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Network Flow Formulation for Cooperative Peer-to-Peer Refueling Strategies

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The problem of a general peer-to-peer refueling strategy for satellites in a circular constellation is addressed. The proposed cooperative egalitarian peer-to-peer strategy allows the satellites participating in a refueling transaction to engage in a cooperative rendezvous, that is, both satellites engaging in a fuel exchange may be active. Furthermore, the active satellites are allowed to interchange their orbital positions during their respective return trips. A mathematical framework to solve this general refueling problem for a large number of satellites is proposed using ideas from network flow theory. The methodology determines the optimal set of maneuvers that achieve fuel-sufficiency for all satellites, while expending the minimum possible fuel during the ensuing orbital transfers. With the help of numerical examples it is shown that the proposed cooperative egalitarian peer-to-peer strategy is the best amongst all known peer-to-peer refueling alternatives to date.

Nomenclature

$\mathcal{C}(\mathcal{M})$	=	cost of a set of feasible assignments $\mathcal{M}$
$\mathcal{C}_{\mathrm{LB}}$	=	optimal value of objective function of the optimization problem
c_{ij}^n	=	cost of an edge in $\mathcal{G}_n$
$c_{0\mu}$	=	characteristic constant for a satellite s_μ
$\mathcal{E}_\ell$	=	set of edges in $\mathcal{G}_\ell$
$\bar{f}_i$	=	maximum fuel capacity of satellite s_i
$\underline{f}_i$	=	minimum fuel requirement by satellite s_i to remain operational
f_i^-	=	initial fuel content of satellite s_i before refueling commences
f_i^+	=	final fuel content of satellite s_i after completion of refueling
$\mathcal{G}_n$	=	constellation network
$\mathcal{G}_\ell$	=	bipartite graph used for calculating lower bound on cost of optimal cooperative egalitarian peer-to-peer solution
g_μ^ν	=	amount of fuel delivered to satellite s_ν by satellite s_μ
$\mathcal{J}$	=	index set for orbital slots
$\mathcal{J}_a$	=	index set for orbital slots of active satellites
$\mathcal{J}_c$	=	index set for orbital slots where fuel exchanges take place
$\mathcal{J}_{d,t}$	=	index set for orbital slots of fuel-deficient satellites at time t
$\mathcal{J}_r$	=	index set of orbital slots available for active satellites to return
$\mathcal{J}_{s,t}$	=	index set for orbital slots of fuel-sufficient satellites at time t
$\mathcal{M}$	=	cooperative egalitarian peer-to-peer solution composed of a set of assignments
$\mathcal{M}^*$	=	optimal cooperative egalitarian peer-to-peer solution
$\mathcal{M}_c^*$	=	optimal cooperative peer-to-peer solution
$\mathcal{M}_e^*$	=	optimal egalitarian peer-to-peer solution
$\mathcal{M}_p^*$	=	optimal (baseline) peer-to-peer solution
$\mathcal{M}_{ce}^H$	=	cooperative egalitarian peer-to-peer solution yield by the optimization problem
$m_{s\mu}$	=	mass of permanent structure of satellite s_μ
n	=	number of satellites
n'	=	number of orbital slots
$\mathcal{P}$	=	set of feasible cooperative egalitarian peer-to-peer assignments in the constellation graph
p_{ij}^μ	=	fuel expenditure required for an orbital transfer by satellite s_μ from slot ϕ_i to slot ϕ_j
$\mathcal{Q}(\mathcal{M})$	=	edge in $\mathcal{E}_\ell$ corresponding to the set of assignments $\mathcal{M}$
s_i	=	satellite with index i
T	=	total time allotted for refueling
x_{ij}	=	binary variable corresponding to an edge
ΔV_{ij}	=	velocity change required for a transfer from slot ϕ_i to slot ϕ_j
η	=	suboptimality of cooperative egalitarian peer-to-peer solution
ϕ_i	=	orbital slot with index i

I. Introduction

On-orbit servicing (OOS) refers to work done in space by man, machine, or both. The primary objectives of such operations include assembly, maintenance and servicing [1]. Although the traditional practice in the space industry is to replace a spacecraft after its design lifetime, there have been instances when on-orbit servicing has proven to be beneficial. Servicing missions have been undertaken for the SkyLab Space Station, Solar Maximum Mission, the Russian Space Station, and, most significantly, for the Hubble Space Telescope [2–7]. Previous OOS studies can be broadly classified into the following categories: 1) studies on economic aspects [8–14], which look into the cost-effectiveness of servicing operations and the value of flexibility offered by OOS missions, 2) studies on enabling technologies [15–23], which look into the problem of automated rendezvous and capture, feasibility of fluid exchange in space, and robotic servicing, 3) studies on architecture and design [8,24–28], which look at the servicing architecture and design modifications for satellites to enable servicing, and 4) studies on servicing strategies [29–36], which look at optimal methods of servicing a system of multiple satellites. Our interest in this paper lies in the latter of the previously mentioned areas.

Refueling is considered to be a vital servicing operation, and several studies have tried to capture the benefits of satellite refueling [9,10,14,37–39], because the mission life of most satellites is driven by the amount of onboard fuel. Satellites need a regular fuel budget

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for station-keeping, and providing fuel-deficient satellites with propellant has significant benefits by extending their lifetime [38]. Apart from lifetime extension, provision of refueling capability would allow for satellites to be launched with less fuel. This may either mean reduced launch costs or additional revenue generation by dedicating the volume and mass (previously occupied by excess fuel) to additional payload [10,39]. Provision of refueling capabilities also allows for new missions. For instance, extremely low-altitude, high-drag orbits for Earth observation satellites would almost surely involve refueling [14]. Similarly, refueling operations would be essential for replenishing an operational chemical laser system [24,40]. Furthermore, refueling is considered to be the most acceptable (from a technological point of view) among all potential servicing operations. Firstly, the present industry practices allow fuel to be loaded in a satellite just before launch because of the high volatility and toxicity of fuel. This means that the fueling operation is not part of a satellite's integration process and hence enabling refueling operations would require minimal design change in existing satellites [14]. Secondly, refueling presents little risk, but provides immense gains because it is performed at the end-of-life of a satellite [9]. Hence, many studies on OOS have focused only on the refueling aspect of servicing operations.

OOS missions that have been undertaken to date considered the servicing of a monolithic spacecraft. However, a servicing operation for a space system, such as formation flying spacecraft, constellation clusters, or fractionated spacecraft may require several satellites to be serviced during a single mission. With the current focus of the space industry being on a system of multiple satellites, rather than a traditional monolithic spacecraft, there is a need to address the problem of servicing of such large systems. The problem of determining the best way of servicing multiple satellites is challenging, even for the simple case of a constellation of satellites moving in a circular orbit. Several studies looked at this problem for the refueling operation. The conventional notion of refueling multiple fuel-deficient satellites is to have a refueling spacecraft visit them one by one and impart fuel to them [41]. We will refer this as the single-service vehicle strategy (SSV). This strategy is depicted in Fig. 1a. However, the SSV strategy may not necessarily be the optimal way of delivering fuel to multiple satellites. It has been shown in [33,42] that, with an increasing number of satellites to be refueled, an alternative refueling strategy known as the mixed refueling strategy is better than a SSV strategy in terms of a lower amount of fuel expended during the refueling mission. During a mixed refueling strategy, an external refueling spacecraft, either launched from Earth or coming from a different orbit, replenishes part (perhaps half) of the satellites, and returns back to its original orbit. The satellites that receive fuel from the external refueling spacecraft distribute the fuel amongst the remaining satellites, by engaging in peer-to-peer (P2P) maneuvers. During a P2P maneuver, a fuel-sufficient and a fuel-deficient satellite engages in a refueling transaction, with one of them (the active satellite) initiating an orbital transfer to rendezvous with the other (the passive satellite). After a fuel exchange takes place, the active satellite returns back to its original position. The passive satellite remains in its original slot throughout the P2P maneuver. Figure 1b depicts this mixed refueling strategy.

Several studies have investigated possible ways of decreasing the fuel expended during the P2P phase of a mixed refueling strategy. These studies have shown that two extensions of the P2P refueling problem are particularly beneficial. One of these extensions is the egalitarian P2P (E-P2P) refueling strategy [36], in which the active satellites are allowed to interchange their orbital positions during their return trips. The basic assumption during such a strategy is that all satellites are identical, performing the same functions, and thereby can replace each other in orbit. The other extension of the P2P refueling problem allows for cooperative rendezvous between satellites engaging in a refueling transaction [43,44]. [43] assumed that all satellites participating in the refueling process are in the same circular orbit. [44] removed this assumption, by allowing the satellites to be in different circular orbits. The E-P2P and the C-P2P strategies can be combined into a single, general strategy of refueling involving cooperative egalitarian P2P (CE-P2P) maneuvers [45]. The CE-P2P refueling strategy is the most general of all P2P refueling strategies known to date.

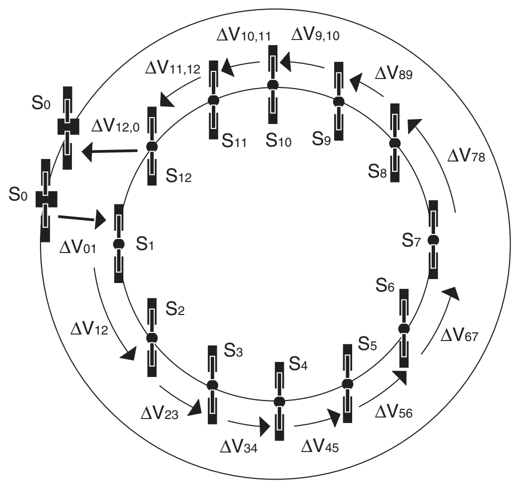
In the current paper the previously introduced baseline P2P and egalitarian P2P strategies are generalized by allowing additional flexibility. Namely, all satellites are now allowed to be active, resulting in potential cooperative rendezvous in intermediate orbital slots. This additional degree of freedom tends to bring the overall refueling expenditure down, at the cost of a more complex problem formulation. This was verified by our numerical investigations, which showed that the proposed CE-P2P refueling strategy is the best strategy when compared with all known P2P refueling alternatives to date. The solution process involves answering the following questions:

1) What is the optimal matching between satellites for a CE-P2P refueling? In other words, which satellites pair up for a refueling transaction?

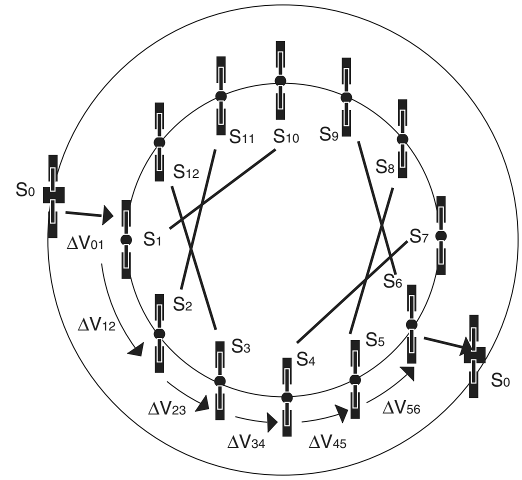
2) Where does the rendezvous for an optimal fuel exchange takes place?

3) Where do the active satellites return to?

It should be clear that the P2P refueling problem has high (combinatorial) complexity. For instance, many orbital transfer problems (in the order of thousands even for a constellation of a dozen satellites) need to be solved, just to set up the discrete optimization problem that yields the optimal set of P2P maneuvers. In [46] it is actually shown that the P2P refueling problem is



a) Single service vehicle strategy



b) Mixed strategy

Fig. 1 Refueling strategies.

nondeterministic polynomial-time hard. One can minimize the computations by assuming that each orbital transfer is composed of just two impulses. This simplification is quite reasonable for chemical propulsion systems, and it leads to slightly suboptimal solutions, which nonetheless can be computed much faster. This simplification is justified by the fact that the P2P refueling problem is a discrete optimization problem. Even if the costs associated with the decision variables are not numerically exact, the optimal matching between satellites most likely will not change. After the optimal matching satellite pairs have been established, one can still compute the actual fuel expenditures using more accurate (i.e., incorporating three or more impulses) optimal orbital transfers, if necessary. In that respect, the proposed problem formulation is very general and it even allows for the incorporation of nonchemical (e.g., continuous, low-thrust, etc.) propulsion systems. Similarly, although the discussion in the sequel will be restricted for simplicity to circular orbits, there is no significant impediment to the extension of the theory to more general orbits, to constellations distributed in several orbital planes, or to the case of multiple satellites flying in formation. The computations of the weights in the corresponding constellation graph (equivalently, the arc costs in the constellation network) will change of course, but other than that the solution framework proposed in this paper can also be applied mutatis mutandis to these more general cases with minimal changes.

II. Problem Formulation

In this section, the mathematical formulation of the general P2P refueling strategy that allows for CE-P2P maneuvers is discussed in detail. The basic notation is introduced and the description of the CE-P2P maneuvers is given. Next, the optimization problem required to determine the optimal set of CE-P2P maneuvers that achieves fuel-sufficiency in the constellation is outlined.

A. Notation

Consider a circular constellation consisting of n satellites, distributed over n orbital slots in a circular orbit. Let the set of n satellites be given by $\{s_i\colon i = 1, 2, \ldots, n\}$ and let the set of $n' \geq n$ slots in the circular orbit, given by the set $\{\phi_i \in [0, 2\pi)\colon i = 1, 2, \ldots, n', \phi_i \neq \phi_j\}$. Out of these n' slots, n are occupied by the satellites. For convenience, let $\mathcal{I} = \{1, 2, \ldots, n\}$ be the index set of the satellites, and $\mathcal{J} = \{1, 2, \ldots, n'\}$ be the index set of all the orbital slots. It is assumed that one has knowledge about the different characteristics of each satellite, mass, fuel, and engine. To this end, denote the initial fuel content of satellite s_i by f_i^- and the final fuel content at the end of the refueling process by f_i^+. Also, $\underline{f}_i$ will denote the minimum amount of fuel for the satellite s_i to remain operational. Fuel-sufficient satellites are those that have at least the required amount of fuel; the remaining satellites are fuel deficient.

The objective of P2P refueling is, therefore, to achieve $f_i^+ \geq \underline{f}_i$ for all $i \in \mathcal{I}$ by expending the minimum amount of fuel during the ensuing orbital transfers. For convenience, let $\mathcal{J}_{s,t}$ denote the index set of orbital slots occupied by fuel-sufficient satellites at time t, and let $\mathcal{J}_{d,t}$ denote the index set of orbital slots occupied by fuel-deficient satellites at time t. Also, let $\mathcal{J}_a \subseteq \mathcal{J}$ denote the index set of orbital slots occupied by the active satellites before any orbital maneuver commences, and let $\mathcal{J}_c \subseteq \mathcal{J}$ denote the set of slots where rendezvous takes place for the various refueling transactions. Then, the set of indices of the orbital slots of the passive satellites is given by $(\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}) \cap \mathcal{J}_c$. Finally, let $\mathcal{J}_r$ denote the set of indices of return slots. Note that the available return slots are the same as the slots initially occupied by the active satellites. It follows that $\mathcal{J}_r = \mathcal{J}_a$. Figure 2 illustrates these notations. The figure depicts eight satellites $s_1, s_2, \ldots, s_8$ in a circular orbit. Hence $\mathcal{I} = \{1, 2, \ldots, 8\}$. If one considers 16 orbital slots in the constellation, with the satellites occupying the slots $\phi_1, \phi_3, \ldots, \phi_{15}$, respectively, one has $n = 8$, $n' = 16$, $\mathcal{J} = \{1, 2, \ldots, 16\}$, $\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} = \{1, 3, \ldots, 15\}$. Figure 2 also shows a CE-P2P maneuver $(s_1, s_4) \to \phi_{12} \to (s_5, s_8)$, in which satellites s_1 and s_4 (initially occupying the orbital slots ϕ_1 and ϕ_7, respectively) rendezvous at the slot ϕ_{12}, and then return back to the slots ϕ_9 and ϕ_{15} originally occupied by the active satellites s_5 and s_8, respectively. Note that this means that satellites s_5 and s_8 also need to be active. Hence, 1, 7, 9, 15 $\in \mathcal{J}_a$, 12 $\in \mathcal{J}_c$, and 1, 7, 9, 15 $\in \mathcal{J}_r$.

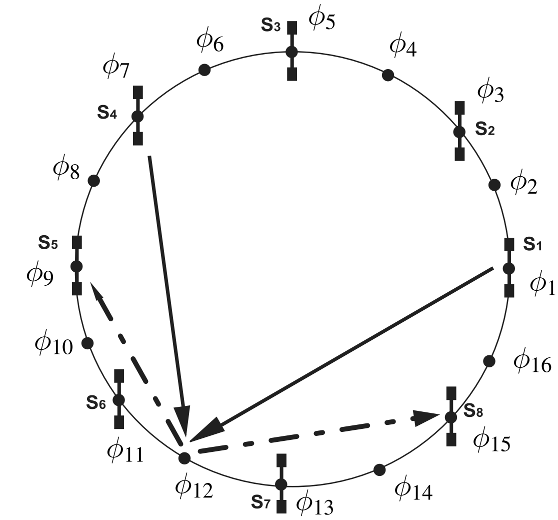


Fig. 2 The typical P2P refueling framework.

B. Problem Description

Consider a CE-P2P maneuver between two satellites s_μ and s_ν, originally occupying the orbital slots ϕ_{i_1} and ϕ_{i_2}, respectively. Without loss of generality, assume that s_μ is the fuel-sufficient satellite and s_ν is the fuel-deficient satellite. Let these satellites engage in a rendezvous at the orbital slot ϕ_j, where $j \in \mathcal{J}_c$. After the refueling transaction, the satellites s_μ and s_ν return to the orbital slots ϕ_{k_1} and ϕ_{k_2}, respectively, where $k_1, k_2 \in \mathcal{J}_r$. Given $i_1, i_2 \in \mathcal{J}_a$, $j \in \mathcal{J}_c$, and $k_1, k_2 \in \mathcal{J}_r$, let (i_1, i_2, j, k_1, k_2) represent an assignment for a CE-P2P maneuver. An assignment (i_1, i_2, j, k_1, k_2) is feasible if the satellites s_μ and s_ν engaging in the CE-P2P refueling transaction end up being fuel sufficient after the maneuver is complete. Let $\mathcal{P}$ denote the set of all feasible CE-P2P assignments in the constellation. As mentioned before, CE-P2P is a general P2P maneuver, and all other P2P maneuvers can be seen as special cases of the CE-P2P maneuver. In a CE-P2P maneuver, if only one of the two satellites is active, while the other satellite stays in its orbital slot throughout the refueling process, then one has an E-P2P maneuver (noncooperative). The set of all feasible E-P2P assignments in the constellation is therefore given by

$$\{(i_1, i_2, j, k_1, k_2) \in \mathcal{P}\colon i_1 = j = k_1\} \cup \{(i_1, i_2, j, k_1, k_2) \in \mathcal{P}\colon i_2 = j = k_2\} \tag{1}$$

Similarly, if the active satellites return back to their original orbital slots, then the maneuver is C-P2P (nonegalitarian). The set of all feasible C-P2P assignments in the constellation is given by

$$\{(i_1, i_2, j, k_1, k_2) \in \mathcal{P}\colon i_1 = k_1, i_2 = k_2\} \tag{2}$$

Moreover, if only one of the satellites is active, and it returns to its original orbital slot, then one has a P2P (baseline) maneuver. The set of all feasible P2P assignments in the constellation is therefore given by

$$\{(i_1, i_2, j, k_1, k_2) \in \mathcal{P}\colon i_1 = j = k_1, i_2 = k_2\} \cup \{(i_1, i_2, j, k_1, k_2) \in \mathcal{P}\colon i_2 = j = k_2, i_1 = k_1\} \tag{3}$$

The goal of a P2P refueling strategy is to expend minimum fuel during all maneuvers that are necessary to refuel all the fuel-deficient satellites. It is assumed that a fuel-sufficient satellite can refuel, at most, one fuel-deficient satellite, so that the total number of maneuvers required for the refueling process is $|\mathcal{J}_{d,0}|$. Let $\mathcal{M}_{ce} \subseteq \mathcal{P}$

denote the set of $|\mathcal{J}_{d,0}|$ feasible CE-P2P assignments such that all of the following conditions hold: 1) all fuel-deficient satellites are included in the assignments, 2) a satellite should engage in at most one refueling transaction, 3) the set of return positions are the same as the original slots occupied by the active satellites, 4) the orbital slots of the passive satellites cannot be the return positions for any of the active satellites, and 5) two different assignments cannot have the same slot for rendezvous. Henceforth we will refer to $\mathcal{M}_{ce}$ as a feasible CE-P2P solution. The cost of a CE-P2P solution is the total fuel expenditure incurred during all the orbital transfers.

Let p_{ij}^{μ} denote the fuel used by satellite s_μ during its transfer from the orbital slot ϕ_i to the slot ϕ_j. Therefore, the cost of the CE-P2P solution $\mathcal{M}_{ce}$ is given by

$$\mathcal{C}(\mathcal{M}_{ce}) = \sum_{(i_1,i_2,j,k_1,k_2)\in\mathcal{M}_{ce}} p_{i_1 j}^{\mu} + p_{i_2 j}^{\nu} + p_{jk_1}^{\mu} + p_{jk_2}^{\nu} \tag{4}$$

Furthermore, let $\mathcal{M}_{ce}^*$ denote the optimal set of assignments that minimizes the fuel expenditure during CE-P2P refueling. It follows that

$$\mathcal{C}(\mathcal{M}_{ce}^*) = \min_{\mathcal{M}_{ce}\subseteq\mathcal{P}} \mathcal{C}(\mathcal{M}_{ce}) \tag{5}$$

Similarly, let $\mathcal{M}_e^*$, $\mathcal{M}_c^*$, and $\mathcal{M}_p^*$ denote the optimal set of assignments for E-P2P, C-P2P, and (baseline) P2P refueling, respectively. Clearly, $\mathcal{M}_e^*$, $\mathcal{M}_c^*$, and $\mathcal{M}_p^*$ are subsets of the set of assignments given by (1–3), respectively.

C. Peer-to-Peer Maneuver Costs

Let us consider a CE-P2P maneuver (i_1, i_2, j, k_1, k_2). During the first phase of the maneuver, the two satellites s_μ and s_ν transfer to the orbital slot ϕ_j. The fuel consumed by the active satellite s_μ to transfer from the orbital slot ϕ_{i_1} to the orbital slot ϕ_j is given by

$$p_{i_1 j}^{\mu} = (m_{s\mu} + f_\mu^-)(1 - e^{-\frac{\Delta V_{i_1 j}}{c_{0\mu}}}) \tag{6}$$

where $m_{s\mu}$ denotes the mass of the permanent structure of the satellite s_μ, $c_{0\mu}$ denotes the characteristic constant for the satellite s_μ, and $\Delta V_{i_1 j}$ denotes the optimal velocity change required for the transfer from the slot ϕ_{i_1} to ϕ_j. The characteristic constant is defined by $c_{0\mu} = g_0 I_{s\mu}$, where g_0 denote the gravitational acceleration on the surface of the Earth, and $I_{s\mu}$ denote the specific thrust of the engine of the satellite s_μ. Similarly, the fuel expenditure for satellite s_ν to transfer from the orbital slot ϕ_{i_2} to the orbital slot ϕ_j is given by

$$p_{i_2 j}^{\nu} = (m_{s\nu} + f_\nu^-)(1 - e^{-\frac{\Delta V_{i_2 j}}{c_{0\nu}}}) \tag{7}$$

The fuel content of satellite s_μ after its forward trip (but before the fuel exchange takes place) is $f_\mu^- - p_{i_1 j}^{\mu}$, and that of satellite s_ν is $f_\nu^- - p_{i_2 j}^{\nu}$. The amount of fuel that s_μ delivers to s_ν is g_μ^ν. Hence, the fuel content of satellite s_μ just after the fuel exchange takes place is $f_\mu^- - p_{i_1 j}^{\mu} - g_\mu^\nu$, whereas that of satellite s_ν is $f_\nu^- - p_{i_2 j}^{\nu} + g_\mu^\nu$. After the fuel exchange, and in the second phase of the P2P maneuver, satellites s_μ and s_ν transfer to the orbital slots ϕ_{k_1} and ϕ_{k_2}, respectively. During the return trip, the fuel expenditure of satellite s_μ to transfer from slot ϕ_j to slot ϕ_{k_1} is given by

$$p_{jk_1}^{\mu} = (m_{s\mu} + f_\mu^- - p_{i_1 j}^{\mu} - g_\mu^\nu)(1 - e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}) \tag{8}$$

whereas that of satellite s_ν to transfer from slot ϕ_j to slot ϕ_{k_1} is given by

$$p_{jk_2}^{\nu} = (m_{s\nu} + f_\nu^- - p_{i_2 j}^{\nu} + g_\mu^\nu)(1 - e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}}) \tag{9}$$

The amount of fuel exchanged affects the return trip fuel expended by the active satellites. To keep the fuel expended during the maneuver to a minimum, there is an optimal amount of fuel that needs to be exchanged between each pair of satellites, as given by the following proposition (a detailed proof of which is given in the Appendix at the end of this paper).

Proposition 1: The amount of fuel exchanged between each pair of satellites engaged in a P2P refueling transaction is given by

$$g_\mu^\nu = \begin{cases} (m_{s\nu} + \underline{f}_\nu)e^{\frac{\Delta V_{jk_2}}{c_{0\nu}}} - (m_{s\nu} + f_\nu^- - p_{i_2 j}^{\nu}), & \text{if } e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} < e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}, \\ (m_{s\mu} + f_\mu^- - p_{i_1 j}^{\mu}) - (m_{s\mu} + \underline{f}_\mu)e^{\frac{\Delta V_{jk_1}}{c_{0\mu}}}, & \text{if } e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} > e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}} \end{cases} \tag{10}$$

If

$$e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} = e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}$$

then

$$(m_{s\nu} + \underline{f}_\nu)e^{\frac{\Delta V_{jk_2}}{c_{0\nu}}} - (m_{s\nu} + f_\nu^- - p_{i_2 j}^{\nu}) \le g_\mu^\nu \le (m_{s\mu} + f_\mu^- - p_{i_1 j}^{\mu}) - (m_{s\mu} + \underline{f}_\mu)e^{\frac{\Delta V_{jk_1}}{c_{0\mu}}} \tag{11}$$

To determine the final fuel content of the satellites when the fuel exchange is optimal, one needs to consider two cases. If

$$e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} < e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}$$

then $f_\nu^+ = \underline{f}_\nu$, which implies that s_ν returns with just enough fuel to be fuel sufficient. On the other hand, if

$$e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} > e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}$$

then $f_\mu^+ = \underline{f}_\mu$, which implies that s_μ returns with the enough fuel to be fuel sufficient. Note that if both satellites have the same engine characteristics, then $c_{0\mu} = c_{0\nu}$, and

$$e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} < e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}$$

or

$$\frac{\Delta V_{jk_2}}{c_{0\nu}} > \frac{\Delta V_{jk_1}}{c_{0\mu}}$$

and hence, $\Delta V_{jk_2} > \Delta V_{jk_1}$. Similarly

$$e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} > e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}$$

implies that $\Delta V_{jk_2} > \Delta V_{jk_1}$. Hence, the following corollary holds.

Corollary 1: If two satellites engaging in a cooperative P2P maneuver (egalitarian or nonegalitarian) have engines with the same specific impulse, then the satellite making the higher ΔV transfer returns with just enough fuel to be fuel sufficient.

D. Constellation Digraph

One can represent a CE-P2P maneuver using a directed graph. To this end, let us define a constellation graph consisting of three partitions $\mathcal{J}_a$, $\mathcal{J}_c$, and $\mathcal{J}_r$. However, it is not known a priori which satellites are active, which are passive, and which slots are used for cooperative rendezvous. That is, the sets $\mathcal{J}_a$, $\mathcal{J}_c$, and $\mathcal{J}_r$ are not known a priori. Hence, let $\mathcal{J}_a = \mathcal{J}_r = \mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}$ and $\mathcal{J}_c = \mathcal{J}$. An orbital transfer will be denoted using a directed edge, with the direction of edge signifying the direction of the orbital transfer. Let an edge (i, j), where $i \in \mathcal{J}_a$ and $j \in \mathcal{J}_c$, denote a forward trip from the slot ϕ_i to the slot ϕ_j, and let the associated cost for this transfer be denoted by c_{ij}^n. Let an edge (j, k), where $j \in \mathcal{J}_c$ and $k \in \mathcal{J}_r$, denote a return trip from the slot ϕ_j to ϕ_k, and let the associated cost for this transfer be denoted by c_{jk}^n. A set of edges (i_1, j), (i_2, j), (j, k_1), and (j, k_2) represents a CE-P2P maneuver.

Figure 3 depicts a constellation digraph, with the four directed edges corresponding to a CE-P2P maneuver. The two edges between the partitions $\mathcal{J}_a$ and $\mathcal{J}_c$ correspond to forward trips of the active satellites, and the edges between $\mathcal{J}_c$ and $\mathcal{J}_a$ correspond to their

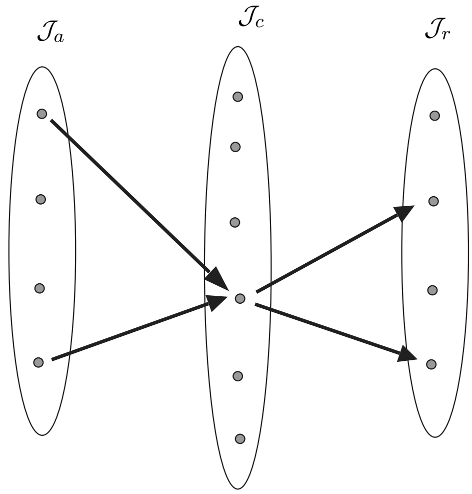


Fig. 3 Directed constellation graph.

return trips. Note that any edge (i, j) having $\phi_i = \phi_j$ does not represent a physical transfer, because this would mean that the active satellite occupies the same orbital slot during its forward/return trip. Naturally, the cost associated with such an edge is zero. Hence, if $\phi_{i_1} = \phi_j$ or $\phi_{i_2} = \phi_j$, then the maneuver is actually noncooperative, because one of the satellites involved in the refueling transaction remains in its orbital slot throughout the maneuver. In other words, the previous representation of a CE-P2P maneuver allows a (noncooperative) E-P2P maneuver to be treated as a special case of a CE-P2P maneuver in which one forward edge and one return edge does not actually represent a maneuver, and each of these edges has a zero cost.

Ideally, the cost of the edges in the digraph has to be the fuel expenditure during the orbital transfers. However, the calculation of the fuel expenditure depends on the mass of the satellite performing the orbital transfer. Because one does not know a priori which satellites are going to pair up during the refueling transactions, the return trip fuel expenditure cannot be uniquely determined for the return trip edges of the constellation graph. Instead of the fuel expenditure, one can use the velocity change ΔV required for the corresponding orbital transfer because the ΔV can be uniquely determined for all edges. The minimization of ΔV would yield suboptimal results because the true objective is to minimize fuel expenditure. However, it was observed in the numerical simulations that solutions are only marginally suboptimal when minimizing ΔV. Furthermore, to avoid solutions in which a fuel-deficient satellite does not have enough fuel to complete the desired rendezvous, only those forward edges (i, j) are allowed for which $p_{ij}^{\mu} < f_{\mu}^{-}$, where s_μ is the satellite that transfers from the orbital slot ϕ_i to the slot ϕ_j.

E. Network Flow Formulation

This section proposes a network flow formulation for the solution of the CE-P2P problem. A constellation network $\mathcal{G}_n$ is set up using the constellation digraph. To this end, one adds a source node s and a sink node t to the constellation digraph. For all $i \in \mathcal{J}_a$, one also adds an arc (s, i) with associated cost $c_{si} = 0$. Let the set of these arcs be denoted by $\mathcal{E}_s$. Similarly, for all $k \in \mathcal{J}_r$, one adds an arc (k, t) with associated cost $c_{kt} = 0$. Let the set of these arcs be denoted by $\mathcal{E}_t$. Consider now two $s \to t$ flows in the network $\mathcal{G}_n$ that pass through the same node $j \in \mathcal{J}_c$. A pair of such flows $s \to i_1 \to j \to k_1 \to t$ and $s \to i_2 \to j \to k_2 \to t$ represents a CE-P2P maneuver (i_1, i_2, j, k_1, k_2). The total cost of the flows equal the total ΔV required for all the orbital transfers during a CE-P2P maneuver. One seeks $|\mathcal{J}_{d,0}|$ pairs of flows in the constellation network with minimum total cost, such that all flows also pass through all the fuel-deficient satellites in the constellation. Note that each assignment (i_1, i_2, j, k_1, k_2) in a CE-P2P solution $\mathcal{M}_{ce}$ corresponds to a set of edges (s, i_1), (s, i_2), (i_1, j), (i_2, j), (j, k_1), (j, k_2), (k_1, j), and (k_2, t) in $\mathcal{G}_n$, and vice versa. The total cost of these edges is, therefore, the total ΔV required for all the orbital transfers corresponding to the assignment (i_1, i_2, j, k_1, k_2). Let the set of edges in the network corresponding to all assignments in the CE-P2P solution $\mathcal{M}_{ce}$ be denoted by $\mathcal{M}$. Also, let the set of slots where the cooperative rendezvous takes place corresponding to the solution $\mathcal{M}_{ce}$ be given by $\mathcal{Y}$.

Corresponding to each edge (i, j), introduce now a flow variable x_{ij} defined by

$$x_{ij} = \begin{cases} 1, & \text{if } x_{ij} \in \mathcal{M}, \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Also, corresponding to each slot for cooperative rendezvous, introduce the decision variables y_j, as follows

$$y_j = \begin{cases} 1, & \text{if } j \in \mathcal{Y} \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

Because $|\mathcal{J}_{d,0}|$ CE-P2P maneuvers are needed to refuel all fuel-deficient satellites, the total flow that goes out of the source node is $2|\mathcal{J}_{d,0}|$, and the flow distributes itself into $|\mathcal{J}_{d,0}|$ fuel-sufficient satellites and $|\mathcal{J}_{d,0}|$ fuel-deficient satellites. Noting that the set of orbital slots initially occupied by the satellites is given by $\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}$, it follows that

$$\sum_{i \in \mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}} x_{si} = 2|\mathcal{J}_{d,0}| \tag{14}$$

and

$$\sum_{i \in \mathcal{J}_{s,0}} x_{si} = |\mathcal{J}_{d,0}| \tag{15}$$

An amount of flow equal to the flow originating from the source must be collected at the sink node, that is

$$\sum_{k \in \mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}} x_{kt} = 2|\mathcal{J}_{d,0}| \tag{16}$$

The flow balance equations at the nodes yield the following constraints:

$$x_{si} = \sum_{j \in \mathcal{J}_c} x_{ij}, \qquad \text{for all } i \in \mathcal{J}_a \tag{17}$$

$$x_{kt} = \sum_{j \in \mathcal{J}_c} x_{jk}, \qquad \text{for all } i \in \mathcal{J}_r \tag{18}$$

and

$$\sum_{i \in \mathcal{J}_a} x_{ij} = \sum_{k \in \mathcal{J}_r} x_{jk}, \qquad \text{for all } j \in \mathcal{J}_c \tag{19}$$

The orbital slots available for return are exactly the orbital slots for the active satellites. Hence

$$x_{si} = x_{it}, \quad \text{for all } i \in \mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} \tag{20}$$

The total number of slots for rendezvous is the total number of CE-P2P maneuvers, which in turn equals the number of fuel-deficient satellites in the constellation and thus

$$\sum_{j \in \mathcal{J}_c} y_j = |\mathcal{J}_{d,0}| \tag{21}$$

If a slot is selected for cooperative rendezvous, two satellites must transfer to that location (unless it is a noncooperative maneuver). Hence, the following constraint holds:

$$\sum_{i \in \mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}} x_{ij} = 2y_j, \quad \text{for all } j \in \mathcal{J}_c \tag{22}$$

The two satellites transferring to the slot ϕ_j must be a fuel-sufficient and a fuel-deficient satellite. In other words, at most one fuel-sufficient satellite ending up in the slot ϕ_j, that is,

$$\sum_{i\in\mathcal{J}_{s,0}} x_{ij} \le 1, \quad \text{for all } j \in \mathcal{J}_c \tag{23}$$

Given the decision variables defined in Eqs. (12) and (13), and the set of constraints given by Eqs. (14) and (23), one is required to minimize the total ΔV for the CE-P2P maneuvers, that is

$$(\mathrm{CE-P2P})\text{: } \min \sum_{(i,j)\in\mathcal{E}_n} c^n_{ij} x_{ij} \tag{24}$$

This cost function and the constraints represent a standard optimization problem involving binary decision variables, and any integer programming solver can be used to solve it. For the numerical examples in this paper, the MATLAB binary integer programming solver bintprog has been used to obtain the CE-P2P maneuvers. This solver is based on the branch and bounds method common in the field of integer programming.

III. Bounds on Optimal Fuel Expenditure

The set of CE-P2P maneuvers obtained by solving the optimization problem (CE-P2P) corresponds to the minimum total ΔV required for the orbital transfers taking place during refueling. Let this solution be denoted by $\mathcal{M}^H_{ce}$. The true objective should be to minimize fuel expenditure, and hence the solution $\mathcal{M}^H_{ce}$ is potentially suboptimal. In this section, we provide a measure of the suboptimality of the solution $\mathcal{M}^H_{ce}$ by deriving bounds on the optimal fuel expenditure for CE-P2P refueling. In particular, it is shown that a (conservative) lower bound on the total fuel expenditure $\mathcal{C}(\mathcal{M}^*_{ce})$ can be obtained by solving a bipartite assignment problem. To this end, consider the undirected bipartite graph $\mathcal{G}_\ell = \{\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}, \mathcal{E}_\ell\}$ (Fig. 4). A P2P maneuver between two satellites will be represented by an undirected edge in the graph $\mathcal{G}_\ell$. In particular, it is said that there exists an (undirected) edge $\langle i_1, i_2\rangle$ between two nodes $i_1 \in \mathcal{J}_{s,0}$ and $i_2 \in \mathcal{J}_{d,0}$ if and only if the satellites s_μ and s_ν, occupying initially the orbital slots ϕ_{i_1} and ϕ_{i_2}, respectively, can engage in a feasible CE-P2P maneuver. That is, the satellites can engage in a rendezvous at a slot ϕ_j, where $j \in \mathcal{J}'$, and return, respectively, to the orbital slots ϕ_{k_1} and ϕ_{k_2}. The set of all such edges in the graph is given by $\mathcal{E}_\ell = hi, ji$, such that there exists $j \in \mathcal{J}_\mathcal{C}, k_1, k_2 \in \mathcal{J}$, and $(i_1, i_2, j, k_1, k_2) \in \mathcal{P}$. To each edge $\langle i_1, i_2\rangle$, we associate a cost $c^\ell_{i_1 i_2}$ that takes into account the fuel expenditure during the forward and return trips of the satellites, among all possible slots for cooperative rendezvous and return positions. The minimum fuel consumption for all possible return slots corresponding to the cooperative rendezvous slot ϕ_j, where $j \in \mathcal{J}$ is given by

$$[p^\mu_{i_1 j} + p^\nu_{i_2 j} + \min_{\substack{k_1,k_2\in\mathcal{J}_{s,0}\cup\mathcal{J}_{d,0}\\ k_1\neq k_2}} (p^\mu_{jk_1} + p^\nu_{jk_2})] \tag{25}$$

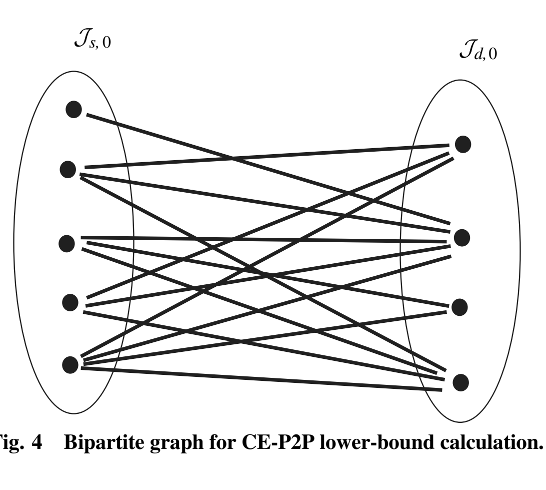


Fig. 4 Bipartite graph for CE-P2P lower-bound calculation.

Therefore, the cost of the edge $\langle i_1, i_2\rangle \in \mathcal{E}_\ell$ is taken as

$$c^\ell_{i_1 i_2} = \min_{j\in\mathcal{J}_c}[p^\mu_{i_1 j} + p^\nu_{i_2 j} + \min_{\substack{k_1,k_2\in\mathcal{J}_{s,0}\cup\mathcal{J}_{d,0}\\ k_1\neq k_2}} (p^\mu_{jk_1} + p^\nu_{jk_2})] \tag{26}$$

This expression represents the minimum possible fuel expenditure if the satellites s_μ and s_ν engage in a CE-P2P maneuver. The set of nodes and edges complete the description of the bipartite graph, shown in Fig. 4. Note the distinction between the two graphs depicted in Figs.3 and 4. Figure 3 depicts forward and return trips possible in the constellation, and Fig. 4 depicts possible pairs (matching) of satellites that can be involved in CE-P2P refueling transactions. The construction of the bipartite graph is crucial to the computation of bounds for the CE-P2P optimization problem. However, it cannot be used (except for a few special cases, as will be elaborated later) to compute solutions for the CE-P2P problem.

To ensure that a satellite can be assigned to only one CE-P2P maneuver it is necessary to work with a subset $\mathcal{M}_\ell$ of $\mathcal{E}_\ell$ with $|\mathcal{J}_{d,0}|$ edges, such that no two edges share the same node. To this end, associate with each edge $\langle i, j\rangle \in \mathcal{E}_\ell$ the binary variable x_{ij} given by

$$x_{ij} = \begin{cases} 1, & \text{if } x_{ij} \in \mathcal{M}_\ell \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

and define the following optimization problem on $\mathcal{G}_\ell$ (CE-P2P-LB stands for CE-P2P lower bound):

$$(\mathrm{CE-P2P-LB})\text{: } \min \sum_{\langle i,j\rangle\in\mathcal{E}_\ell} c^\ell_{ij} x_{ij} \tag{28}$$

subject to

$$\sum_{j\text{: } \langle i,j\rangle\in\mathcal{E}_\ell} x_{ij} \le 1 \text{ for all } i \in \mathcal{J}_{s,0} \tag{29}$$

$$\sum_{i\text{: } \langle i,j\rangle\in\mathcal{E}_\ell} x_{ij} = 1 \text{ for all } j \in \mathcal{J}_{d,0} \tag{30}$$

Constraint Eq. (29) implies that each fuel-sufficient satellite can be assigned to, at most, one fuel-deficient satellite, whereas the constraint Eq. (30) implies that each fuel-deficient satellite has to be assigned to a fuel-sufficient satellite. Let the optimal solution to the problem (CE-P2P-LB) be $\mathcal{M}^*_\ell$, and the optimal value of the objective given in Eq. (28) be denoted by $\mathcal{C}_{\mathrm{LB}}$. It follows that

$$\mathcal{C}_{\mathrm{LB}} = \sum_{\langle i,j\rangle\in\mathcal{M}^*_\ell} c^\ell_{ij} \tag{31}$$

One can now state the following theorem (a detailed proof of which is given in the Appendix at the end of this paper).

Theorem 1: The total fuel expenditure $\mathcal{C}(\mathcal{M}^*_{ce})$ corresponding to the optimal CE-P2P solution $\mathcal{M}^*_{ce}$ is bounded below by the optimal value $\mathcal{C}_{\mathrm{LB}}$ of the objective function in the bipartite assignment problem (CE-P2P-LB). Moreover, $\mathcal{C}(\mathcal{M}^*_{ce})$ is bounded above by the optimal fuel expenditure $\mathcal{C}(\mathcal{M}^*_e)$ obtained via E-P2P refueling or $\mathcal{C}(\mathcal{M}^*_c)$ obtained via C-P2P refueling, whichever is smaller. Therefore, $\mathcal{C}_{\mathrm{LB}} \le \mathcal{C}(\mathcal{M}^*_{ce}) \le \min\{\mathcal{C}(\mathcal{M}^*_e), \mathcal{C}(\mathcal{M}^*_c)\}$.

The fuel expenditure associated with the (CE-P2P) solution, obtained by solving the optimization problem (CE-P2P), is given by $\mathcal{C}(\mathcal{M}^H_{ce})$. Because $\mathcal{M}^H_{ce}$ might be a suboptimal solution, one has $\mathcal{C}(\mathcal{M}^H_{ce}) \ge \mathcal{C}(\mathcal{M}^*_{ce})$. Considering the bounds given by Theorem 1, one obtains an estimate of suboptimality of these results. Specifically, define the maximum percentage of suboptimality of $\mathcal{M}^H_{ce}$ by the following expression

$$\eta = \frac{\mathcal{C}(\mathcal{M}^H_{ce}) - \mathcal{C}_{\mathrm{LB}}}{\mathcal{C}_{\mathrm{LB}}} \times 100\% \tag{32}$$

Note that because the solution of the CE-P2P-LB problem may correspond to an infeasible CE-P2P solution, η is a worst-case (conservative) estimate of the suboptimality of $\mathcal{M}^H_{ce}$. One can thus guarantee that the solution is no worse than η, but it could also be better. In fact, there are cases in which the solution of the (CE-P2P-LB) does indeed lead to a feasible solution. In such cases, the solution is globally optimal. Such cases will be illustrated in the next section. Finally, it should be noted that in [36], the E-P2P fuel expenditure has an upper bound given by the optimal P2P (baseline) fuel expenditure. Note also from (1) and (2), that the set of (baseline) P2P assignments is a subset of the set of C-P2P assignments. Hence, the C-P2P fuel expenditure also has an upper bound given by the optimal P2P (baseline) fuel expenditure. Hence, the next corollary to Theorem 1 follows.

Corollary 2: $\mathcal{C}_{\mathrm{LB}} \leq \mathcal{C}(\mathcal{M}_{ce}^*) \leq \min\{\mathcal{C}(\mathcal{M}_e^*), \mathcal{C}(\mathcal{M}_c^*)\} \leq \mathcal{C}(\mathcal{M}_p^*)$.

This corollary provides a comparison of all the P2P refueling alternatives in terms of the fuel expended during the refueling process. The following section provides a comparison of the strategies via numerical examples.

IV. Numerical Examples

In this section numerical examples are presented that show the benefits of a cooperative refueling strategy for several satellite constellations. These constellations vary in the number of satellites, the mass and fuel content of the satellites, and the constellation orbit. The details of these constellations are given in Table 1.

Example 1: CE-P2P strategy for a constellation of 10 satellites.

Consider the constellation C_1 given in Table 1. This constellation consists of 10 satellites evenly distributed in a circular orbit. The initial fuel content of the satellites $s_1, s_2, \ldots, s_{10}$ are 30, 30, 6, 6, 6, 6, 6, 30, 30, 30 units, respectively. The maximum allowed time for refueling is $T = 12$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 12$ units, and the maximum amount of fuel for each satellite is $\bar{f}_i = 30$ units. Each satellite has a permanent structure of $m_{si} = 70$ units, and a characteristic constant of $c_0 = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 2, 8, 9, 10\}$ and those of the fuel-deficient satellites are $\mathcal{I}_{d,0} = \{3, 4, 5, 6, 7\}$. Let us consider a set of 20 evenly distributed slots, out of which 10 are occupied by the satellites. Hence $\mathcal{J} = \{1, 2, \ldots, 20\}$, and the satellites occupy the slots with indices $\{1, 3, \ldots 19\}$, respectively, that is, satellite s_i occupies the slot ϕ_{2i-1} for all $i \in \{1, 2, \ldots, 10\}$. An E-P2P strategy for this constellation yields the following optimal assignments: $s_1 \to s_3 \to s_2$, $s_2 \to s_4 \to s_5$, $s_5 \to s_8 \to s_9$, $s_7 \to s_{10} \to s_1$, $s_9 \to s_6 \to s_7$, where the assignment $s_1 \to s_3 \to s_2$ implies that the satellite s_1 undergoes an orbital transfer to rendezvous with s_3, exchanges fuel, and then returns to the orbital slot originally occupied by the satellite s_2. Figure 5a depicts these E-P2P maneuvers. The fuel expenditure during the E-P2P refueling process is 19.11 units.

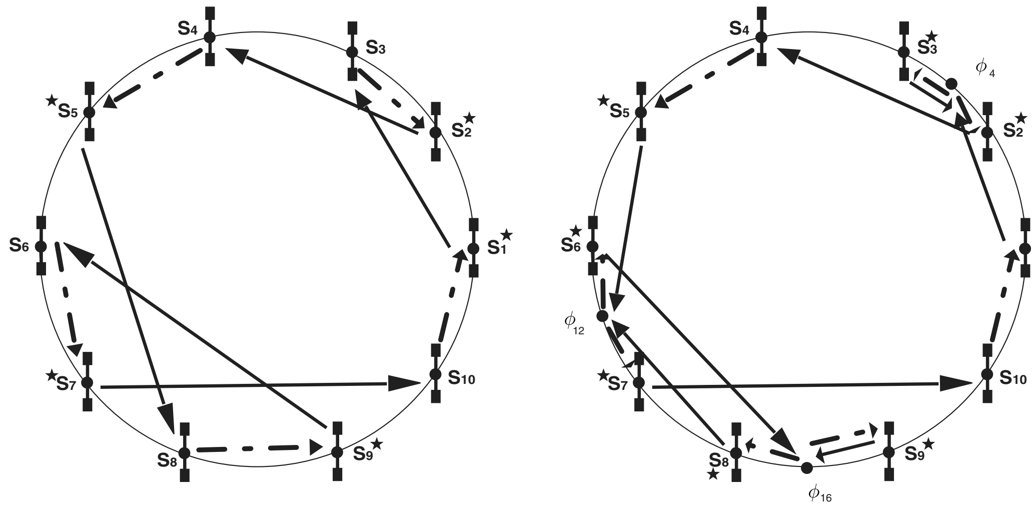
This represents 10.62% of the total initial fuel in the constellation. Figure 5a shows the optimal assignments for the E-P2P case. A C-P2P strategy for this constellation yields a higher fuel expenditure than the E-P2P case. Consider now a CE-P2P strategy for this constellation. First, recall the solution provided by the problem (CE-P2P-LB). The lower bound on CE-P2P is found to be $\mathcal{C}_{\mathrm{LB}} =$ 17.05 units. The corresponding optimal matching consists of the following satellite pairs: $s_1 \leftrightarrow s_4$, $s_2 \leftrightarrow s_3$, $s_8 \leftrightarrow s_5$, $s_9 \leftrightarrow s_6$, and $s_{10} \leftrightarrow s_7$ with their preferred slots for rendezvous being ϕ_1, ϕ_3, ϕ_{15}, ϕ_{17}, and ϕ_{19}, respectively. Note that in all of these matchings between the fuel-sufficient and fuel-deficient satellites, the fuel-deficient satellite performs a noncooperative rendezvous with the corresponding fuel-sufficient satellite. The preferred return locations for these active satellites are ϕ_3, ϕ_7, ϕ_{17}, ϕ_{19}, and ϕ_1, respectively. All these are slots adjacent to the corresponding rendezvous slot. Note that these slots are occupied by the passive satellites and it is not possible for all of the active satellites to return to their most preferred choice of orbital slots. Hence, the solution of (CE-P2P-LB) is not a feasible CE-P2P solution. The optimization problem (CE-P2P) yields the following assignments: $(s_1, s_3) \to \phi_4 \to (s_2, s_3)$, $s_2 \to s_4 \to s_5$, $(s_5, s_8) \to \phi_{12} \to (s_6, s_7)$, $(s_6, s_9) \to \phi_{16} \to (s_8, s_9)$, and $s_7 \to s_{10} \to s_1$. Figure 5b depicts this solution. Note that, similarly to the E-P2P case, all active satellites transfer to available slots in their vicinity during their return trips. The fuel expenditure during the cooperative E-P2P refueling process is 18.65 units, which represents 2.5% fuel savings over the E-P2P refueling strategy. This example demonstrates the utility of the CE-P2P refueling strategy in reducing the fuel expenditure incurred during a (noncooperative) E-P2P strategy or a (nonegalitarian) C-P2P strategy. The CE-P2P solution determined is potentially suboptimal. Comparing with the lower bound on the fuel expenditure it results in a value of $\eta = 9.38\%$. This means that the solution is at most 9.38% suboptimal. Furthermore, looking at the CE-P2P solution, it is found that two of the maneuvers are actually noncooperative E-P2P maneuvers. Satellites s_2, s_4, s_7, and s_{10} engage in (noncooperative) E-P2P maneuvers, whereas the remaining transactions are all cooperative. Hence, s_4 and s_{10} are the passive satellites for the CE-P2P refueling strategy, that is, they remain in their orbital slots throughout the refueling process.

Example 2: Global minimum in the case of a constellation of 16 satellites.

Consider the constellation C_3 in Table 1 consisting of 16 satellites, evenly distributed in a circular orbit. The fuel content of satellites $s_1, s_2, \ldots, s_{16}$ are 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, and 10, respectively. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 3, 5, 7, 9, 11, 13, 15\}$ and those of the fuel-deficient satellites are $\mathcal{I}_{d,0} = \{2, 4, 6, 8, 10, 12, 14, 16\}$. Let us consider a set of 32 orbital slots evenly distributed on the orbit, out of which 16 are initially occupied by the satellites. Hence, $\mathcal{J} = \{1, 2, \ldots, 32\}$. The satellites occupy the slots $\phi_1, \phi_3, \ldots \phi_{31}$, respectively, so that s_i occupies the orbital slot ϕ_{2i-1} for all $i \in \{1, 2, \ldots, 16\}$. By the solution of the (CE-P2P-LB), the lower bound on the CE-P2P fuel expenditure is computed to be $\mathcal{C}_{\mathrm{LB}} = 9.08$ units of fuel. The optimal matching yielded by (CE-P2P-LB) are the following satellites pairs: $s_1 \leftrightarrow s_{16}$, $s_2 \leftrightarrow s_3$, $s_4 \leftrightarrow s_5$, $s_6 \leftrightarrow s_7$, $s_{10} \leftrightarrow s_{11}$, $s_{12} \leftrightarrow s_{13}$, and $s_{14} \leftrightarrow s_{15}$. For all of these matchings, the fuel-deficient satellite performs a noncooperative rendezvous with the corresponding fuel-sufficient satellite, and returns to an orbital slot previously occupied by a different active satellite. Furthermore, the active satellites rendezvous with their preferred choice of fuel-sufficient satellite in their vicinity, and return to their preferred choice of orbital slots

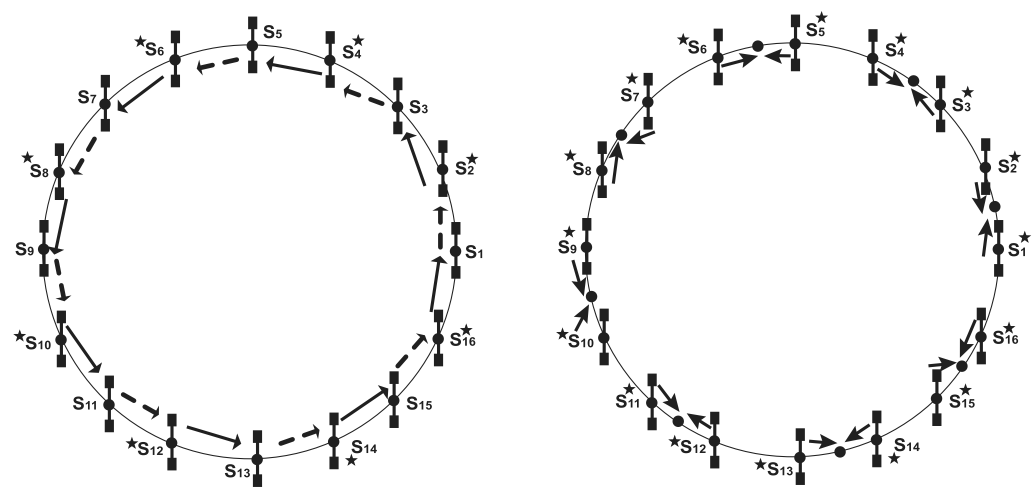
Table 1 Sample constellations

Label	Description
C_1	10 satellites, altitude =35, 786 Km, $T = 12$ f_i^-: 30, 30, 6, 6, 6, 6, 6, 30, 30, 30 $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{si} = 70$ for all satellites
C_2	16 satellites, Altitude = 1, 200 Km, $T = 30$ f_i^-: 30, 30, 30, 30, 30, 30, 10, 10, 10, 10, 10, 10, 10, 10, 30, 30 $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_3	16 satellites, Altitude Altitude = 1, 200 Km, $T = 30$ f_i^-: 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10 $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} =$ 70 for all satellites
C_4	16 satellites, Altitude = 1, 200 Km, $T = 30$ f_i^-: 30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4 $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{si} = 70$ for all satellites
C_5	12 satellites, Altitude = 1, 200 Km, $T = 20$ f_i^-: 25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8 $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_6	14 satellites, Altitude = 1, 400 Km, $T = 35$ f_i^-: 25, 25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8, 8 $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_7	14 satellites, Altitude = 30, 000 Km, $T = 15$ f_i^-: 1.2, 1.2, 1.2, 1.2, 1.2, 1.2, 1.2, 25, 25, 25, 25, 25, 25, 25 $\bar{f}_i = 25$, $\underline{f}_i = 10$, $m_{si} = 75$ for all satellites



a) E-P2P strategy **b) CE-P2P strategy**

Fig. 5 Optimal assignments.



a) Example 2: Fuel-deficient satellites can initiate non-cooperative rendezvous **b) Example 3: Fuel-deficient satellites cannot initiate non-cooperative rendezvous**

Fig. 6 Global minimum for a constellation of 16 satellites.

without any conflict. Thus, the solution of (CE-P2P-LB) yields a feasible, and hence the global optimum, CE-P2P solution.

Figure 6a depicts this global minimum. The solid arrows indicate the forward trip of the active satellites, and the broken arrows indicate their return trips. In particular, it is found that the global minimum is also the optimal (noncooperative) E-P2P solution. The (non-egalitarian) C-P2P solution has a higher fuel expenditure (10.34 units) in this case.

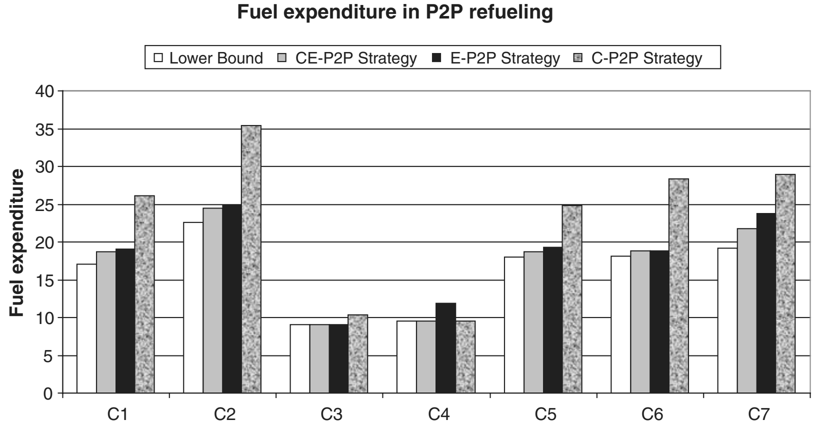


Fig. 7 Refueling expenditures.

Example 3: Fuel-deficient satellites have insufficient fuel to engage in noncooperative rendezvous.

Consider the constellation C_4 given in Table 1. This is similar to the constellation C_3, except that now the fuel-deficient satellites have a much smaller amount of fuel so that they cannot engage in a noncooperative rendezvous. If we solve (CE-P2P-LB), the optimal matching obtained is the following set of satellite pairs: $s_1 \leftrightarrow s_2$, $s_3 \leftrightarrow s_4$, $s_5 \leftrightarrow s_6$, $s_7 \leftrightarrow s_8$, $s_9 \leftrightarrow s_{10}$, $s_{11} \leftrightarrow s_{12}$, $s_{13} \leftrightarrow s_{14}$, and $s_{15} \leftrightarrow s_{16}$. The lower bound obtained is $C_{\mathrm{LB}} = 9.48$ units of fuel. In each of these assignments, the fuel-deficient satellite engages in a cooperative rendezvous with a neighboring fuel-sufficient satellite and, after undergoing a fuel exchange, returns to its original orbital slot. For each pair of active satellites engaging in a fuel exchange, the slot for cooperative rendezvous is midway between the original slots of the satellites. In fact, all fuel-deficient satellites rendezvous with their preferred choice of fuel-sufficient satellite and return to their preferred orbital slot, without any conflict. The solution of (CE-P2P-LB) is, therefore, a feasible CE-P2P solution and, hence, also the global optimal solution. Figure 6b depicts this optimal solution. The forward trips of the active satellites are shown by arrows. For each pair of satellites involved in a refueling transaction, the satellites meet midway between their orbital slots for a fuel exchange, and then return back to their original positions. The global minimum in this case is the optimal C-P2P solution. For this constellation, the (noncooperative) E-P2P solution has a higher fuel expenditure (11.85 units).

Figure 7 provides a comparison of the CE-P2P, E-P2P, and C-P2P refueling strategies for the constellations depicted in Table 1. The same figure also shows the lower bound given by the (CE-P2P-LB) solution for all constellations. In general, it is observed that the CE-P2P strategy provides an improvement over both the E-P2P and the C-P2P strategies.

V. Conclusions

In this paper general cooperative P2P refueling strategies for satellites in a circular constellation are studied. The proposed strategy incorporates the ideas of cooperative and egalitarian P2P maneuvers. A network flow formulation is proposed for determining the optimal set of P2P maneuvers in the constellation, and a lower bound on the optimal fuel expenditure during the refueling process is computed. The bound is determined by solving a bipartite assignment problem, the solution of which may or may not correspond to a feasible P2P solution. In case it does, one obtains the globally optimal solution. Otherwise, the bound helps in estimating the suboptimality of the solution obtained by the proposed methodology. Among all possible P2P refueling alternatives, the CE-P2P strategy is found to be the best in terms of fuel expended during the refueling process. It is shown that whenever two satellites engage in a cooperative rendezvous to exchange fuel, the one that makes the higher-ΔV transfer during the forward trip returns with just enough fuel to be fuel sufficient. If the fuel-deficient satellites have enough fuel to engage in noncooperative rendezvous, then these are the active satellites (because of their smaller mass, they consume lesser fuel). Otherwise, the satellites engage in cooperative rendezvous. As a consequence, cooperative maneuvers tend to be beneficial when the fuel-deficient satellites do not have enough fuel to perform a noncooperative rendezvous. In this case, the fuel-deficient satellite usually moves as close as possible to the fuel-sufficient satellites by using up its onboard fuel. Also, allowing for the active satellites to interchange their orbital positions during their return trips results in the active satellites performing smaller ΔV transfers to nearby available positions during their return trips. The CE-P2P refueling strategy has both these benefits and thereby provides the minimum fuel expenditure among all previously known P2P refueling strategies.

Appendix

I. Proof of Proposition 1

Proof: The final fuel content of satellite s_μ after the CE-P2P maneuver is given by $f_\mu^+ = f_\mu^- - p_{i_1 j}^\mu - g_\mu^\nu - p_{jk_1}^\mu$, whereas that of satellite s_ν is given by $f_\nu^+ = f_\nu^- - p_{i_2 j}^\nu + g_\mu^\nu - p_{jk_2}^\nu$. Using Eqs. (11) and (12), the total fuel content of the satellites at the end of the maneuver can be written as

$$f_\mu^+ + f_\nu^+ = (m_{s\mu} + f_\mu^- - p_{i_1 j}^\mu)e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}} - g_\mu^\nu e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}} + (m_{s\nu} + f_\nu^- - p_{i_2 j}^\nu)e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} + g_\mu^\nu e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} - (m_{s\mu} + m_{s\nu}) \quad \text{(A1)}$$

Clearly, minimizing the fuel expenditure during the C-P2P maneuver is the same as maximizing the total fuel content $f_\mu^+ + f_\nu^+$ of the satellites after the maneuver. From Eq. (A1), $f_\mu^+ + f_\nu^+$ is maximized when

$$g_\mu^\nu e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} - g_\mu^\nu e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}} = g_\mu^\nu \left(e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} - e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}\right)$$

is maximized. Recall that both satellites need to be fuel sufficient after the P2P maneuver. The satellite s_μ will be fuel-sufficient if

$$f_\mu^+ \geq \underline{f}_\mu$$

which yields

$$g_\mu^\nu \leq (m_{s\mu} + f_\mu^- - p_{i_1 j}^\mu) - (m_{s\mu} + \underline{f}_\mu)e^{\frac{\Delta V_{jk_1}}{c_{0\mu}}}$$

Also, the satellite s_ν will be fuel-sufficient if

$$f_\nu^+ \geq \underline{f}_\nu$$

which implies

$$g_\mu^\nu \geq (m_{s\nu} + \underline{f}_\nu)e^{\frac{\Delta V_{jk_2}}{c_{0\nu}}} - (m_{s\nu} + f_\nu^- - p_{i_2 j}^\nu)$$

The conditions of fuel sufficiency on the satellites provide us with a lower bound on the amount of fuel exchange given by

$$(m_{s\nu} + \underline{f}_\nu)e^{\frac{\Delta V_{jk_2}}{c_{0\nu}}} - (m_{s\nu} + f_\nu^- - p_{i_2 j}^\nu) \quad \text{(A2)}$$

and also provides an upper bound on the amount of fuel exchange given by

$$(m_{s\mu} + f_\mu^- - p_{ik}^\mu) - (m_{s\mu} + \underline{f}_\mu)e^{\frac{\Delta V_{i_1 j}}{c_{0\mu}}} \quad \text{(A3)}$$

To maximize

$$g_\mu^\nu \left(e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} - e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}\right)$$

and depending on the sign of the expression

$$\left(e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} - e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}\right)$$

g_μ^ν must attain its upper or lower bound to maximize the total final fuel of the satellites. If

$$e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} = e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}$$

g_μ^ν can assume any value in the interval defined by its bounds. □

II. Proof of Theorem 1

Proof: The optimal CE-P2P solution $\mathcal{M}_{ce}^*$ consists of $|\mathcal{J}_{d,0}|$ assignments. For an assignment given by $(i_1, i_2, j, k_1, k_2) \in \mathcal{M}_{ce}^*$, the satellites $s_\mu = \sigma_0(\phi_{i_1})$ and $s_\nu = \sigma_0(\phi_{i_2})$ represent the fuel-sufficient and fuel-deficient satellites, respectively. Because $\mathcal{M}_{ce}^* \subseteq \mathcal{P}$, s_μ and s_ν can engage in a feasible CE-P2P maneuver, which implies that the edge $\langle i_1, i_2 \rangle$ exists in $\mathcal{G}_\ell$. Define the mapping $\mathcal{Q}$: $\mathcal{P} \mapsto \mathcal{E}_\ell$ giving an edge in $\mathcal{E}_\ell$ for every assignment in $\mathcal{P}$. For instance, $\mathcal{Q}(i_1, i_2, j, k_1, k_2) = \langle i_1, i_2 \rangle$. Note that the CE-P2P solution $\mathcal{M}_{ce}^*$ corresponds to $|\mathcal{J}_{d,0}|$ distinct fuel sufficient and all $|\mathcal{J}_{d,0}|$ fuel-deficient satellites involved in refueling transactions (refer to

Eqs. (20) and (21)). Consider now the following assignment in $\mathcal{G}_\ell$: $x_{qr} = 1$ for all $\langle q, r\rangle \in \mathcal{Q}(\mathcal{M}^*)$ and 0 otherwise. For all the $|\mathcal{J}_{d,0}|$ fuel-sufficient satellites included in CE-P2P solution $\mathcal{M}^*_{ce}$

$$\sum_{r:\ \langle q,r\rangle\in\mathcal{E}_\ell} x_{qr} = 1$$

whereas for the remaining $|\mathcal{J}_{s,0}| - |\mathcal{J}_{d,0}|$ fuel-sufficient satellites not included in any refueling transaction

$$\sum_{r:\ \langle q,r\rangle\in\mathcal{E}_\ell} x_{qr} = 0$$

Combining the previous two equations, it follows

$$\sum_{r:\ \langle q,r\rangle\in\mathcal{E}_\ell} x_{qr} \le 1 \quad \text{for all}\ \ q \in \mathcal{J}_{s,0}$$

All the fuel-deficient satellites are included in the CE-P2P solution and each of them engages in a refueling transaction with a distinct fuel-sufficient satellite (refer to (14), (15), and (23)). Hence,

$$\sum_{q:\ \langle q,r\rangle\in\mathcal{E}_\ell} x_{qr} = 1 \quad \text{for all}\ \ r \in \mathcal{J}_{d,0}$$

The optimal CE-P2P solution $\mathcal{M}^*_{ce}$ corresponds to a feasible solution $\mathcal{Q}(\mathcal{M}^*_{ce})$ for the optimization problem (CE-P2P-LB). Hence

$$\sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}^*_{ce})} c^\ell_{qr} \ge \sum_{\langle q,r\rangle\in\mathcal{M}^*_\ell} c^\ell_{qr} \tag{A4}$$

Now, let us consider the fuel expenditure $\mathcal{C}(\mathcal{M}^*_{ce})$. It follows that

$$\begin{aligned}\mathcal{C}(\mathcal{M}^*_{ce}) &= \sum_{(i_1,i_2,j,k_1,k_2)\in\mathcal{M}^*_{ce}} p^\mu_{i_1 j} + p^\nu_{i_2 j} + (p^\mu_{jk_1} + p^\mu_{jk_2})\\ &\ge \sum_{\{i_1,i_2,j\}:\ (i_1,i_2,j,k_1,k_2)\in\mathcal{M}^*_{ce}} [p^\mu_{i_1 j} + p^\nu_{i_2 j} + \min_{\substack{k_1,k_2\in\mathcal{J}_{s,0}\cup\mathcal{J}_{d,0}\\ k_1\ne k_2}} (p^\mu_{jk_1} + p^\mu_{jk_2})]\\ &\ge \sum_{\{i_1,i_2\}:\ (i_1,i_2,j,k_1,k_2)\in\mathcal{M}^*_{ce}} [\min_{j\in\mathcal{J}_c}(p^\mu_{i_1 j} + p^\nu_{i_2 j} + \min_{\substack{k_1,k_2\in\mathcal{J}_{s,0}\cup\mathcal{J}_{d,0}\\ k_1\ne k_2}} (p^\mu_{jk_1} + p^\mu_{jk_2}))]\end{aligned} \tag{A5}$$

Using Eq. (26), and by virtue of Eq. (A5), it follows that

$$\mathcal{C}(\mathcal{M}^*_{ce}) \ge \sum_{\langle q,r\rangle\in\mathcal{Q}(\mathcal{M}^*_{ce})} c^\ell_{qr} \tag{A6}$$

Finally, comparing Eqs. (A4) and (A6), yields

$$\mathcal{C}(\mathcal{M}_{ce}) \ge \mathcal{C}_{\text{LB}} \tag{A7}$$

For the upper bound, recall that the set of E-P2P [given by Eq. (1)] and C-P2P maneuvers [given by Eq. (2)] are subsets of the set $\mathcal{P}$ of CE-P2P maneuvers. Hence

$$\mathcal{C}(\mathcal{M}^*_{ce}) \le \mathcal{C}(\mathcal{M}_c) \quad \text{and} \quad \mathcal{C}(\mathcal{M}^*_{ce}) \le \mathcal{C}(\mathcal{M}_e) \tag{A8}$$

The inequalities Eqs. (A7) and (A8) give the desired result.

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