Peer-to-Peer Refueling for Circular Satellite Constellations

Haijun Shen* and Panagiotis Tsiotras†

In this paper, we study the scheduling problem arising from refueling multiple satellites in a circular constellation. It is assumed that there is no fuel delivered to the constellation from an external source. Instead, all satellites in the constellation are assumed to be capable of refueling each other (peer-to-peer refueling). The total time to complete the rendezvous maneuvers including the refueling itself is specified. During the refueling period each satellite may conduct a fuel exchange with at most one other satellite. Whenever two satellites perform a fuel transfer only one is active, that is, only one of the two satellites initiates an orbital transfer to rendezvous with the inactive satellite. After the transfer of fuel is completed, the active satellite returns to its original orbital slot. The goal is to equalize the fuel among the satellites in the constellation after one refueling period, while minimizing the total fuel consumed during the orbital transfers. It is shown that this problem can be formulated as a maximum-weight matching problem on the reduced constellation graph, which can be solved using standard numerical methods. Numerical results indicate the benefits of a P2P strategy over a single-spacecraft refueling strategy for constellations with a large number of satellites.

Introduction

The current practice when the fuel on-board a satellite is exhausted is to simply replace the satellite with a new one. As a result, the lifespan of a satellite is limited – typically to a few years – depending on its initial amount of fuel. Replacing old satellites with new ones incurs significant cost in the production and launching of satellites, not to mention the addition of space debris. An alternative to replacing a satellite when its fuel is depleted, is to create a satellite architecture having the capability of refueling the satellites when needed. Under this concept, a satellite may be refueled after it runs low on fuel, thus extending its operation. Satellites in a constellation can be refueled either from a vehicle launched from the Earth for that purpose, or from other satellites in the constellation.

During the past two decades, both NASA and the DOD, as well as some individual companies and organizations have conducted numerous studies of servicing and refueling in space.[1–9] These studies have shown that fluid resupply of on-orbit spacecraft is feasible and would allow for extended spacecraft utilization.

Although refueling capability is currently not part of the standard spacecraft operational requirements, the technology required for spacecraft refueling on orbit is quickly becoming available.[5,6,10–12] In fact, it is envisioned that in the near future satellite refueling (or servicing for that matter) will become routine.[13] Several studies by NASA as well as private industry have shown the economic and operational benefits of satellite servicing, refurbishing and refueling.[14,15] By the same token, decentralized optimal refueling algorithms will require satellites with significantly more autonomy and decision-making capabilities than current ones in order to implement refueling algorithms with minimal ground intervention. The recent trend to increase onboard processing power and autonomy of orbiting satellites[16,17] seems therefore to go hand-in-hand with the approach proposed in this paper. Finally, we emphasize the fact that propellant resupply in space not only extends the life of a satellite, but it can also be used to enhance the payload capability of an orbital transfer vehicle. As shown in Ref. 18, for instance, a manned mission to Mars can benefit from the periodic refueling of the manned ship from fuel tankers launched in advance for that purpose. Similarly, the operational capabilities of a lunar mission can be enhanced manyfold by refueling the transfer vehicle in LEO via an orbiting fuel tanker, before it sets off for its journey to the Moon.[19,20]

Much of the previous work on satellite refueling has been limited to hardware design and/or feasibility

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[*] Senior Project Engineer, Analytical Mechanics Associates, Inc., Hampton, VA. Tel: (757) 865-0944, ext. 203. Fax: (757) 865-1881. Email: `shen@ama-inc.com`. Member AIAA.

[†] Professor, School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA. Tel: (404) 894-9526, Fax: (404) 894-2760, Email: `p.tsiotras@ae.gatech.edu`. Associate Fellow AIAA.

studies for transferring liquid in space. A brief overall conceptual study of this topic can be found in Ref. 12. Only recently the scheduling problem arising from refueling multiple satellites has received attention. Shen and Tsiotras in Ref. 21 studied the optimal scheduling strategy for refueling or servicing multiple satellites in a circular orbit using a single servicing spacecraft. Integer programming was proposed in Ref. 21 to obtain the best schedule of refueling the satellites in a given order. A heuristic study suggested that the best sequence to visit all satellites can be chosen from the sequences that assume the minimum of the so-called total sweep angle.[21] On a slightly different note, in Ref. 22, the authors considered the optimal scheduling problem for servicing multiple satellites in a geosynchronous orbit by a single service vehicle. The orbits of the satellites are assumed to have small inclinations. The problem was to find the order of satellite visits such that the total fuel consumption is minimized. In Ref. 22 the total time for the transfers was assumed to be sufficiently long, so that the fuel consumption of in-plane maneuvers is negligible. In such a case the fuel consumption is mainly due to the plane changes. It is shown that owing to the small inclination, the fuel consumption is proportional to the distance between the projections on the equatorial plane of the angular momentum vectors of the orbits of the two satellites. Thus, the minimum-fuel ordering is transformed into the classical traveling salesman problem (TSP), for which numerous algorithms exist.[23]

Both Refs. 21 and 22 studied refueling/servicing problems for which a single satellite visits all the satellites in the constellation. In this paper, we study a different scenario which arises from the need to redistribute fuel within a satellite constellation without a designated refueling spacecraft. It is therefore assumed that there is no extra fuel delivered to the constellation. Instead, all satellites in the constellation are capable of refueling each other. Consequently, satellites with excess fuel must deliver fuel to the satellites which are depleted of, or are low on fuel. The purpose of refueling is to redistribute the amount of fuel or propellant equally among all the satellites in the constellation so as to extend the lifespan of the constellation while minimizing the fuel consumed during the transfers.

A refueling scenario that seeks fuel equalization amongst the satellites in the same constellation using multiple satellite fuel transfers within the constellation will be hereto called the *Peer-to-Peer (P2P) refueling problem.* The blanket assumption here is, of course, that the constellation is operational only if *all* satellites have an acceptable minimum amount of fuel. Unequal fuel distribution amongst the satellites in a constellation may be the result of failures, diverse operational requirements, slightly different orbits, etc. It could also be the result of a prior refueling of a small number of the satellites in the constellation by an external refueler, in a mixed (single/multiple satellite) refueling strategy. By mixed refueling strategy we mean a strategy which involves at least two stages. During the first stage a single spacecraft refuels only part (perhaps half) of the satellites. During the second stage the satellites that received fuel during the first stage act as go-betweens, and distribute the fuel to the rest of the constellation in a P2P manner. We discuss such a scenario in the Numerical Examples section. There it is shown that mixed refueling strategies may outperform single-spacecraft refueling especially as the number of satellites in the constellation increases.

Incidentally, the criterion of fuel equalization used in this paper as the overall performance objective is not restrictive, and it is considered only for the sake of simplicity. Unequal fuel distribution between satellites having distinct operational requirements can be easily accommodated in the proposed framework; see the discussion after Eq. (2).

In our initial investigation of the P2P refueling problem[24] we assumed that the total cost is dominated by the refueling transactions and not by the cost incurred during the rendezvous transfer. This assumption is valid only if the times of transfer between the satellites are long.[25, 26] Although in practice binding time constraints will invalidate this assumption, nevertheless by neglecting the delivery cost of the transfers, one allows to establish a market between the satellites in the constellation. In this market, satellites having an amount of fuel above the constellation average are designated as the *sellers*, whereas satellites in the constellation having an amount of fuel below the constellation average are designated as the *buyers*. Within this market, fuel is treated as a commodity that is bought and sold between buyers and sellers in order to reach an equilibrium (i.e., fuel equidistribution). The *a priori* separation of the constellation satellites into sellers and buyers induces naturally a bipartite graph, and well-known methods[27, 28] can be used to solve the resulting maximum-weight matching problem on this graph.

In this paper we relax the assumption on the negligibility of the rendezvous cost. Indeed, in practice, the cost of the orbital transfers may dominate the total cost, especially for severely time-constrained refueling scenarios. An a priori separation of satellites into sellers and buyers is no longer possible in this case. Hence the problem cannot be reduced to a bipartite graph. Nonetheless, we show that by a proper use of the performance objective, the problem can still be formulated as a maximum weight matching problem in the constellation graph. The solution to this problem provides the optimal satellite pairs. In this scenario, owing to the cost incurred during the rendezvous maneuver it is possible to have two satellites with excess fuel be involved in a fuel exchange. We demonstrate these observations via numerical examples.

The results in the paper are restricted to circular satellite constellations in the same orbital plane. Multi-plane orbit constellations are not considered here since they require plane changes which are known to be extremely fuel-inefficient. In general, the cost (fuel) required to perform a plane change maneuver will dominate the fuel required for an in-plane maneuver unless the difference in the orbit inclinations is small. In the case of a multi-plane constellation the results of this paper thus have to be implemented on a plane by plane basis. Hence, only satellite pairs within the same orbital plane are allowed. This restriction can be easily imposed during the formulation of the problem as it is shown later.

The paper is organized as follows. First, the general description of the P2P refueling problem is outlined. Then the constellation graph is introduced. After the weights are assigned on each edge of the constellation graph, the reduced constellation graph is derived by removing all edges which are not feasible or not cost-effective. The P2P refueling problem is then formulated as a maximum-weight matching problem and it is solved using standard methods from linear and integer programming. The computations for the fuel consumed during a rendezvous maneuver associated with each fuel transaction are also given in detail. Finally, two numerical examples demonstrating the effectiveness of the proposed algorithm are presented, followed by some conclusions and suggestions for future extensions of the proposed approach.

The P2P Refueling Problem

Given a constellation of $n \geq 3$ satellites $\mathcal{C} = \{s_1, \ldots, s_n\}$ with unequal amounts of fuel, we wish to develop a strategy to distribute the fuel in the constellation so that at the end of this process all satellites will have an almost equal amount of fuel. The satellites with fuel greater than the average amount of fuel are termed *fuel-sufficient* satellites, whereas the satellites with fuel less than or equal the average amount of fuel in the constellation are termed the *fuel-deficient* satellites.

In a P2P refueling scenario, the satellites in the constellation perform rendezvous with each other for the purpose of exchanging/transfering fuel. In this paper, the combination of the two orbital transfers and the actual fuel transfer between two satellites is called a *fuel transaction*. Hence, a P2P refueling scenario consists of a set of fuel transactions within the constellation. It will be assumed that during a refueling transaction, only one satellite, called the *seller*, gives fuel to the other satellite involved in the fuel transaction. The latter satellite is called the *buyer*. The set of seller satellites will be denoted by $\mathcal{S}$ and the set of buyer satellites will be denoted by $\mathcal{B}$. Depending on the amount of fuel between the two, either of these two satellites can initiate a fuel transaction, i.e., perform a rendezvous with the other satellite, exchange fuel and return to its original orbital slot. The former satellite is said to be the *active* satellite and the latter satellite is said to be the *passive* satellite. The set of active satellites will be denoted by $\mathcal{A}$ and the set of passive satellites will be denoted by $\mathcal{P}$. Note that, in general, $\mathcal{S} \cup \mathcal{B} \subseteq \mathcal{C}$ since not all satellites may be involved in fuel transactions. Similarly, $\mathcal{A} \cup \mathcal{P} \subseteq \mathcal{C}$ for the same reason. Clearly, $\mathcal{S} \cap \mathcal{B} = \varnothing$ and $\mathcal{A} \cap \mathcal{P} = \varnothing$. Also note that it is not necessarily true that $\mathcal{S} = \mathcal{A}$ or that $\mathcal{B} = \mathcal{P}$, although this typically may be the case. For instance, it may happen that a satellite, say s_i, initiating a fuel transaction receives fuel (i.e., $s_i \in \mathcal{A} \cap \mathcal{B}$) or that a passive satellite is the seller ($s_i \in \mathcal{P} \cap \mathcal{S}$). We say that this "market" of buyer/seller satellites reaches an equilibrium when the fuel distributed among all satellites is (approximately) equal.

It is assumed that all transfers for the rendezvous between each seller/buyer pair are two-impulse, multi-revolution transfers. As shown in Ref. 25, allowing multi-revolution orbital transfers may reduce the fuel significantly. Allowing more than two impulses, on the other hand, offers little improvement.[26]

Problem Formulation

Let $\mathcal{I} = \{1, 2, \cdots, n\}$ denote the index set of the n satellites in the constellation and let f_i, where $i \in \mathcal{I}$, denote the fuel stored in each satellite. Similarly, let f_i^M and f_i^m ($i \in \mathcal{I}$), denote the maximum fuel capacity and minimum required fuel for each satellite. Satellite s_i is considered operational if and only if $f_i^m \leq f_i \leq f_i^M$ ($i \in \mathcal{I}$). We are also given a time period T within which all fuel transactions must take place simultaneously. We also assume that within the given time frame each satellite can (but is not required to) be involved in a single fuel transaction with at most one other satellite. That is, no two seller-buyer pairs share a common satellite during a single refueling period.

We will assume that for each pair of satellites engaged in a fuel transaction, say s_i and s_j, only one is the active satellite which initiates the fuel transaction. For instance, if satellite $s_i \in \mathcal{A}$, it applies impulses to travel and rendezvous with satellite $s_j \in \mathcal{P}$; it then exchanges fuel with s_j, before traveling back to its originally designated orbital slot. During the whole process, satellite s_j remains at its pre-assigned orbital slot. Thus, only the active satellite consumes fuel during the rendezvous maneuver. One can preselect a subset of satellites to be inactive (due to, say, operational restrictions). These can either be satellites which do not have enough fuel to perform rendezvous maneuvers, or they may be satellites that are required to stay in their orbit in order to maintain normal operation of the constellation. It should be noted that the proposed active/passive P2P refueling scheduling method is unaffected even if cooperative rendezvous are allowed between each pair of satellites. However, the challenge in this case is to find a computationally efficient way to calculate the minimum-cost for the cooperative rendezvous maneuvers. Therefore cooperative rendezvous will not be pursued in this paper.

Let p_i^j denote the fuel consumed by satellite s_i in order to rendezvous with satellite s_j and then return to its designated orbital slot. Similarly, let p_j^i denote the fuel consumed by satellite s_j if it is the active satellite. Note that, in general, $p_i^j \neq p_j^i$ (see Cost of Fuel Transaction section below). Also note that during a fuel transaction between s_i and s_j either one can be the active satellite, provided that it has enough amount of fuel to rendezvous with the inactive satellite and return to its original orbital slot. Hence, the fuel cost assigned to a single rendezvous between satellites s_i and s_j is given by

$$p_{ij} = \begin{cases} p_i^j, & \text{if } s_i \text{ can be active but } s_j \text{ cannot be active,} \\ p_j^i, & \text{if } s_j \text{ can be active but } s_i \text{ cannot be active,} \\ \min\{p_i^j, p_j^i\}, & \text{if both } s_i \text{ and } s_j \text{ can be active,} \\ \infty, & \text{neither } s_i \text{ nor } s_j \text{ can be active.} \end{cases} \tag{1}$$

Let f_i^- and f_i^+ denote the amount of fuel onboard satellite s_i before and after a fuel transaction, respectively. Given satellites s_i and s_j, let g_i^j denote the amount of fuel transferred from $s_i \in \mathcal{S}$ to $s_j \in \mathcal{B}$ during the fuel transaction. If, on the other hand, $s_i \in \mathcal{B}$ and $s_j \in \mathcal{S}$, then $g_i^j = -g_j^i$. It follows that $f_i^+ = f_i^- - p_i^j - g_i^j$ and $f_j^+ = f_j^- + g_i^j$ if $s_i \in \mathcal{A}$, and $f_i^+ = f_i^- - g_i^j$ and $f_j^+ = f_j^- - p_j^i + g_i^j$ if $s_j \in \mathcal{A}$.

In order to achieve fuel equalization after one refueling period we further assume that whenever two satellites conduct a fuel transaction, the fuel is redistributed such that the two satellites have the same amount of fuel after the completion of the fuel transaction. That is, if satellites s_i and s_j conduct a fuel transaction, we impose that

$$f_i^+ = f_j^+ = \frac{f_i^- + f_j^- - p_{ij}}{2}. \tag{2}$$

We hasten to point out that although this is a natural choice of an objective, it is by no means restrictive. Unequal fuel distribution can be accommodated via proper weighting of the fuel before and after the fuel exchange. We will not elaborate any further on the unequal distribution problem in this paper. Instead, we will assume that (2) holds after each fuel transaction. Similarly, and without loss of generality, henceforth we assume that all satellites can carry the same maximum amount of fuel and that any amount of fuel is acceptable for the satellite to be fully operational, that is, we assume for simplicity that $f_i^M = f_j^M$ and $f_i^m = 0$ for all $1 \leq i,\ j \leq n$.

Formulation of P2P Refueling as a Maximum-Weight Matching Problem

The Constellation Graph

In this section, we formulate the P2P refueling problem as a maximum-weight matching problem[28] in a graph derived from the satellite constellation.

Given the set $\mathcal{C}$ we may construct a graph $\mathcal{G}$ having as nodes (or vertices) the satellites of $\mathcal{C}$. We call $\mathcal{G}$ the constellation graph. Associated with $\mathcal{G}$ is a set of vertices $\mathcal{V} = \{1, 2, \ldots, n\}$ and a set of edges $\mathcal{E} = \{\langle i, j\rangle : i, j \in \mathcal{V}\}$ connecting the nodes of $\mathcal{G}$. In the graph $\mathcal{G}$, an edge between two vertices exists if a fuel transaction between the corresponding satellites is permissible. Without loss of generality, we will enumerate the vertices such that $i \leftrightarrow s_i$ for all $1 \leq i \leq n$. This, allows us in the sequel to refer to "vertex" s_i or vertex i without the danger of confusion. To each edge $\langle i, j \rangle \in \mathcal{V}$ we will assign the minimum fuel required between the two transfers $s_i \to s_j$ and $s_j \to s_i$ via (1). Hence, we make no distinction between the edge $\langle i, j \rangle$ and the edge $\langle j, i \rangle$ and thus $\mathcal{G}$ is an undirected graph. The constellation graph $\mathcal{G}$ is then completely described by the doublet $(\mathcal{V}, \mathcal{E})$. We summarize the above in the following definition.

Definition 1. *The constellation graph, denoted by $\mathcal{G}$, is a graph having the constellation satellites as its vertices. An edge between two vertices exists if either of the two vertex-satellites can initiate a rendezvous and carry out a fuel transaction with the other.*

The set of vertices connected to vertex i is called the set of neighbors of i, and it is denoted by $\mathcal{N}_i$. The edge neighborhood of i is defined by $\mathcal{Q}_i = \{\langle i, j \rangle \in \mathcal{E} : j \in \mathcal{N}_i\}$. Note that if i has no neighbors then no edges are connected to this vertex and $\mathcal{Q}_i = \varnothing$. For example, we may impose that certain satellites are not involved in any fuel transactions due to operational constraints.

If there are no restrictions on the satellite pairs, and each satellite in the constellation has enough fuel to complete the rendezvous maneuvers, the constellation graph is a complete graph.[28] If, on the other hand, there are restrictions on certain satellite pairs due to operational requirements, the constellation graph may not be a complete graph. For example, in order to maintain a minimum level of operation for the constellation, a subset of satellites may be required to remain in their original orbital slots, while the rest are engaged in fuel transactions. Reflected in the constellation graph, this implies that there are no edges between the former group of satellites. Obviously, if any of these satellites is involved in a fuel transaction, it can only be passive. By removing all satellites, which are known a priori that will not be involved in fuel transactions due to operational restrictions, we get the *core constellation graph* $\mathcal{G}^c$. The developments that follow still hold if we replace $\mathcal{G}$ with $\mathcal{G}^c$. For simplicity, in the sequel we assume that $\mathcal{G} = \mathcal{G}^c$.

Cost Selection

Let $\bar{f}$ denote the average fuel storage among the satellites in the constellation, that is,

$$\bar{f} = \frac{1}{n} \sum_{i=1}^{n} f_i.$$

Equalizing fuel is equivalent to minimizing the total deviation of the fuel from the average. Thus, the objective can be written as one of maximizing

$$\mathcal{J} = -\sum_{i \in \mathcal{I}} \left| f_i^+ - \bar{f}^- \right|. \tag{3}$$

Ideally, one would like to minimize the total deviation of the fuel from the average after refueling is complete. That is, maximize

$$\mathcal{J}' = -\sum_{i \in \mathcal{I}} \left| f_i^+ - \bar{f}^+ \right| \tag{4}$$

instead of (3). The latter cost, however, solely concentrates on fuel equalization and it does not penalize the fuel consumption associated with the rendezvous maneuvers. In fact, fuel equalization and minimum transfer cost are conflicting objectives.

The reason we define the cost as in (3) is to account for situations where satellites consume excessive amount of fuel during the rendezvous transfers just for the sake of achieving better fuel equalization. To avoid such undesirable cases, the objective function should reflect a balance between achieving fuel equalization and minimizing the total fuel consumption. The objective function in (3) achieves such an objective. Since the total fuel stored in all satellites after refueling is less than the fuel before refueling, i.e., $\bar{f}^+ < \bar{f}^-$, maximizing the cost (3) ensures us that fuel equalization is achieved without sacrificing too much of the total fuel.

An additional reason behind the choice of (3) is that it results in a computationally tractable solution algorithm. For instance, suppose one chooses the following alternative formulation that seeks to maximize

$$\mathcal{J}' = -\alpha \sum_{i \in \mathcal{I}} \left| f_i^+ - \bar{f}^+ \right| - (1 - \alpha) \sum_{\langle i, j \rangle \in \mathcal{M}} p_{ij}, \tag{5}$$

with $0 \leq \alpha \leq 1$ and $\mathcal{M} \subseteq \mathcal{E}$ represents the satellite pairs that engage in fuel transactions. This cost is a weighted (convex) sum of (4) and the total fuel consumption. With this choice, the total fuel consumption is explicitly penalized. It can be easily shown, however, that the use of (5) does not lead to a computationally efficient optimization problem.

The Maximum-Weight Matching Problem

In graph theory, a matching in a graph is defined as a collection of edges such that no two edges in the collection share a common vertex. Recall that no two seller-buyer pairs share a common satellite during a refueling period, which implies that the edges associated with satellite pairs which conduct fuel transactions do not share common vertices. Therefore, the collection of these edges form a matching[28] in the constellation graph. As a result, the search for satellite pairs to achieve fuel equalization is equivalent to the search for a matching in the constellation graph, such that $\mathcal{J}$ in Eq. (3) is maximized. In the following, we will utilize this analogy to model and solve the P2P refueling problem in the framework of a maximum-weight matching problem on the constellation graph. That is, we will assign a weight on each edge, and seek the matching that maximizes the sum of the weights of all edges in this matching.

To this end, we associate a binary variable x_{ij} with each edge $\langle i,j\rangle \in \mathcal{E}$, defined by

$$x_{ij} = \begin{cases} 1, & \text{if } \langle i,j\rangle \in \mathcal{M}, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Then, the condition for $\mathcal{M}$ to be a matching can be written as follows[28]

$$\sum_{\langle i,j\rangle \in \mathcal{Q}_i} x_{ij} \leq 1, \qquad \forall\, i \in \mathcal{V}. \tag{7}$$

Now, let us further elaborate on the objective function in Eq. (3). In general, not every satellite is involved in a fuel transaction. For example, if the number of satellites is odd, at least one satellite will be left unmatched. Moreover, suppose that satellite s_i is matched with satellite s_j. Then after the fuel transaction, the fuel stored between the two is averaged out. Therefore, the contribution of satellite s_i to the objective function, denoted by c_{ij}, is given by

$$c_{ij} = -\left| \frac{f_i^- + f_j^- - p_{ij}}{2} - \bar{f}^- \right|.$$

This is the same as the contribution from satellite s_j. On the other hand, if a satellite, say satellite s_k, is not matched with any other satellite, then its fuel remains the same throughout the refueling period. Thus, its contribution to the objective function is

$$-\left| f_k^- - \bar{f}^- \right|.$$

Utilizing the binary variables x_{ij}, we can write the contribution to the objective function of all matched satellites as[26]

$$\sum_{i\in\mathcal{I}} \sum_{\langle i,j\rangle \in \mathcal{Q}_i} \left(-c_{ij}x_{ij}\right), \tag{8}$$

Similarly, we can write the contributions to the objective function from all unmatched satellites as[26]

$$\sum_{i\in\mathcal{I}} \left(1 - \sum_{\langle i,j\rangle \in \mathcal{Q}_i} x_{ij}\right)\Big(-\left|f_i^- - \bar{f}^-\right|\Big). \tag{9}$$

Then, the objective function in Eq. (3) is the sum of (8) and (9). It follows that

$$\mathcal{J} = \sum_{i\in\mathcal{I}} \sum_{\langle i,j\rangle \in \mathcal{Q}_i} \Big(\left|f_i^- - \bar{f}^-\right| - c_{ij}\Big)x_{ij} - \sum_{i\in\mathcal{I}}\Big(\left|f_i^- - \bar{f}^-\right|\Big).$$

Since the last term in the previous equation is constant, it can be removed from the previous expression without affecting the optimal solution. Then, we may rewrite $\mathcal{J}$ as a sum over all edges, to obtain

$$\mathcal{J} = \sum_{\langle i,j\rangle \in \mathcal{E}} \Big(\left|f_i^- - \bar{f}^-\right| + \left|f_j^- - \bar{f}^-\right| - \left|f_i^- + f_j^- - p_{ij} - 2\bar{f}^-\right|\Big)x_{ij}. \tag{10}$$

In the following, we will use π_{ij} to denote the coefficient of x_{ij} in (10), i.e.,

$$\pi_{ij} = \left|f_i^- - \bar{f}^-\right| + \left|f_j^- - \bar{f}^-\right| - \left|f_i^- + f_j^- - p_{ij} - 2\bar{f}^-\right|, \qquad \langle i,j\rangle \in \mathcal{E}. \tag{11}$$

Therefore, the P2P refueling problem can be formulated as a maximum-weight matching problem (MW-MP) in terms of a zero-one integer program as follows

$$\text{(MW-MP):} \quad \begin{cases} \text{Maximize} & \sum_{\langle i,j\rangle\in\mathcal{E}} \pi_{ij}\, x_{ij}, \\ \text{Subject to} & \text{the matching conditions (6) – (7).} \end{cases}$$

The Reduced Constellation Graph

The number of edges in the constellation graph can be reduced according to the signs of the weights π_{ij} in (11). Namely, the edges with weights $\pi_{ij} \leq 0$ can be removed from the constellation graph because any optimal solution to (MW-MP) does not contain those edges. Doing so reduces the effort for solving (MW-MP). The resulting graph is called a *reduced constellation graph*, and it is denoted by $\mathcal{G}_r$.

Definition 2. *The reduced constellation graph $\mathcal{G}_r$ results from the constellation graph $\mathcal{G}$ after all edges with weights less than or equal to zero have been removed.*

For an edge $\langle i,j\rangle$, if $f_i^- \leq \bar{f}^-$ and $f_j^- \leq \bar{f}^-$, it follows from Eq. (11) that $\pi_{ij} \leq 0$. Therefore, the reduced constellation graph does not contain edges between fuel deficient satellites. However, for an edge $\langle i,j\rangle$ with $f_i^- > \bar{f}^-$ or $f_j^- > \bar{f}^-$, it is not obvious whether $\pi_{ij} > 0$ or $\pi_{ij} \leq 0$. In general, it may be beneficial in terms of the chosen objective function to have two fuel-sufficient satellites conduct a fuel transaction. Thus, the reduced constellation graph may contain some edges between fuel-sufficient satellites. Note that this is different from the case when the rendezvous cost is negligible. In that case, all edges between fuel-sufficient satellites may be removed from the constellation graph.[24]Therefore, unlike the case in Ref. 24 where the reduced constellation graph is a bipartite graph with only edges between fuel-sufficient and fuel-deficient satellites, here the reduced constellation graph cannot be reduced to a bipartite graph.

After the reduced constellation graph $\mathcal{G}_r$ is obtained, the solution to the P2P refueling problem can be obtained by solving (MW-MP) defined on $\mathcal{G}_r$. An efficient polynomial-time algorithm to solve (MW-MP) is the one by Edmonds and Johnson.[28] Its computational complexity is $\mathcal{O}(n^3)$.

Cost of Fuel Transaction

Before solving (MW-MP), one first needs to calculate the fuel consumption, p_{ij} in Eq. (2), for the two satellites to conduct a fuel transaction. To this end, let the weight of the permanent structure of the two satellites be m_{si} and m_{sj}, respectively, and let I_{spi} and I_{spj} denote the specific impulses of the propulsion system for the two satellites. In order to calculate p_{ij}, two cases need to be considered. In the first case, satellite s_i is the active satellite; in the second case, satellite s_j is the active satellite.

Case 1, $s_i \in \mathcal{A}$. In this case, satellite s_i initiates the fuel transaction. Let ΔV_{ij}^i be the velocity change required for satellite s_i to rendezvous with satellite s_j, and ΔV_{ji}^i be the velocity change required for satellite s_i to depart from satellite s_j and return to its designated orbital slot. Then the amount of fuel p_{ti} consumed by satellite s_i to rendezvous with s_j is given by[29]

$$p_{ti} = (m_{si} + f_i^-)\left(1 - e^{-\frac{\Delta V_{ij}^i}{g_0 I_{spi}}}\right)$$

where g_0 is the gravitational acceleration at sea level.

If $p_{ti} > f_i^-$, then satellite s_i does not have enough fuel to complete the rendezvous with s_j and thus, it cannot initiate the rendezvous. In this case, $s_j \in \mathcal{A}$. Otherwise, if $p_{ti} \leq f_i^-$, then satellite s_i can complete the rendezvous with s_j, and after the rendezvous, the amounts of fuel onboard the two satellites, denoted by f_{i1} and f_{j1}, are

$$f_{i1} = f_i^- - p_{ti}, \quad \text{and} \quad f_{j1} = f_j^-. \tag{12}$$

Before satellite s_i starts the return maneuver, the amounts of fuel in each of the two satellites, denoted by f_{i2} and f_{j2}, are

$$f_{i2} = f_{i1} - g_i^j, \quad \text{and} \quad f_{j2} = f_{j1} + g_i^j. \tag{13}$$

Since satellite s_i has to return to its original designated orbital slot, it has to perform a second maneuver for the return trip. As mentioned earlier, the velocity change of the returning maneuver is ΔV_{ji}^i. Thus, the fuel consumption p_{bi} for this returning maneuver is given by

$$p_{bi} = (m_{si} + f_{i1} - g_i^j) \left(1 - e^{-\frac{\Delta V_{ji}^i}{g_0 I_{spi}}}\right). \tag{14}$$

Therefore, after satellite s_i returns to its original orbital slot, the amount of fuel onboard each satellite is given by

$$f_i^+ = f_{i2} - p_{bi}, \quad \text{and} \quad f_j^+ = f_{j2}. \tag{15}$$

Recall that the requirement is that the two satellites have the same amount of fuel after the fuel transaction, that is, $f_i^+ = f_j^+$. This requirement, along with Eqs. (12), (13) and (15), allows us to solve for g_i^j as

$$g_i^j = \frac{f_i^- - f_j^- - p_{ti} - p_{bi}}{2}. \tag{16}$$

Substituting g_i^j into Eq. (14), we get the explicit expression for p_{bi} as

$$p_{bi} = (2m_{si} + f_i^- + f_j^- - p_{ti}) \frac{1 - e^{-\frac{\Delta V_{ji}^i}{g_0 I_{spi}}}}{1 + e^{-\frac{\Delta V_{ji}^i}{g_0 I_{spi}}}}. \tag{17}$$

With p_{ti} and p_{bi} available, we obtain g_i^j from Eq. (16) and f_{i2} from Eq. (13). If $p_{bi} > f_{i2}$, then satellite s_i does not have enough fuel to return to its original orbital slot. It follows that $s_i \notin \mathcal{A}$. On the other hand, if $p_{bi} \leq f_{i2}$, satellite s_i has enough fuel to return, and thus, can be the active satellite. In that case, the fuel stored in each of the two satellites after the fuel transaction is obtained from Eq. (15).

In addition, the total fuel expense for the fuel transaction when satellite s_i is active is given by

$$p_i^j = p_{ti} + p_{bi}. \tag{18}$$

Case 2, $s_j \in \mathcal{A}$. Similar results can be derived for the case when satellite s_j is the active satellite. In this case, let ΔV_{ji}^j be the velocity change required for satellite s_j to rendezvous with satellite s_i, and ΔV_{ij}^j be the velocity change required for satellite s_j to depart from satellite s_i and return to its designated orbital slot.

Suppose satellite s_j has enough fuel to complete the go-and-return maneuvers. Then the amount of fuel that is necessary for s_j to rendezvous with s_i is given by

$$p_{tj} = (m_{sj} + f_j^-) \left(1 - e^{-\frac{\Delta V_{ji}^j}{g_0 I_{spj}}}\right),$$

and the amount of fuel needed for the return trip by satellite s_j can be calculated as

$$p_{bj} = (2m_{sj} + f_i^- + f_j^- - p_{tj}) \frac{1 - e^{-\frac{\Delta V_{ij}^j}{g_0 I_{spj}}}}{1 + e^{-\frac{\Delta V_{ij}^j}{g_0 I_{spj}}}}. \tag{19}$$

Thus, the total fuel expense for the fuel transaction when s_j is active is given by

$$p_j^i = p_{tj} + p_{bj}. \tag{20}$$

In addition, the amount of fuel transferred from satellite s_j to s_i can be calculated as

$$g_j^i = \frac{f_j^- - f_i^- - p_{tj} - p_{bj}}{2},$$

and the fuel stored in the two satellites after the refueling is given by

$$f_i^+ = f_j^+ = f_j^- - p_{tj} - g_j^i - p_{bj}.$$

Finally, the cost of the fuel transaction between satellites s_i and s_j is given from Eq. (1). It is evident from the previous analysis that, in general, $p_i^j \neq p_j^i$. Note that this is true despite the fact that the total velocity changes for satellite s_i to go to s_j and return, and for satellite s_j to go to s_i and return, are the same (assume same characteristics for both satellites). This is due to the fact that the fuel consumption for the two satellites is different if the satellites initially contain unequal amounts of fuel.

Numerical Examples

In this section, we provide two numerical examples to demonstrate some of the characteristics of the P2P refueling problem. The first example deals with a constellation of 14 satellites in a circular orbit. The objective of this example is to provide a step-by-step explanation how to formulate the P2P refueling problem as a maximum-weight matching problem. For the benefit of the reader, all relevant intermediate matrices needed to compute the edge weights are explicitly given.

In the second example we investigate the refueling of a constellation of 12 satellites evenly distributed in a circular orbit. We compare two different refueling options for this constellation. The first option involves the standard scenario of a single spacecraft refueling the whole constellation. The total refueling time is given and the optimal time allocation between each rendezvous is computed using the approach outlined in Ref. 21. The second option involves a mixed strategy which is composed of two steps. During the first step a single service vehicle refuels only half of the total number of satellites in the constellation. During the second step these satellites, in turn, deliver the fuel to the rest of the satellites in the constellation, in a P2P fashion. The total time to complete the refueling is the same as in the first strategy. It is shown that a mixed strategy may lead to a more efficient refueling than the single-refueler case. It is also shown that the gains from the use of a mixed strategy increase as the number of satellites increases.

Example 1

In this example, we consider a circular constellation of 14 satellites as shown in Figure 1. The satellites are evenly distributed along the circular orbit at an altitude of 500 km. The initial amount of fuel is shown next to each satellite in the figure. It is assumed that each satellite with a full tank of fuel has a mass of 100 units, and the permanent structure of each satellite weighs 60 units. Therefore, the maximum amount of fuel stored onboard a satellite is 40 units. The average fuel stored in the constellation before refueling is $\bar{f}^- = 20.4$ units. It is also assumed that all satellites have identical propulsion systems, and the specific impulse for satellite s_i is $I_{spi} = 300$ seconds ($i = 1, 2, \ldots, 14$). No additional operational constraints are imposed. Subsequently, each satellite is allowed to pair up with any other satellite. It follows that the constellation graph is a complete graph.

The rendezvous between two satellites is assumed to be a minimum-ΔV two-impulse, multi-revolution rendezvous transfer. The velocity change for each rendezvous is calculated according to the method presented in Ref. 25. Only multi-revolution rendezvous trajectories whose perigees are higher than the radius of the earth are considered valid. The total time for a fuel transaction is selected to be $T = 12$ orbital periods of the circular orbit. Half of the time is allotted for the active satellites to rendezvous with the passive satellites, and the remaining half is allotted for the active satellites to return to their original locations. The time it takes to transfer fuel between any two satellites is assumed to be negligible compared to the time it takes for the rendezvous maneuvers.

Given the above assumptions, the fuel expenditure between active satellite s_i and inactive satellite s_j, denoted by p_i^j, can be calculated according to Eqs. (18) and (20). The values p_i^j for each satellite pair can be conveniently represented by a matrix, denoted by P_1, such that $P_1(i,j) = p_i^j$. For the underlying example,

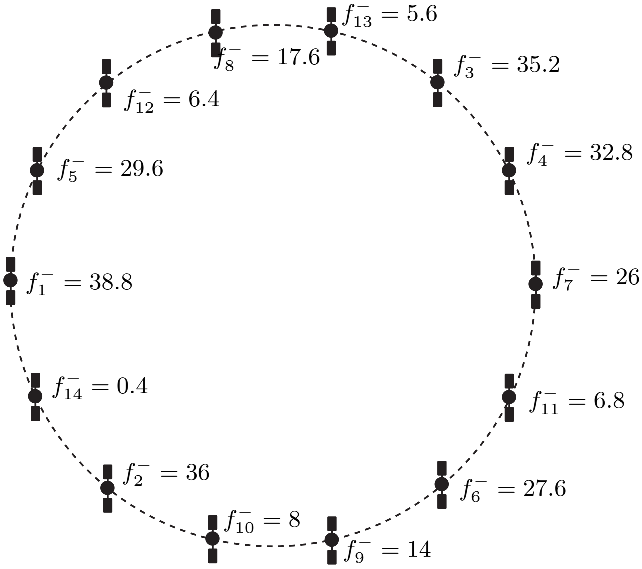


Fig. 1 The refueling scenario with fourteen satellites for Example 1.

P_1 is calculated as follows.

$$P_1 = \begin{array}{c|cccccccccccccc|}
 & \overset{s_1}{\downarrow} & \overset{s_2}{\downarrow} & \overset{s_3}{\downarrow} & \overset{s_4}{\downarrow} & \overset{s_5}{\downarrow} & \overset{s_6}{\downarrow} & \overset{s_7}{\downarrow} & \overset{s_8}{\downarrow} & \overset{s_9}{\downarrow} & \overset{s_{10}}{\downarrow} & \overset{s_{11}}{\downarrow} & \overset{s_{12}}{\downarrow} & \overset{s_{13}}{\downarrow} & \overset{s_{14}}{\downarrow} \\
s_1 \rightarrow & \times & 8.65 & 20.75 & 24.47 & 4.11 & 20.36 & 27.73 & 12.19 & 15.85 & 11.82 & 22.73 & 8.01 & 15.52 & 3.78 \\
s_2 \rightarrow & 8.50 & \times & 27.80 & 23.98 & 12.30 & 12.25 & 19.88 & 19.35 & 7.94 & 3.78 & 15.16 & 15.19 & 22.14 & 3.71 \\
s_3 \rightarrow & 20.44 & 27.69 & \times & 4.03 & 16.12 & 16.00 & 8.19 & 7.97 & 19.02 & 22.15 & 11.49 & 11.42 & 3.72 & 21.63 \\
s_4 \rightarrow & 23.80 & 23.57 & 3.98 & \times & 19.54 & 11.92 & 3.89 & 11.59 & 15.11 & 18.32 & 7.61 & 14.74 & 7.53 & 24.40 \\
s_5 \rightarrow & 3.92 & 11.93 & 15.61 & 19.17 & \times & 22.45 & 22.36 & 7.63 & 18.15 & 14.41 & 24.22 & 3.57 & 10.90 & 7.20 \\
s_6 \rightarrow & 19.18 & 11.68 & 15.43 & 11.63 & 22.18 & \times & 7.71 & 21.40 & 3.59 & 7.29 & 3.50 & 23.76 & 17.36 & 13.85 \\
s_7 \rightarrow & 26.13 & 18.90 & 7.86 & 3.76 & 22.07 & 7.65 & \times & 14.51 & 10.89 & 14.08 & 3.48 & 17.28 & 10.59 & 20.09 \\
s_8 \rightarrow & 10.89 & 17.55 & 7.19 & 10.59 & 7.13 & 20.19 & 13.71 & \times & 22.24 & 18.91 & 15.86 & 3.18 & 3.18 & 12.55 \\
s_9 \rightarrow & 13.76 & 6.97 & 16.95 & 13.61 & 16.54 & 3.30 & 10.12 & 21.76 & \times & 3.09 & 6.35 & 18.12 & 18.08 & 9.20 \\
s_{10} \rightarrow & 9.84 & 3.18 & \mathrm{CI} & 15.88 & 12.60 & 6.47 & 12.58 & \mathrm{CI} & 2.96 & \times & 8.84 & 14.33 & \mathrm{CI} & 5.82 \\
s_{11} \rightarrow & \mathrm{CI} & 12.72 & 9.74 & 6.51 & \mathrm{CI} & 3.06 & 3.07 & \mathrm{CI} & 6.03 & 8.74 & \times & \mathrm{CI} & 11.44 & \mathrm{CI} \\
s_{12} \rightarrow & 6.63 & 12.81 & 9.57 & 12.53 & 3.08 & \mathrm{CI} & \mathrm{CI} & 2.94 & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \times & 5.82 & \mathrm{CR} \\
s_{13} \rightarrow & 12.84 & \mathrm{CI} & 3.10 & 6.34 & 9.42 & \mathrm{CI} & 9.22 & 2.93 & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & 5.80 & \times & \mathrm{CI} \\
s_{14} \rightarrow & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \mathrm{CI} & \times
\end{array} \tag{21}$$

Each element of Eq. (21) gives the fuel consumption of the rendezvous scenario where the satellite with the row index is active and the satellite with the column index is passive. In Eq. (21), ‘CI’ stands for ‘Cannot Initiate’, and ‘CR’ stands for ‘Cannot Return’. Thus, in P_1, an entry of ‘CI’ implies that the satellite of the row index cannot initiate the fuel transaction with the satellite of the column index; an entry of ‘CR’ implies that the satellite of the row index can rendezvous with the satellite of the column index, but it cannot return to its original orbital slot. For example, consider satellites s_6, s_{12}, and s_{14}. Since satellite s_6 initially has a large amount of fuel, it can rendezvous with satellite s_{12} and return to its original location, and the total fuel expense for this go-and-return maneuver is 23.76; i.e., $P_1(6, 12) = 23.76$ in Eq. (21). On the other hand, satellite s_{12} initially does not have enough fuel to rendezvous with satellite s_6, so $P_1(12, 6) =$ ‘CI’. Therefore, satellite s_{12} cannot be the active satellite if it is paired up with satellite s_6. However, satellite s_{12} has enough fuel to rendezvous with satellite s_{14}, but the fuel consumption for this rendezvous maneuver is so large that satellite s_{12} does not have enough fuel left to complete the return maneuver. In addition, satellite s_{14} does not have enough fuel for satellite s_{12}. Thus, even though satellite s_{12} can rendezvous with s_{14}, it does not have enough fuel to return to its original slot. Therefore, in Eq. (21), $P_1(12, 14) =$ ‘CR’.

After computing the matrix P_1, we can identify the cost for a fuel transaction between two satellites according to Eq. (1). Similarly to p_i^j, the values of p_{ij} can also be represented compactly as the elements of a matrix, denoted by P_2, such that $P_2(i, j) = p_{ij}$. For the underlying example, P_2 can be calculated as

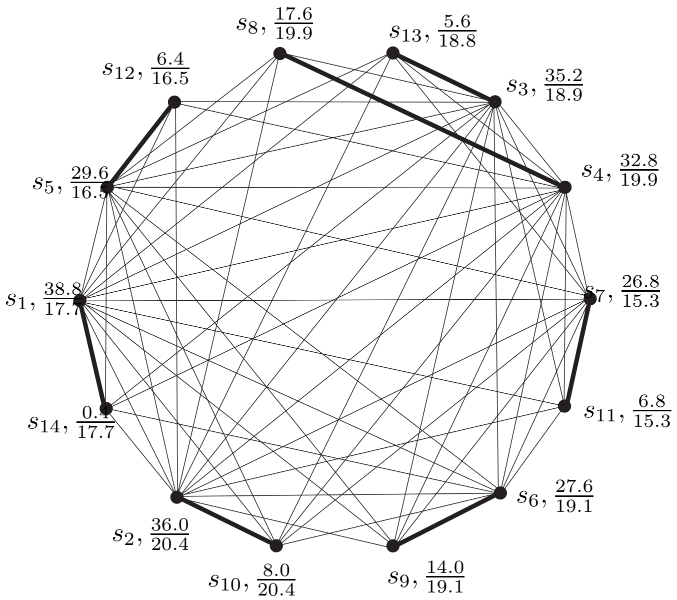


Fig. 2 The reduced constellation graph with the optimal matching for Example 1.

follows

$$P_2 = \begin{array}{cc}
\begin{array}{cccccccccccccc}
s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} & s_{12} & s_{13} & s_{14} \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow
\end{array} & \\
\left(\begin{array}{cccccccccccccc}
\times & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.73 & 0 & 0 & 3.78 \\
8.50 & \times & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.14 & 3.71 \\
20.44 & 27.69 & \times & 0 & 0 & 0 & 0 & 0 & 0 & 22.15 & 0 & 0 & 0 & 21.63 \\
23.80 & 23.57 & 3.98 & \times & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 24.40 \\
3.92 & 11.93 & 15.61 & 19.17 & \times & 0 & 0 & 0 & 0 & 0 & 24.22 & 0 & 0 & 7.20 \\
19.18 & 11.68 & 15.43 & 11.63 & 22.18 & \times & 0 & 0 & 0 & 0 & 0 & 23.76 & 17.36 & 13.85 \\
26.13 & 18.90 & 7.86 & 3.76 & 22.07 & 7.65 & \times & 0 & 0 & 0 & 0 & 17.28 & 0 & 20.09 \\
10.89 & 17.55 & 7.19 & 10.59 & 7.13 & 20.19 & 13.71 & \times & 0 & 18.91 & 15.86 & 0 & 0 & 12.55 \\
13.76 & 6.97 & 16.95 & 13.61 & 16.54 & 3.30 & 10.12 & 21.76 & \times & 0 & 0 & 18.12 & 18.08 & 9.20 \\
9.84 & 3.18 & 0 & 15.88 & 12.60 & 6.47 & 12.58 & 0 & 2.96 & \times & 0 & 14.33 & \mathrm{NA} & 5.82 \\
0 & 12.72 & 9.74 & 6.51 & 0 & 3.06 & 3.07 & 0 & 6.03 & 8.74 & \times & \mathrm{NA} & 11.44 & \mathrm{NA} \\
6.63 & 12.81 & 9.57 & 12.53 & 3.08 & 0 & 0 & 2.94 & 0 & 0 & \mathrm{NA} & \times & 0 & \mathrm{NA} \\
12.84 & 0 & 3.10 & 6.34 & 9.42 & 0 & 9.22 & 2.93 & 0 & \mathrm{NA} & 0 & 5.80 & \times & \mathrm{NA} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{NA} & \mathrm{NA} & \mathrm{NA} & \times
\end{array}\right) &
\begin{array}{c}
\leftarrow s_1 \\ \leftarrow s_2 \\ \leftarrow s_3 \\ \leftarrow s_4 \\ \leftarrow s_5 \\ \leftarrow s_6 \\ \leftarrow s_7 \\ \leftarrow s_8 \\ \leftarrow s_9 \\ \leftarrow s_{10} \\ \leftarrow s_{11} \\ \leftarrow s_{12} \\ \leftarrow s_{13} \\ \leftarrow s_{14}
\end{array}
\end{array} \tag{22}$$

It can be seen that each element of P_2 is either a positive value, zero, or ‘NA’. An element of P_2 being positive or zero conveys the information about which satellite is active. Namely, if $P_2(i,\, j) > 0$ and $i > j$, then s_i is active. In this case, $P_2(j,\, i) = 0$. If $P_2(i,\, j) > 0$ and $i < j$, then s_j is active, and in this case, $P_2(j,\, i) = 0$. The symbol ‘NA’ implies that for the corresponding pair of satellites, neither one can be active. Therefore, using matrix P_2 we can remove from the constellation graph the edges of satellite pairs of which neither satellite can be active.

The weight π_{ij} assigned to edge $\langle i, j\rangle$ is calculated according to Eq. (11). The edge weights can also be represented in a matrix form. A matrix denoted by Π is created, such that $\Pi(i,\, j) = \pi_{ij}$. Since $\Pi(i,\, j) = \Pi(j,\, i) = \pi_{ij}$, Π is a symmetric matrix. Thus, the upper triangular part of Π is sufficient to represent the edge weights. The matrix Π for the underlying example is given by

$$\Pi = \begin{array}{cc}
\begin{array}{cccccccccccccc}
\overset{s_1}{\downarrow} & \overset{s_2}{\downarrow} & \overset{s_3}{\downarrow} & \overset{s_4}{\downarrow} & \overset{s_5}{\downarrow} & \overset{s_6}{\downarrow} & \overset{s_7}{\downarrow} & \overset{s_8}{\downarrow} & \overset{s_9}{\downarrow} & \overset{s_{10}}{\downarrow} & \overset{s_{11}}{\downarrow} & \overset{s_{12}}{\downarrow} & \overset{s_{13}}{\downarrow} & \overset{s_{14}}{\downarrow}
\end{array} & \\
\left(\begin{array}{cccccccccccccc}
\times & 8.50 & 20.44 & 23.80 & 3.92 & 19.18 & 23.47 & 16.49 & 23.04 & 26.96 & 14.07 & 30.17 & 23.96 & 33.02 \\
 & \times & 27.69 & 23.57 & 11.93 & 11.68 & 18.90 & 13.65 & 19.77 & 27.98 & 18.48 & 18.39 & 9.06 & 27.49 \\
 & & \times & 3.98 & 15.61 & 15.43 & 7.86 & 12.79 & 12.65 & 7.45 & 19.86 & 20.03 & 26.50 & 7.97 \\
 & & & \times & 19.17 & 11.63 & 3.76 & 14.21 & 11.19 & 8.92 & 18.29 & 12.27 & 18.46 & 0.40 \\
 & & & & \times & 10.62 & 9.13 & 11.27 & 1.86 & 5.80 & -5.82 & 15.32 & 8.98 & 11.20 \\
 & & & & & \times & 7.65 & -5.79 & 11.10 & 7.93 & 11.34 & -9.36 & -2.96 & 0.55 \\
 & & & & & & \times & -0.91 & 2.68 & 0.22 & 9.73 & -4.48 & 3.58 & -7.29 \\
 & & & & & & & \times & -21.76 & -18.91 & -15.86 & -2.94 & -2.93 & -12.55 \\
 & & & & & & & & \times & -2.96 & -6.03 & -18.12 & -18.08 & -9.20 \\
 & & & & & & & & & \times & -8.74 & -14.33 & \mathrm{NA} & -5.82 \\
 & & & & & & & & & & \times & \mathrm{NA} & -11.44 & \mathrm{NA} \\
 & & & & & & & & & & & \times & -5.80 & \mathrm{NA} \\
 & & & & & & & & & & & & \times & \mathrm{NA} \\
 & & & & & & & & & & & & & \times
\end{array}\right) &
\begin{array}{c}
\leftarrow s_1 \\ \leftarrow s_2 \\ \leftarrow s_3 \\ \leftarrow s_4 \\ \leftarrow s_5 \\ \leftarrow s_6 \\ \leftarrow s_7 \\ \leftarrow s_8 \\ \leftarrow s_9 \\ \leftarrow s_{10} \\ \leftarrow s_{11} \\ \leftarrow s_{12} \\ \leftarrow s_{13} \\ \leftarrow s_{14}
\end{array}
\end{array} \tag{23}$$

Evidently, the weights are less than zero for edges between fuel-deficient satellites which are satellites from s_8 to s_{14}. In addition, there are other edges with negative weights. These are among the edges where one end-vertex is a fuel-sufficient satellite and the other end-vertex is a fuel-deficient satellite. As mentioned earlier, edges with negative weights are removed from the constellation graph. After removing all these edges, we obtain the reduced constellation graph shown in Figure 2.

Once the reduced constellation graph and the weights of the edges are obtained, we can solve the maximum-weight matching problem. The optimal matching for this example is shown in Figure 2, depicted by thick solid lines. The optimal matching has seven edges. This implies that every satellite is engaged in a fuel transaction with another satellite. The fuel stored onboard each satellite before and after refueling is also shown next to each vertex in the form of a fraction. The value of the numerator is the amount of fuel before refueling, and the value of the denominator is the amount of fuel after refueling. It can be seen that after the whole refueling process is completed, the fuel is much more evenly distributed than prior to refueling. Indeed, before refueling, $\sum_{i=1}^{14} |f_i^- - \bar{f}^-| = 168$, while after refueling $\sum_{i=1}^{14} |f_i^+ - \bar{f}^-| = 30.137$. The total fuel spent to perform the orbital transfers of the active satellites is 30.105.

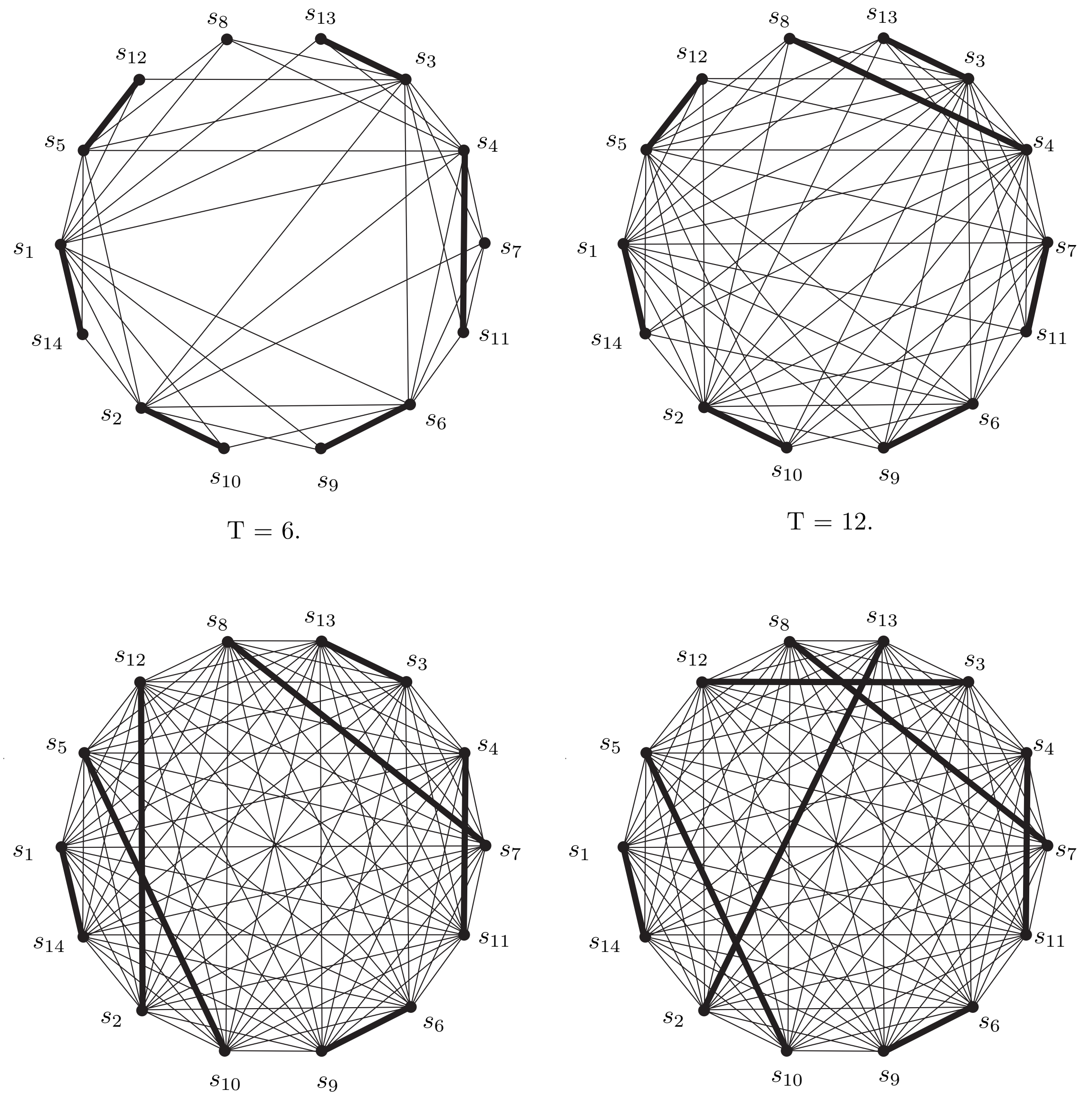
The fact that the rendezvous cost is taken into consideration in the formulation of the problem has a large impact on the optimal satellite pairs. In Ref. 24 the P2P refueling problem was considered, where the rendezvous cost was assumed to be negligible. Therein, a symmetric matching is defined as a collection of satellite pairs such that the satellite with the most fuel and the satellite with the least fuel pair up, the satellite with the second most fuel and the satellite with the second least fuel pair up, etc. In Ref. 24, it was shown that for cases where each satellite pair in the constellation is allowed to conduct a fuel transaction, a symmetric matching is the optimal matching. In the constellation shown in Figure 1, the symmetric matching is given by the collection of the following edges

$$\big\{\langle 1,\ 14\rangle,\ \langle 2,\ 13\rangle,\ \langle 3,\ 12\rangle,\ \langle 4,\ 11\rangle,\ \langle 5,\ 10\rangle,\ \langle 6,\ 9\rangle,\ \langle 7,\ 8\rangle\big\}.$$

As can be seen from Figure 2, the optimal matching when the rendezvous costs are taken into account is not the symmetric matching. This is true despite the fact that in this example each satellite pair is allowed to conduct a fuel transaction. In fact, one of the edges in the symmetric matching, namely $\langle 7,\ 8\rangle$, does not even exist in the reduced constellation graph. It has been removed from the constellation graph because the weight for this edge is negative.

However, there is a reason to believe that if we can reduce the rendezvous costs in Example 1, the solution to the matching problem will change and eventually, as the rendezvous costs become negligible, the symmetric matching will become the optimal matching. It is shown in Ref. 25 that the rendezvous cost between two satellites in the same circular orbit decreases monotonically as the total time to conduct the rendezvous increases. As a result, the number of edges involved in the optimal matching changes, depending on the refueling period.

Figure 3 shows how the solution to the P2P refueling problem evolves as a function of the total refueling period. Four matching solutions are presented in Figure 3. The total times for the four solutions are $T = 6,\ 12,\ 48$ and 140, as indicated in the figure. In each case, half of the time is spent by the active satellites to rendezvous with the passive satellites, and the other half is spent for the active satellites to depart the passive satellites and return to their original orbital slots. Notice that the solution for $T = 12$ is precisely the one in Figure 2. As it can be seen in Figure 3, as the total time is reduced from 12 to 6, the



T = 6.

T = 12.

T = 48.

T = 140.

Fig. 3 The evolution of solution with respect to the total refueling time.

reduced constellation graph contains fewer edges, and the satellites s_7 and s_8 become unmatched. This is due to the fact that increased rendezvous costs cause a great amount of edges to have negative weights. On the other hand, as the total time is increased from 12 to 48, it is seen that the constellation graph becomes a complete graph, and the solution starts to take the shape of the symmetric matching. In fact, the solution contains all edges from the symmetric matching except $\langle 2, 13\rangle$ and $\langle 3, 12\rangle$. As the total time is increased even further, from 48 to 140, it is seen that the solution tends to the symmetric matching.

Example 2

In this example we consider a constellation of 12 satellites. For the sake of simplicity we also assume that initially all satellites have no fuel. We wish to refuel all of the satellites in the constellation such that at the end of the refueling period T all satellites have the same amount of fuel. We investigate two alternative refueling scenarios. In the first scenario, a single servicing vehicle s_0 refuels all satellites in the constellation. We will refer to this as the single-refueler strategy. In the second scenario the satellite s_0 delivers fuel to six of the satellites in the constellation. Subsequently, these satellites refuel the remaining six satellites in the P2P fashion. We will refer to the latter as the mixed (single-refueler/P2P) refueling strategy. The single-refueler and the mixed refueling strategies are depicted schematically in Figure 4. Our objective is to minimize the

Seg. No.	ΔV_i	p_i
1	0.182	35.9811
2	0.182	32.1340
3	0.182	28.5647
4	0.182	25.2531
5	0.182	22.1805
6	0.182	19.3297
7	0.182	16.6847
8	0.182	14.2307
9	0.182	11.9538
10	0.182	9.8412
11	0.3804	15.8278

Table 1 Optimal ΔVs and fuel consumption for refueling with a single-spacecraft.

Seg. No.	ΔV_i	p_{ij}
1	0.182	33.222
2	0.182	26.8151
3	0.182	20.8707
4	0.182	15.3554
5	0.182	10.2382

Table 2 Optimal ΔVs and fuel consumption during the first step of the mixed refueling strategy.

total fuel consumption during the ensuing orbital transfers. Equivalently, we want to maximize the total amount of fuel delivered to the constellation.

We assume that the 12 satellites are in a circular orbit at an altitude of 500 km, and have the same physical characteristics as the satellites in Example 1. We also assume that the refueling vehicle s_0 has the same characteristics and same I_{sp} as before, with the exception of a much larger fuel tank. We will assume that s_0 carries an initial amount of fuel $f_0^- = 500$ units, and at the end of the refueling process it is left with $f_0^+ = 10$ units of fuel; i.e., a maximum of 490 units of fuel is to be delivered to the constellation. Of course, the 490 units of fuel also contains the fuel to be consumed during the refueling process.

Spacecraft s_0 is initially at a higher circular orbit than the constellation orbit. It is required to return to the same orbit after completing the refueling process. Since for both refueling strategies the initial ΔV_0 for s_0 to reach the constellation orbit and the final ΔV_f for s_0 to return to its original orbit are the same, we do not consider the initial and final transfer maneuvers of s_0 as part of the optimization process. For this problem, these values can be calculated as $\Delta V_0 = \Delta V_\mathrm{f} = 0.2$ and are the same for both refueling strategies. Here the velocity unit is the orbital velocity of the constellation orbit divided by 2π. These ΔVs correspond to fuel consumption of $p_0 = 44.262$ and $p_\mathrm{f} = 6.009$ units, respectively. The total time allowed for refueling for both cases is $T = 20$.

The singe-refueler strategy involves 11 rendezvous segments as shown in Figure 4(a). Table 1 shows the ΔV and the corresponding fuel consumption for each rendezvous segments obtained from the solution to the single-refueler scheduling problem. At the end of the refueling process each satellite will end up with an equal amount of fuel $f_i^+ = 17.31$ $(1 \le i \le 12)$. The total amount of fuel consumed during the refueling process is $490 - 12 \times 17.31 = 282.28$. Note that although the ΔVs for the first 10 rendezvous segments are equal, the corresponding fuel consumed is progressively decreased, as the mass of the refueling vehicle is reduced after each rendezvous segment.

For the mixed refueling scenario, the first step (s_0 refueling satellites s_1 to s_6) is completed in $T_1 = 9.55$ units of time and the second step (P2P refueling) is completed in $T_2 = 20 - 9.55 = 10.45$ units of time. After the first step, satellites $s_1, s_2, \ldots, s_6$ will end up with an equal amount of fuel $f_i(T_1^-) = 55.53$ $(1 \le i \le 6)$. The first step has five rendezvous segments. The ΔV and fuel consumption during each segment are given in Table 2.

The remaining satellites $s_7, \ldots, s_{12}$ are refueled during the second step of the mixed refueling strategy using an optimal pairing with satellites $s_1, \ldots, s_6$ by solving the corresponding P2P refueling problem. The optimal pairs, along with the corresponding values of ΔVs and fuel consumptions are shown in Table 3.

After the second step is completed, the final amounts of fuel in each satellite are as follows: $f_1^+ = f_2^+ =$

Pairs ($i \leftrightarrow j$)	ΔV_{ij}^i	p_{ti}	ΔV_{ji}^i	p_{bi}	p_{ij}
$1 \leftrightarrow 10$	0.2023	9.2317	0.2208	7.5531	16.7848
$2 \leftrightarrow 11$	0.2023	9.2317	0.2208	7.5531	16.7848
$3 \leftrightarrow 12$	0.2023	9.2317	0.2208	7.5531	16.7848
$4 \leftrightarrow 7$	0.2208	10.0386	0.2023	6.8871	16.9257
$5 \leftrightarrow 8$	0.2208	10.0386	0.2023	6.8871	16.9257
$6 \leftrightarrow 9$	0.2208	10.0386	0.2023	6.8871	16.9257

Table 3 Optimal pairs and corresponding ΔVs and fuel consumption for the P2P stage of the mixed refueling strategy.

$f_3^+ = f_{10}^+ = f_{11}^+ = f_{12}^+ = 19.37, f_4^+ = f_5^+ = f_6^+ = f_7^+ = f_8^+ = f_9^+ = 19.30$, leading to an average fuel of $\bar{f}^+ = 19.34$. The total fuel spent during the orbital transfers is $490 - 12 \times 19.34 = 257.92$. Thus, the mixed single-vehicle/P2P strategy is more efficient than the pure single-vehicle strategy in this case.

Of course it is not the case that the mixed refueling strategy will always outperform the single-refueler strategy. The relative merit between the two refueling strategies relies upon the number of satellites and the total refueling time. Figure 5 depicts the results from the comparison between these two refueling strategies as the number of satellites in the constellation varies, while keeping the total refueling time constant ($T = 20$). Figure 5 shows that the mixed strategy leads to better results as the number of satellites increases.

Conclusions

In this paper, we have studied the problem of optimal fuel equalization and distribution amongst satellites in a circular constellation. We have introduced and investigated the case when no fuel is delivered to the constellation from an external source but instead each satellite has the capability of receiving or delivering fuel to any other satellite in the constellation. This so-called peer-to-peer (P2P) refueling problem has been formulated as a maximum-weight matching problem and solved using standard methods from linear and integer programming. Numerical examples indicate that a refueling strategy that incorporates a P2P step may lead to reduced delivery costs, especially as the number of satellites increases.

Several generalizations to the proposed P2P baseline refueling scenario are immediate, and are currently under investigation. First, unequal fuel distribution (arising, say, from diverse operational requirements for each satellite) can easily be accommodated using weighted averages in the problem formulation. Asynchronous implementations for each P2P rendezvous segment (where the refueling time for each satellite pair as well as the go-and-return portions of each transfer are not equal) are also possible, and indeed lead to more efficient refueling than the synchronous P2P implementation discussed here.[30] Relaxing the assumption of a single refueling period is also straightforward. More involved is the relaxation of the assumption that each satellite is either active or passive during a single refueling period. For instance, it may be beneficial to consider scenarios where a satellite takes fuel from one satellite and delivers it to another satellite within a single refueling period. The incorporation of such *traders* may result in a more efficient market, that is, in a smaller final deviation from the desired equilibrium, as well as in faster convergence. This remains to be shown, however.

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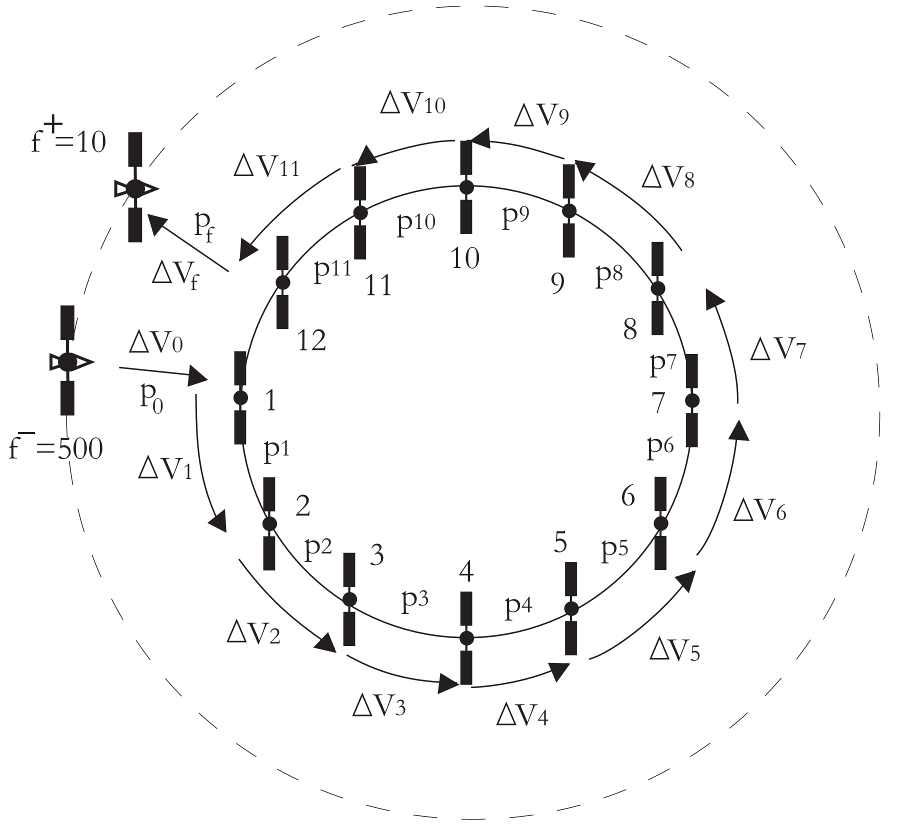
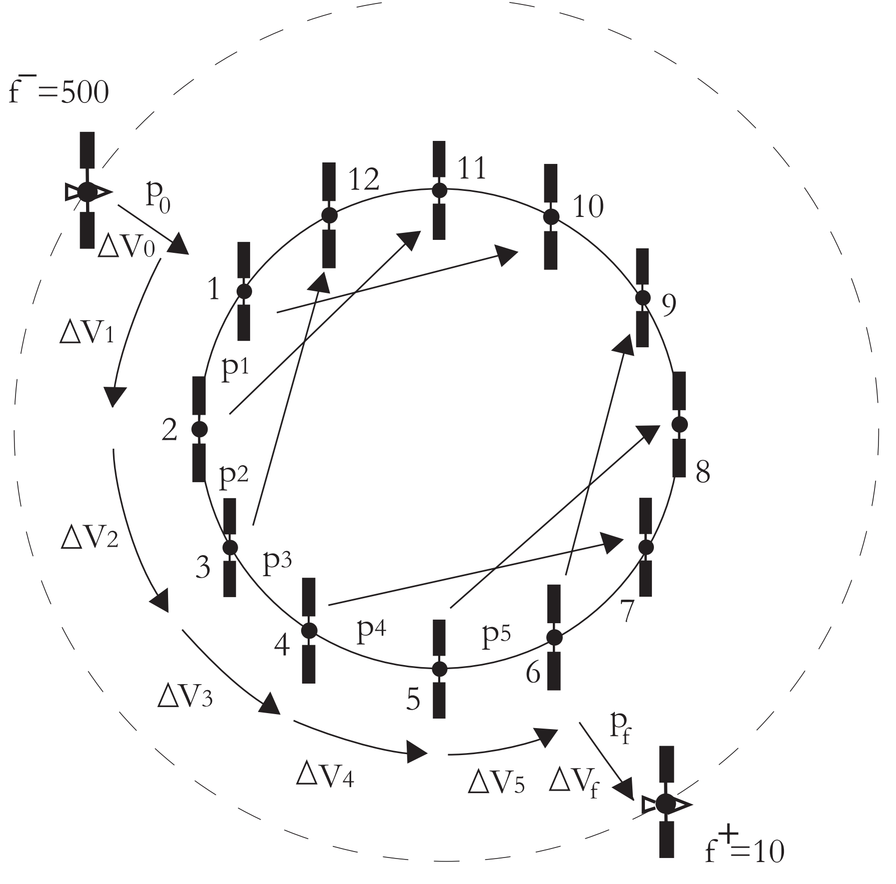


a) Single-Refueler Strategy



b) Mixed Refueling Strategy

Fig. 4 Example 2. Constellation with 12 evenly distributed satellites in a circular orbit. Comparison of two alternative refueling strategies.

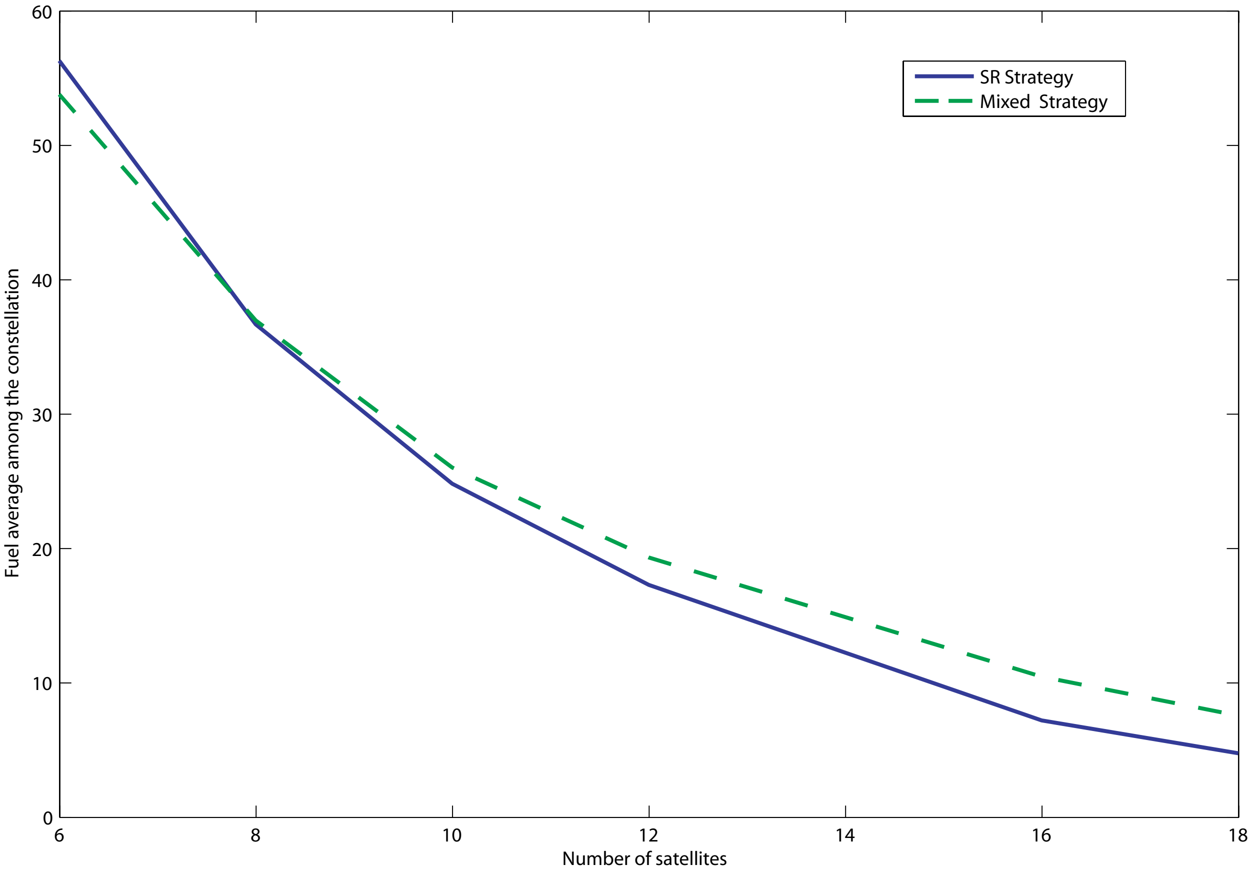


Fig. 5 **Comparison between the single-refueler and mixed refueling strategies for various numbers of satellites in the constellation.**