $\mathcal{M}_{\text{IP}}$ is itself the local optimum. If a neighbor $\mathcal{M}'$ with a lower cost is found, then the current E-P2P solution $\mathcal{M}_{\text{IP}}$ is updated, and a new search in its neighborhood is initialized. The local search terminates when no neighbor of the current solution has a lower cost. We denote by $\mathcal{M}_H$ the final solution that is obtained by the application of the local search method on $\mathcal{M}_{\text{IP}}$.

IV. Bounds on the Optimal Fuel Expenditure

In this section, we provide a measure of the suboptimality of the solution $\mathcal{M}_H$ by deriving the bounds on the optimal fuel expenditure for E-P2P refueling. We show that the lower bound on the total fuel expenditure $\mathcal{C}(\mathcal{M}^*)$ can be obtained by solving a bipartite assignment problem. To this end, let us consider the bipartite graph $\mathcal{G}_\ell = \{\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}, \mathcal{E}_\ell\}$. There exists an (undirected) edge $\langle i, j\rangle$ between two nodes $i \in \mathcal{J}_{s,0}$ and $j \in \mathcal{J}_{d,0}$ if and only if the satellites s_μ and s_ν, occupying initially the orbital slots ϕ_i and ϕ_j, respectively, can engage in a feasible E-P2P maneuver. The set $\mathcal{E}_\ell$ of all such edges in the graph can therefore be given by $\mathcal{E}_\ell = \{\langle i, j\rangle\colon \text{there exists } k \in \mathcal{J} \text{ such that either } (i, j, k) \in \mathcal{T} \text{ or } (j, i, k) \in \mathcal{T}\}$. To each edge $\langle i, j\rangle$, we associate a cost c^ℓ_{ij} that takes into account the fuel expenditure during the forward transfer and the minimum fuel expenditure among all possible return costs. Therefore, if the fuel-sufficient satellite s_μ is active, then the fuel consumption for the related E-P2P maneuver (forward trip plus cheapest return trip) is given by

$$p^\mu_{ij} + \min_{k\in\mathcal{J}\setminus\{j\}} p^\mu_{jk}$$

On the other hand, if the fuel-deficient satellite s_ν is active, then the fuel consumption for the related E-P2P maneuver (forward trip plus cheapest return trip) is given by

$$p^\nu_{ji} + \min_{k\in\mathcal{J}\setminus\{i\}} p^\nu_{ik}$$

Therefore, the cost of the edge $\langle i, j\rangle \in \mathcal{E}_\ell$ is taken as

$$c^\ell_{ij} = \min\{p^\mu_{ij} + \min_{k\in\mathcal{J}\setminus\{j\}} p^\mu_{jk}, p^\nu_{ji} + \min_{k\in\mathcal{J}\setminus\{i\}} p^\nu_{ik}\} \tag{14}$$

Note that corresponding to each edge $\langle i, j\rangle \in \mathcal{E}_\ell$ we have an associated active satellite, a passive satellite, and a returning slot for the active satellite. Because this assignment corresponds to a feasible E-P2P maneuver, an edge $\langle i, j\rangle \in \mathcal{E}_\ell$ corresponds to a feasible triplet in $\mathcal{T}$.

We are interested in a subset $\mathcal{M}_\ell$ of $\mathcal{E}_\ell$ with $|\mathcal{J}_{d,0}|$ elements (edges), such that no two edges share the same nodes. This ensures that a satellite can be assigned to only one P2P maneuver. Let us associate with each edge $\langle i, j\rangle \in \mathcal{E}_\ell$ the binary variable x_{ij} given by

$$x_{ij} = \begin{cases} 1, & \text{if } x_{ij} \in \mathcal{M}_{\ell,} \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

We now define the following assignment problem–lower bound (AP-LB) optimization problem on $\mathcal{G}_\ell$:

$$\text{(AP-LB)}\colon \ \min \sum_{\langle i,j\rangle\in\mathcal{E}_\ell} c^\ell_{ij} x_{ij} \tag{16}$$

subject to

$$\sum_{j\colon \langle i,j\rangle\in\mathcal{E}_\ell} x_{ij} \leq 1 \quad \text{for all } i \in \mathcal{J}_{s,0} \tag{17}$$

$$\sum_{i\colon \langle i,j\rangle\in\mathcal{E}_\ell} x_{ij} = 1 \quad \text{for all } j \in \mathcal{J}_{d,0} \tag{18}$$

The constraint (17) implies that each fuel-sufficient satellite can be assigned to, at most, one fuel-deficient satellite, whereas the constraint (18) implies that each fuel-deficient satellite has to be assigned to a fuel-sufficient satellite. Let the optimal solution to the problem (AP-LB) be $\mathcal{M}^*_\ell$ and the optimal value of the objective given in Eq. (16) be denoted by $\mathcal{C}_{\text{LB}}$. We then have

$$\mathcal{C}_{\text{LB}} = \sum_{\langle i,j\rangle\in\mathcal{M}^*_\ell} c^\ell_{ij} \tag{19}$$

We now state the following theorem.

Theorem 1: The total fuel expenditure $\mathcal{C}(\mathcal{M}^*)$ corresponding to the optimal E-P2P solution $\mathcal{M}^*$ is bounded below by the optimal value $\mathcal{C}_{\text{LB}}$ of the objective function in the bipartite assignment problem (AP-LB). Moreover, $\mathcal{C}(\mathcal{M}^*)$ is bounded above by the optimal fuel expenditure $\mathcal{C}(\mathcal{M}_{\text{P2P}})$ obtained via P2P refueling. Therefore, $\mathcal{C}_{\text{LB}} \leq \mathcal{C}(\mathcal{M}^*) \leq \mathcal{C}(\mathcal{M}_{\text{P2P}})$.

The proof of this theorem is given in the Appendix.

Recall now that the edge $\langle q, r\rangle \in \mathcal{E}_\ell$ corresponds to a feasible triplet in $\mathcal{T}$. Hence, from the solution $\mathcal{M}_\ell^*$ of the optimization problem (AP-LB), we can construct a set $\mathcal{T}_\ell^*$ of $|\mathcal{J}_{d,0}|$ triplets. In general, this set of triplets $\mathcal{T}_\ell^*$ does not correspond to a feasible E-P2P solution because we may not necessarily know a priori that the indices corresponding to the orbital slots of the active satellite are the same as the indices of the orbital slots of the allowable return positions in the orbit. In other words, either more than one active satellite would compete for the same return position, or an active satellite would try to return to an orbital slot occupied by a passive satellite. In case the condition is met, the set of triplets $\mathcal{T}_\ell^*$ represents a feasible E-P2P solution. This observation, along with Theorem 1, leads to the following corollary.

Corollary 1: Let $\mathcal{T}_\ell^*$ be the set of triplets obtained from the solution $\mathcal{M}_\ell^*$ of the optimization problem (AP-LB). If the indices of $\mathcal{T}_\ell^*$ that correspond to the orbital slots of the active satellite are the same as the indices of the orbital slots of the allowable return positions in the orbit, then $\mathcal{T}_\ell^*$ is the globally optimal solution for the E-P2P problem.

In case the condition of Corollary 1 is not met, the set of triplets $\mathcal{T}_\ell^*$ do not correspond to a feasible E-P2P solution. We will use the lower bound given in Theorem 1 to estimate the level of suboptimality of the results obtained using the network flow formulation.

The fuel expenditure associated with the E-P2P solution is given by $\mathcal{C}(\mathcal{M}_H)$. Considering the bounds given by Theorem 1, we obtain an estimate of suboptimality of these results. Specifically, we may define the maximum percentage of suboptimality of $\mathcal{M}_H$ by the following expression:

$$\eta = \frac{\mathcal{C}(\mathcal{M}_H) - \mathcal{C}_{\text{LB}}}{\mathcal{C}_{\text{LB}}} \times 100\% \tag{20}$$

Note that because the solution of the AP-LB problem may correspond to an infeasible assignment, η is a worst case (conservative) estimate of the suboptimality of $\mathcal{M}_H$. That is, we can guarantee that the solution is no worse than η, but it could also be better. In fact, there are indeed cases (as it is demonstrated in Example 3 below) in which the solution of the AP-LB does lead to a feasible solution. In this case, this bound is tight. In fact, if the AP-LB leads to a feasible solution, then this solution is globally optimal.

In the next section, we will use this figure of merit to compare the results obtained from our proposed numerical optimization methodology.

V. Numerical Examples

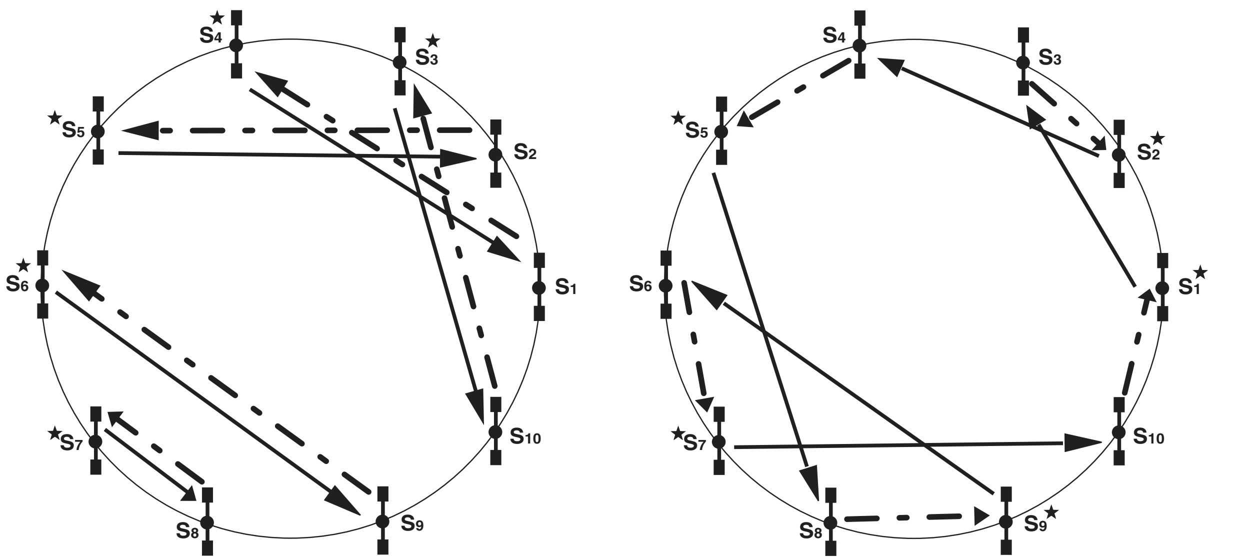
In this section, we apply our proposed method to determine the optimal assignments for E-P2P refueling of several sample constellations. These constellations are given in Table 1. To obtain the optimal assignments, the integer program (EP2P-IP) is solved using the binary integer programming solver `bintprog` of MATLAB. This solver uses branch-and-bound to solve integer programs. We also compare the results against the baseline P2P strategy, in which the active satellites are constrained to return to their original orbital slots. With the help of numerical examples, we show that the E-P2P refueling strategy leads to considerable reduction in the fuel expenditure. We also calculate the level of suboptimality of the results using Eq. (20).

Example 1: Consider the constellation C_1 given in Table 1. This constellation consists of 10 satellites evenly distributed in a circular orbit. The maximum allowed time for refueling is $T = 12$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 12$ units, whereas the maximum amount of fuel each satellite can hold is $\bar{f}_i = 30$ units. Each satellite has a permanent structure of $m_{si} = 70$ units, and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{J}_{s,0} = \{1, 2, 8, 9, 10\}$ and those of the fuel-deficient satellites are $\mathcal{J}_{d,0} = \{3, 4, 5, 6, 7\}$. For the baseline P2P refueling strategy, the optimal assignment is $s_4 \to s_1$, $s_5 \to s_2$, $s_7 \to s_8$, $s_6 \to s_9$, $s_3 \to s_{10}$, and the total fuel consumption for all P2P maneuvers is 26.07 units. This represents 14.48% of the total initial fuel in the constellation. The indices of the active satellites in this case are $\mathcal{J}_a = \{3, 4, 5, 6, 7\}$. Note that $\mathcal{J}_a = \mathcal{J}_{d,0}$, that is, the fuel-deficient satellites are the active ones for the baseline P2P refueling strategy. The baseline P2P assignment is shown in Fig. 6a. The active satellites are marked by a star. The forward trips are marked by solid arrows, and the return trips are marked by dotted arrows.

We now consider the case in which the active satellites are allowed to interchange their orbital slots. The lower bound on the fuel expenditure obtained by solving (AP-LB) is 17.05 units. The solution of (AP-LB) corresponds to the following assignment of satellites for E-P2P refueling: $s_4 \to s_1 \to s_2$, $s_3 \to s_2 \to s_3$, $s_5 \to s_8 \to s_9$, $s_6 \to s_9 \to s_{10}$, $s_7 \to s_{10} \to s_1$. Evidently, this does not correspond to a feasible E-P2P solution. The solution of (EP2P-IP) yields the following optimal assignment for E-P2P refueling: $s_1 \to s_3 \to s_2$, $s_2 \to s_4 \to s_5$, $s_5 \to s_8 \to s_9$, $s_7 \to s_{10} \to s_1$, $s_9 \to s_6 \to s_7$. The fuel expenditure during the E-P2P refueling process is 19.11 units, which is less than the fuel expenditure for the baseline P2P case. This represents 10.62% of the total initial fuel in the constellation. Figure 6b shows the optimal assignments for the E-P2P case. For the optimal assignment for the proposed E-P2P refueling strategy, it is observed that each active satellite, after undergoing a fuel transaction with the corresponding passive satellite, returns to an available orbital slot in the vicinity of the passive satellite with which it was involved in the transaction. For instance, satellite s_1 undergoes a fuel transaction with satellite s_3, and then returns to the orbital slot initially occupied by active satellite s_2. Moving to an orbital slot in the vicinity involves an orbital transfer through a smaller transfer angle, and thereby it likely results in a lesser fuel expenditure during the return trip. Hence, the active satellites, having the freedom to return to any available orbital slot, opt to move to a nearby slot during the return trip. In the baseline P2P strategy, such freedom is not available, and some of the active satellites have to perform orbital transfers that incur higher cost.

Table 1 Sample Constellations

Label	Description
C_1	10 satellites, altitude = 35,786 km, $T = 12$ f_i^-: 30, 30, 30, 6, 6, 6, 6, 30, 30, 30 $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{si} = 70$ for all satellites
C_2	16 satellites, altitude = 1200 km, $T = 30$ f_i^-: 30, 30, 30, 30, 30, 30, 10, 10, 10, 10, 10, 10, 10, 10, 30, 30 $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_3	12 satellites, altitude = 2000 km, $T = 30$ f_i^-: 30, 30, 30, 10, 10, 10, 10, 10, 10, 30, 30, 30 $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_4	18 satellites, altitude = 6000 km, $T = 25$ f_i^-: 25, 25, 25, 25, 25, 25, 25, 25, 25, 6, 6, 6, 6, 6, 6, 6, 6, 6 $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_5	12 satellites, altitude = 12,000 km, $T = 20$ f_i^-: 25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8 $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_6	14 satellites, altitude = 1400 km, $T = 35$ f_i^-: 25, 25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8, 8 $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_7	16 satellites, altitude = 30,000 km, $T = 15$ f_i^-: 10, 10, 10, 10, 10, 10, 10, 10, 28, 28, 28, 28, 28, 28, 28, 28 $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_8	16 satellites, altitude = 1200 km, $T = 30$ f_i^-: 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10 $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites



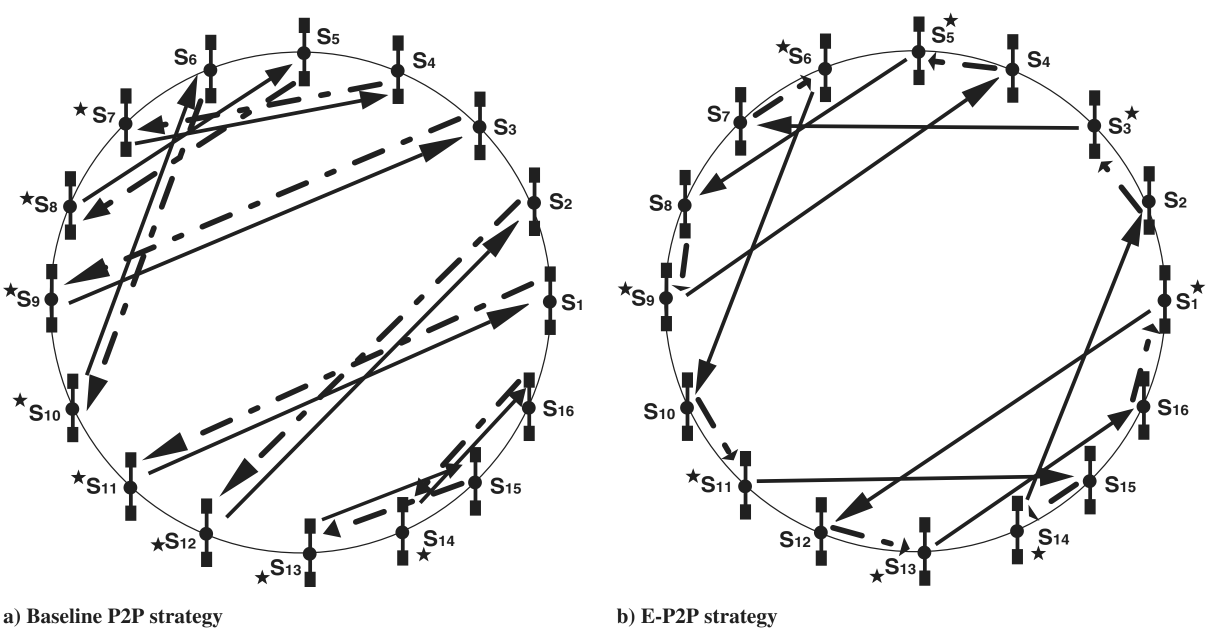
a) Baseline P2P strategy

b) E-P2P strategy

Fig. 6 Optimal assignments for constellation C_1.

Another observation is the fact that some of the active satellites are also fuel-sufficient. For instance, satellites s_1, s_2, and s_9 are fuel-sufficient and active. Figure 6b shows that the optimal solution comprises a Hamiltonian cycle $\{s_1 \to s_3 \to s_2 \to s_4 \to s_5 \to s_8 \to s_9 \to s_6 \to s_7 \to s_{10} \to s_1\}$ in the constellation.

Example 2: In this example, we consider the constellation C_2 given in Table 1. This is a constellation of 16 satellites, evenly distributed in a circular orbit. The maximum allowable time for refueling is $T = 30$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 15$ units, a maximum fuel capacity of $\bar{f}_i = 30$ units, a permanent structure of $m_{si} = 70$ units, and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{J}_{s,0} = \{1, 2, 3, 4, 5, 6, 15, 16\}$, whereas those of the fuel-deficient satellites are $\mathcal{J}_{d,0} = \{7, 8, 9, 10, 11, 12, 13, 14\}$. If the active satellites are constrained to return to their original orbital slots after refueling, the optimal assignment is $s_{11} \to s_1$, $s_{12} \to s_2$, $s_9 \to s_3$, $s_7 \to s_4$, $s_8 \to s_5$, $s_{10} \to s_6$, $s_{13} \to s_{15}$, $s_{14} \to s_{16}$, and the total fuel consumption is 37.46 units. This represents 11.71% of the total initial fuel in the constellation. In this case, only the fuel-deficient satellites are the active ones, that is, $\mathcal{J}_a = \{7, 8, 9, 10, 11, 12, 13, 14\} = \mathcal{J}_{d,0}$. This is similar to the previous example. The standard P2P assignment for C_2 is shown in Fig. 7a. We now consider the case when the active satellites are allowed to interchange their orbital slots. The lower bound on the fuel expenditure obtained by solving (AP-LB) is 22.63 units. The solution of (AP-LB) corresponds to the following assignment of satellites for E-P2P refueling: $s_{13} \to s_1 \to s_2$, $s_{14} \to s_2 \to s_3$, $s_{10} \to s_3 \to s_4$, $s_9 \to s_4 \to s_5$, $s_8 \to s_5 \to s_6$, $s_7 \to s_6 \to s_7$, $s_{11} \to s_{15} \to s_{16}, s_{12} \to s_{16} \to s_1$. Clearly, this does not correspond to a feasible E-P2P solution. The solution of (EP2P-IP) yields the following optimal assignment for E-P2P refueling: $s_1 \to s_{12} \to s_{13}$, $s_3 \to s_7 \to s_6$, $s_5 \to s_8 \to s_9$, $s_6 \to s_{10} \to s_{11}$, $s_9 \to s_4 \to s_5$, $s_{11} \to s_{15} \to s_{14}$, $s_{13} \to s_{16} \to s_1$, $s_{14} \to s_2 \to s_3$. Here, $\mathcal{J}_a = \{1, 3, 5, 6, 9, 11, 13, 14\}$. Relaxing the return orbital position constraint reduces the fuel expenditure to 24.82 units. This represents 7.76% of the total initial fuel in the constellation. Figure 7b shows the constellation and the optimal assignments for the E-P2P case. The active satellites are marked by a star. Similar to Example 1, it is



a) Baseline P2P strategy

b) E-P2P strategy

Fig. 7 Optimal assignments for constellation C_2.

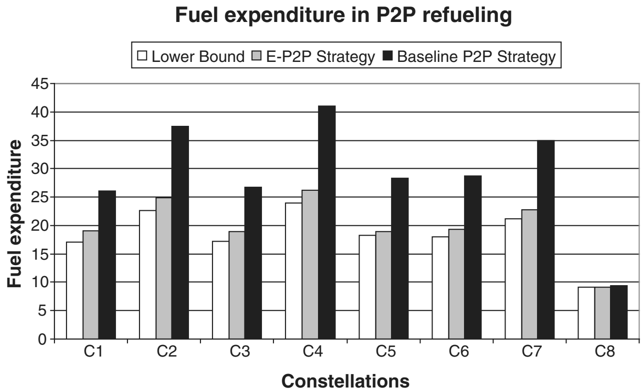


Fig. 8 Comparison of E-P2P and baseline P2P refueling strategies.

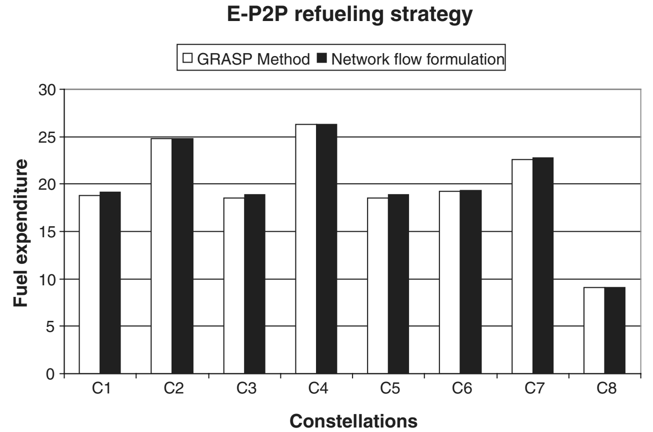


Fig. 9 Comparison with GRASP method.

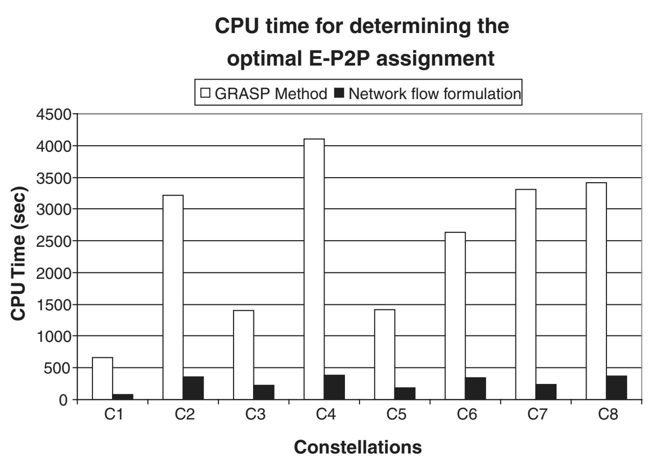


Fig. 10 Comparison of CPU time with GRASP method.

observed that the active satellites, after undergoing fuel transactions with the corresponding passive satellites, return to an available orbital slot in their vicinity. For instance, satellite s_1 undergoes a fuel transaction with satellite s_{12}, and then returns to the orbital slot occupied by active satellite s_{13}. Also note that the active satellites include fuel-sufficient ones. Here s_1, s_3, s_5, and s_6 are fuel-sufficient and active. Figure 7b shows that the optimal solution corresponds to three cycles in the constellation, namely, $\{s_1 \to s_{12} \to s_{13} \to s_{16} \to s_1\}$, $\{s_3 \to s_7 \to s_6 \to s_{10} \to s_{11} \to s_{15} \to s_{14} \to s_2 \to s_3\}$, and $\{s_5 \to s_8 \to s_9 \to s_4 \to s_5\}$.

We have also tested the proposed methodology on several other constellations as depicted in Table 1. The optimal assignments for these constellations show considerable reduction in fuel consumption against the baseline P2P strategy. For instance, for constellation C_3, the baseline P2P refueling strategy yields an optimal assignment $s_4 \to s_1, s_5 \to s_2, s_7 \to s_{10}, s_6 \to s_3, s_{11} \to s_8, s_9 \to s_{12}$, with a fuel expenditure of 26.73 units, with the fuel-deficient satellites being the active ones. Our proposed methodology yields the optimal assignment $s_1 \to s_4 \to s_5$, $s_3 \to s_6 \to s_7$, $s_5 \to s_2 \to s_3$, $s_7 \to s_{10} \to s_{11}$, $s_9 \to s_{12} \to s_1$, $s_{11} \to s_8 \to s_9$, which reduces the fuel expenditure to 18.87 units. The optimal solution consists of the Hamiltonian cycle $\{s_1 \to s_4 \to s_5 \to s_2 \to s_3 \to s_6 \to s_7 \to s_{10} \to s_{11} \to s_8 \to s_9 \to s_{12} \to s_1\}$. Similarly for the other constellations, the fuel expenditure reduces from 41.06 to 26.26 units in the case of C_4, from 28.38 to 18.86 in the case of C_5, from 28.77 to 19.26 units in the case of C_6, and from 34.97 to 22.75 units in the case of C_7. Figure 8 summarizes these results. The figure also shows the lower and upper bounds for the optimal fuel expenditure for E-P2P refueling.

As already mentioned, the E-P2P solution $\mathcal{M}_H$, determined by our methodology, is suboptimal. Using Eq. (20), we can have a measure of the suboptimality of our results. Table 2 summarizes the level of suboptimality for the sample constellations of Table 1. Apart from the suboptimality of $\mathcal{M}_H$, Table 2 also provides a measure of the suboptimality of the solution generated by the GRASP method. It should be noted here that the lower bound $\mathcal{C}_{\text{LB}}$ does not, in general, correspond to a feasible E-P2P solution, in which case there might exist a better (i.e., higher) lower bound.

Our final example is one in which the optimal solution (AP-LB) does indeed correspond to a feasible E-P2P solution.

Table 2 Suboptimality of results

Constellation	Suboptimality of $\mathcal{M}_H$	Suboptimality of GRASP solution
C_1	12.1%	10.05%
C_2	9.69%	9.69%
C_3	9.26%	7.17%
C_4	9.70%	9.70%
C_5	3.06%	1.09%
C_6	7.23%	7.22%
C_7	7.25%	6.36%
C_8	0.00%	0.00%

Example 3: In this example, we consider the constellation C_8 given in Table 1. This is a constellation of 16 satellites, evenly distributed in a circular orbit. The maximum allowable time for refueling is $T = 30$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 15$ units, a maximum fuel capacity of $\bar{f}_i = 30$ units, a permanent structure of $m_{si} = 70$ units, and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{J}_{s,0} = \{1, 3, 5, 7, 9, 11, 13, 15\}$, whereas those of the fuel-deficient satellites are $\mathcal{J}_{d,0} = \{2, 4, 6, 8, 10, 12, 14, 16\}$. The solution of (AP-LB) corresponds to the following assignment of satellites for E-P2P refueling: $s_{16} \to s_1 \to s_2$, $s_2 \to s_3 \to s_4$, $s_4 \to s_5 \to s_6$, $s_6 \to s_7 \to s_8$, $s_8 \to s_9 \to s_{10}$, $s_{10} \to s_{11} \to s_{12}$, $s_{12} \to s_{13} \to s_{14}$, $s_{14} \to s_{15} \to s_{16}$. Clearly, this is a feasible E-P2P solution and, by Corollary 1, this is the globally optimal E-P2P solution.

Finally, Fig. 9 provides a comparison of the results with those obtained using the GRASP method. As shown in this figure, the GRASP results are marginally better than those given by our current methodology. This is encouraging, given the fact that the network flow formulation minimizes total ΔV rather than actual fuel consumption. Nonetheless, as shown in Fig. 10, the network flow formulation introduced in this paper generates the solution much faster than the GRASP method.

VI. Conclusions

In this paper, we have studied the egalitarian peer-to-peer refueling strategy for satellite constellations, in which the active satellites are allowed to interchange orbital positions during their return trips. The problem is formulated as a minimum cost network flow problem that minimizes the total ΔV during the ensuing maneuvers. A local search method is described to improve the solution with respect to fuel expenditure incurred during the orbital maneuvers. We also provide bounds for the optimal fuel expenditure during E-P2P refueling. The lower bound provides a measure of suboptimality of the solution generated by our proposed methodology. With the help of numerical examples, it is shown that the proposed E-P2P strategy results in considerable reduction in the fuel expenditure incurred during the refueling process. It is also shown that each active satellite opts to return to an available orbital slot in the vicinity of the passive satellite with which it is involved in a fuel transaction. The proposed network flow formulation of the E-P2P refueling problem provides accurate solutions several orders of magnitude faster than other standard methods, such as GRASP, for solving three-index assignment problems.

Appendix: Proof of Theorem 1

The optimal E-P2P solution $\mathcal{M}^*$ consists of $|\mathcal{J}_{d,0}|$ triplets. Each triplet corresponds to a feasible E-P2P maneuver between a fuel-sufficient and a fuel-deficient satellite. Therefore, for each triplet $(i,j,k) \in \mathcal{M}^* \subseteq \mathcal{T}$, there exists an edge $\langle q,r\rangle \in \mathcal{E}_\ell$ such that satellites s_α and s_β, occupying the orbital slots ϕ_q and ϕ_r, respectively, correspond to the fuel-sufficient and fuel-deficient satellites for the triplet (i,j,k). Let us define the mapping $\mathcal{Q}\colon \mathcal{T} \mapsto \mathcal{E}_\ell$ that gives an edge in $\mathcal{E}_\ell$ for every triplet in $\mathcal{T}$. Note that the E-P2P solution $\mathcal{M}^*$ corresponds to $|\mathcal{J}_{d,0}|$ distinct fuel-sufficient and all $|\mathcal{J}_{d,0}|$ fuel-deficient satellites involved in refueling transactions. Let us now consider the following assignment in $\mathcal{G}_\ell$: $x_{qr} = 1$ for all $\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}^*)$ and 0 otherwise. For all the $|\mathcal{J}_{d,0}|$ fuel-sufficient satellites included in E-P2P solution $\mathcal{M}^*$

$$\sum_{r\colon \langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} = 1$$

whereas for the remaining $|\mathcal{J}_{s,0}| - |\mathcal{J}_{d,0}|$ fuel-sufficient satellites not included in any refueling transaction

$$\sum_{r\colon \langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} = 0$$

Combining the preceding two equations, we have

$$\sum_{r\colon \langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} \leq 1 \quad \text{for all } q \in \mathcal{J}_{s,0}$$

All the fuel-deficient satellites are included in the E-P2P solution and each of them engages in a refueling transaction with a distinct fuel-sufficient satellite. We therefore have

$$\sum_{q\colon \langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} = 1 \quad \text{for all } r \in \mathcal{J}_{d,0}$$

Hence, the optimal E-P2P solution $\mathcal{M}^*$ corresponds to a feasible solution $\mathcal{Q}(\mathcal{M}^*)$ for the optimization problem (AP-LB). Hence, we have

$$\sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}^*)} c^\ell_{qr} \geq \sum_{\langle q,r\rangle \in \mathcal{M}^*_\ell} c^\ell_{qr} \tag{A1}$$

Now, let us consider the fuel expenditure $\mathcal{C}(\mathcal{M}^*)$. Using Eq. (2), we have

$$\mathcal{C}(\mathcal{M}^*) = \sum_{(i,j,k)\in\mathcal{M}^*} (p^\mu_{ij} + p^\mu_{jk}) \geq \sum_{\langle i,j\rangle\colon (i,j,k)\in\mathcal{M}^*} \left(p^\mu_{ij} + \min_{k\in\mathcal{J}\setminus\{j\}} p^\mu_{jk}\right) \tag{A2}$$

Now, consider $\langle q,r\rangle = \mathcal{Q}(i,j,k)$. Also, let s_α and s_β occupy the orbital slots ϕ_q and ϕ_r, respectively. Further, note that we have two cases: either $q=i$, $r=j$, or $q=j$, $r=i$. In the first case, when the fuel-sufficient satellite is active, $\mu=\alpha$ and the right-hand side of the inequality in Eq. (A2) reduces to

$$\sum_{\langle i,j\rangle\colon (i,j,k)\in\mathcal{M}^*} \left(p^\mu_{ij} + \min_{k\in\mathcal{J}\setminus\{j\}} p^\mu_{jk}\right) = \sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}^*)} \left(p^\alpha_{qr} + \min_{k\in\mathcal{J}\setminus\{r\}} p^\alpha_{rk}\right) \tag{A3}$$

In the second case, when the fuel-deficient satellite is active, $\mu=\beta$ and the right-hand side of the inequality in Eq. (A2) reduces to

$$\sum_{\langle i,j\rangle\colon (i,j,k)\in\mathcal{M}^*} \left(p^\mu_{ij} + \min_{k\in\mathcal{J}\setminus\{j\}} p^\mu_{jk}\right) = \sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}^*)} \left(p^\beta_{rq} + \min_{k\in\mathcal{J}\setminus\{q\}} p^\beta_{qk}\right) \tag{A4}$$

Using Eqs. (A3) and (A4) and observing the definition of cost of edges in $\mathcal{E}_\ell$ given by Eq. (14), we have

$$\sum_{\langle i,j\rangle\colon (i,j,k)\in\mathcal{M}^*} \left(p^\mu_{ij} + \min_{k\in\mathcal{J}\setminus\{j\}} p^\mu_{jk}\right) \geq \sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}^*)} \min\left\{c^\alpha_{qr}, c^\beta_{qr}\right\} = \sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}^*)} c^\ell_{qr} \tag{A5}$$

Using Eq. (A5), we have from Eq. (A2) that

$$\mathcal{C}(\mathcal{M}^*) \geq \sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}^*)} c^\ell_{qr} \tag{A6}$$

Finally, comparing Eqs. (A1) and (A6), we have

$$\mathcal{C}(\mathcal{M}^*) \geq \mathcal{C}_{\mathrm{LB}} \tag{A7}$$

For the upper bound, recall that the P2P refueling is a special case of E-P2P and therefore the optimal P2P solution given by $\mathcal{M}_{\mathrm{P2P}}$ is a feasible E-P2P solution. Hence,

$$\mathcal{C}(\mathcal{M}^*) \leq \mathcal{C}(\mathcal{M}_{\mathrm{P2P}}) \tag{A8}$$

The inequalities (A7) and (A8) give the desired result.

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