Comparison Between Peer-to-Peer and Single-Spacecraft Refueling Strategies for Spacecraft in Circular Orbits

Panagiotis Tsiotras* and Arnaud de Nailly†

The lifespan of satellite constellations can be extended by periodic refueling and/or servicing of the satellites in the constellation. The traditional approach for satellite refueling/servicing is for a single satellite to refuel all satellites in the constellation in a sequential manner. Recently, an alternative, decentralized refueling approach has been proposed, namely, peer-to-peer (P2P) refueling. In a P2P refueling architecture, there is no a priori designated refueling satellite. Instead, all satellites in the constellation can play the role of the refueling/servicing spacecraft. In this paper, we compare single-spacecraft and mixed P2P refueling scenarios. We show that a mixed strategy with a P2P component may result in better overall fuel consumption.

I. Introduction

Satellite constellations offer a robust, flexible alternative to the operation of single, large satellites in orbit. The redundancy provided by a large number of satellites provides distinct advantages, and for several missions (e.g., interferometry) the coordinated operation of several satellites is the only option. Regardless of the specific application, it appears that the current trend of operating a large number of satellites in unison (in lieu of large monolithic spacecraft) is likely to continue for the foreseeable future, especially for LEO and MEO missions.

Maintaining a large number of satellites in orbit introduces several challenges. Ideally–and in order to keep ground operations to a minimum–one would require the constellation to have at least some on-board autonomy. Decentralization of several regular orbit maintenance operations should be part of the overall constellation capabilities. Moreover, for a LEO constellation the lifespan of the satellites is severely limited by the amount of available onboard fuel, which is depleted due to orbit correction maneuvers that have to be performed on a regular basis to keep the satellites in their original orbital slots. Once the onboard fuel is depleted the satellite has to be replaced by another one launched from Earth. Replacing all the satellites in the constellation every few years is an expensive proposition. Refueling the satellites offers a great alternative for lengthening the lifetime of the constellation. On-orbit refueling can increase the constellation's profitability and survivability. In fact, recent studies have shown that just the incorporation of refueling capability in each satellite (without even implementing *actual* refueling in orbit) can decrease insurance costs by 50%. A second advantage stems from the option to refuel the satellite at a later stage after launch. Therefore, a refuel-capable satellite may carry lesser propellant during launch. This decrease in propellant mass at launch time translates directly to increased payload mass.

It becomes clear from the previous discussion that adding refueling capability to the next generation of spacecraft offers many advantages, and several recent initiatives (for example, Refs. 1 and 2) show that indeed the time has arrived for this technology to become reality. Nonetheless, and despite the obvious advantages of refueling spacecraft in orbit, very few studies have been conducted thus far in order to determine which are the best (i.e., most fuel-efficient) refueling strategies.[3–5] In fact, most results that have appeared in the open literature thus far deal mainly with technological issues, such as, safe docking, transferring fuel in a microgravity environment, etc.[6–8]

*Professor, School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 3032-0150, USA. E-mail: p.tsiotras@ae.gatech.edu

†Graduate student, School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 3032-0150, USA. E-mail: gtg894n@mail.gatech.edu

The few studies that have looked at the problem of optimal refueling or servicing of several satellites in orbit, have assumed that a single spacecraft undertakes the whole task of refueling the constellation. In this scenario a single service spacecraft plays the role of the sole supplier of fuel. The service spacecraft is situated in the same or a different orbit and transfers all necessary supplies from this orbit to the constellation orbit.[3,7,9] Although this is the simplest and most natural method to equally distribute fuel amongst a number of satellites, it is not the only one. It will be shown later in this paper that it is also not necessarily the best one. In fact, for refueling under limited-time constraints the single-satellite strategy may not even be a feasible option. We elaborate on this statement later on.

Recently, an alternative scenario for distributing fuel amongst a large number of satellites has been proposed.[10–12] In this scenario, no single spacecraft per se is in charge of the whole refueling process. Instead, all satellites share the responsibility of refueling on an equal footing. We call this the peer-to-peer (P2P) refueling strategy. This distribution of "refueling responsibilities" leads to some very interesting results. First, a P2P strategy is distributed. As a result, a P2P strategy offers a greater degree of robustness to failures. A single spacecraft failure will have almost no impact to the refueling of the rest of the constellation. On the contrary, a failure of the service vehicle in the single-spacecraft scenario will lead to the inability to complete the refueling task. For a P2P scenario the orbital transfers can be executed *in parallel* whereas in the single-vehicle case they must necessarily be executed *in series*. Therefore, a single spacecraft strategy will require much shorter transfers between each satellite than a P2P strategy (assuming that the total allowed time to refuel all the satellites in the constellation is the same for both cases). Since the cost of orbital transfers is related inversely to the transfer time, one can then easily envision a case where the single-vehicle strategy will be infeasible (in the sense that it will lead to non-elliptic/extremely fuel inefficient orbital transfers), whereas a P2P strategy will manage to achieve the distribution of fuel within the total allowed time. By the same token, if for some reason sending a spacecraft from Earth is not possible (for technological, scheduling, or other reasons) the only option to refuel satellites low on fuel, or equally distribute fuel amongst the satellites in the constellation, is a P2P strategy.

Although a stand-alone P2P scenario may seem unrealistic or unconventional at first glance (the fuel has to be delivered to the fuel sufficient satellites *somehow* in the first place), it arises almost naturally as an essential component of a mixed refueling strategy. That is, P2P refueling can be the final distribution phase of a single-vehicle refueling strategy. Therefore, a P2P approach offers greater flexibility in the design and execution of a cost-efficient refueling strategy.

In this paper, we provide a comparison between two distinct baseline refueling scenarios for a satellite constellation in a circular orbit. We show that a mixed strategy that incorporates a P2P component may indeed lead to fuel savings as the number of satellites increases. Assuming for simplicity that all satellites are evenly distributed along the circular orbit, we compare two different approaches to replenish the satellites with propellant from a service vehicle sent from Earth for this purpose. The objective is to equally distribute a given amount of fuel among the satellites, and to do this in the most effective manner, i.e., by spending as little propellant for the orbital transfers as possible. The problem is complicated by the fact that the time to complete the refueling of the whole constellation is given.

In the first scenario, the service vehicle performs a series of rendezvous with each satellite in the constellation, one at a time. In the second scenario, the main service vehicle refuels only half of the total number of satellites in the constellation. These satellites, in turn, deliver the fuel to the rest of the satellites in the constellation, in a P2P fashion. In order to provide a fair comparison, we restrict the total time allowed for the refueling of the whole constellation to be the same for both cases. For the sake of simplicity, we also allow only single-pairings between the satellites. That is, satellites can rendezvous only once with at most one other satellite in the constellation. No satellite is allowed to rendezvous with more than one other satellite. Also, for each fuel transaction only one satellite is active. These restrictions can be relaxed (and may lead to further improvements) at the cost of increased complexity.

Our analysis shows that for a large number of satellites, a refueling architecture that incorporates a P2P component leads to reduced fuel consumption. The use of a P2P refueling strategy is therefore justified both by the flexibility and robustness it provides to failures, as well as by the resulting fuel savings.

All orbital transfers in this work are assumed to be elliptic, two-impulse, time-limited transfers. This restriction leads naturally to study of the celebrated Lambert's problem, which we briefly review in the next section. The paper is therefore structured as follows. In the next section we review the Lambert problem for two-impulse transfers. In particular, we present the results for a somewhat different version of the classical Lambert problem, namely one that allows multi-revolution transfers. In this case the calculation of the optimal number of revolutions and the corresponding geometry of the transfer orbit (semi-major axis, eccentricity, etc) is a bit more involved. Despite the added complexity, the study of multi-revolution Lambert transfers cannot be bypassed, as it has been shown[13] that it leads to significant fuel savings. We then summarize the optimal scheduling for a single-spacecraft refueling scenario. The problem involves the solution of two separate subproblems, that is, the determination of the optimal sequence of satellites in the constellation, and the optimal allocation of the total allotted transfer time to each rendezvous segment. The P2P refueling problem is formulated afterwards as a maximum matching problem over the so-called constellation graph. Standard methods from the literature can then be used to find the optimal satellite pairs by solving this maximum matching problem. A comparison between the single-spacecraft and the mixed P2P refueling methodologies is given next for a satellite constellation in a circular orbit with all the satellites in the constellation uniformly distributed along the orbit. Two different configurations are analyzed in detail. A parametric analysis by varying the number of satellites in the constellation suggests that a mixed refueling strategy incorporating a P2P component may indeed be more cost-effective. We conclude with some general remarks and by offering a few natural extensions that promise a further improvement of the P2P approach.

II. Time-Constrained, Fuel-Optimal, Two-Impulse Transfers

The problem of optimal orbital transfers has been studied for a long time. See, for instance, the classical books of Lawden[14] and Marec.[15] By formulating the orbital transfer problem using optimal control theory, the optimal thrusting profile is readily determined from the time history of the *primer vector*, which is the co-state corresponding to the velocity vector in the adjoint system of equations. The problem even admits a closed-form solution for some special cases. No analytic solutions are known to exist for the general case, however. Moreover, the optimal solution strategy depends on the rocket engine used (chemical, electrical, etc).[15]

In this work we deal with time-critical, relatively short duration maneuvers. For such cases chemical rockets are the only option given the current state of technology, despite their relative low $I_{\rm sp}$ compared to, say, electrical propulsion (e.g., ion-thruster) engines. The high thrust and the relatively short duration of the thrusting period of rocket engines allow the approximation of each thrusting arc by an impulse of infinite magnitude and arbitrarily small duration. The approximation of the transfer orbit by a series of coasting (Keplerian) arcs separated by impulses simplifies the problem significantly. Nonetheless, the number and location of impulses is not known a priori. Both can be determined from the magnitude of the primer vector[16] using, for instance, the approach of Ref. 17. The final answer nonetheless requires the solution of a parameter optimization problem.

By restricting to two-impulse orbital transfers we can use the solutions to the well-known Lambert problem.[18] Moreover, as it is shown in Ref. 13 the fuel savings gained by the use of three or more impulses may not be as great so as to justify the use of more than two impulses. Most importantly, several efficient methods exist for solving the Lambert problem.[19] Recently, Prussing[20] (see also Refs. 21 and 22) has shown that significant fuel savings may result from the use *multiple revolution* Lambert transfers. The solution to the multi-revolution Lambert problem is more involved than the zero-revolution Lambert problem, however. In fact, the former exhibits a multitude of solutions. As shown in Ref. 20, between any two fixed points, there are actually $2N_{\rm max}+1$ solutions to the multi-revolution Lambert problem where $N_{\rm max}$ is the number of maximum number of revolutions which are allowed for the chasing satellite. In general, one has to compute all possible $2N_{\rm max}+1$ candidates before choosing the one that results in the most fuel-efficient transfer (that is, the lowest ΔV).

Reference 22 developed an efficient method to quickly calculate the optimal ΔV by comparing only two out of all possible $2N_{\rm max}+1$ candidates. The optimal solution depends on the given transfer time $t_{\rm f}$. Figure 1 shows the ΔV vs $t_{\rm f}$ curve (dashed line) for a transfer between two satellites in the same circular orbit. As shown in Fig. 1, an orbital maneuver between two points strongly depends on the transfer time. Fuel-efficient transfers occur only at distinct points along the ΔV vs $t_{\rm f}$ curve, namely close to the relative minima of the dashed line in Fig. 1. This sequence of relative minima corresponds to phasing maneuvers (that is, maneuvers that are performed by tangential initial and final burns). The situation can be improved by the use of final coasting arcs. These correspond to horizontal line segments in the ΔV vs. $t_{\rm f}$ plot. Therefore, an optimal transfer between satellites in the same circular orbit is composed, in general, by a phasing maneuver plus a final coasting.

For transfers that include final coasting, the ΔV monotonically decreases as a function of $t_{\rm f}$ (solid line

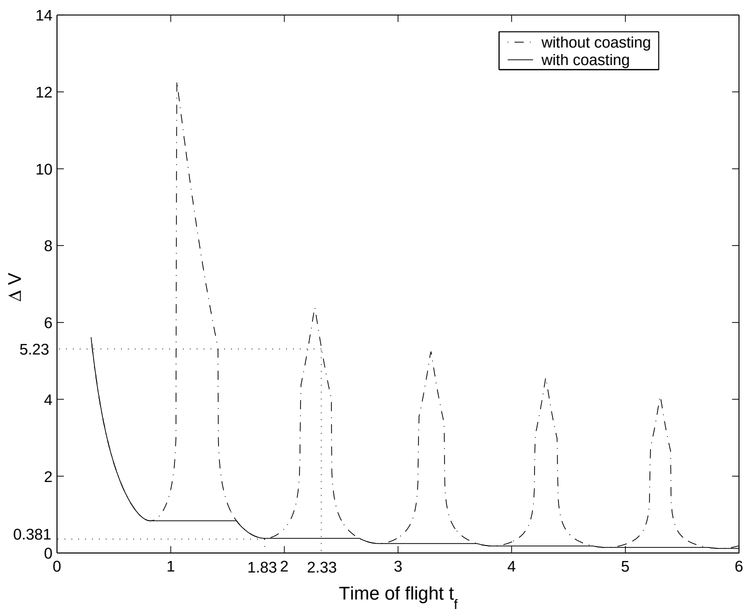


Figure 1. ΔV vs. t_f when $r_1 = r_2$ and $\theta_0 = 60°$, with (solid line) and without (dashed line) final coasting. For the given time $t_\mathrm{f} = 2.33$ a transfer with a final coasting of $t_\mathrm{c} = 0.5$ yields a value of $\Delta V = 0.381$, whereas without coasting it yields a value of $\Delta V = 5.23$. The use of a final coasting arc leads to a reduction of ΔV by 92% in this case; from Ref. 22.

in Fig. 1). The decrease is not strictly monotonic however, due to the coasting arcs. This creates some complications when trying to compute the optimal time allocation for a multi-segment rendezvous. We will elaborate on this issue later on, when we also propose a solution to the optimal time-allocation problem using integer programming.

In the sequel, a constellation of $n \geq 2$ satellites, s_i $(i = 1, \ldots, n)$ at time t_0 will be denoted by $\mathcal{S}_n(\theta(t_0))$, where $\theta(t_0) = (\theta_1(t_0), \ldots, \theta_n(t_0))^\intercal \in (0, 2\pi)^n$ is the vector of initial satellite separation angles defined by $\theta_i(t_0) = \nu(s_{i+1}; t_0) - \nu(s_i; t_0)$ $(i = 1, \ldots, n-1)$ and $\nu(s_i; t_0)$ denotes the true anomaly for satellite s_i at initial time t_0. The satellites are numbered sequentially along the direction of the orbit. Note that, by definition, $\theta_n(t_0) = 2\pi - \sum_{i=1}^{n-1} \theta_i(t_0)$. All angles are measured positivewise in the direction of the orbit. Since we assume that all satellites are in the same circular orbit, we have that $\theta_i(t) = \theta_i(t_0)$ for all $t \geq t_0$. We can therefore drop the argument t_0 and write simply θ_i. Let also $\theta_{ij} \in [-\pi, +\pi]$ denote the *lead angle* between satellites s_i and s_j. For each ordered pair of satellites (s_i, s_j) $(i \neq j)$ we assign a time interval t_{ij} within which the rendezvous of satellite s_i with satellite s_j has to be completed. In the pair (s_i, s_j) satellite s_i will be the *active* satellite and s_j will be the *passive* satellite.

Note that if the orbital frequency of the constellation is ω_0 then the two-impulse transfer between satellites s_i and s_j can be formulated as a multi-revolution Lambert problem with transfer time t_{ij} and separation angle $\varphi_{ij} = \mod(\theta_{ij} + t_{ij}\,\omega_0, 2\pi)$.

III. Optimal Single Service Vehicle Refueling

Given a constellation $\mathcal{S}_n(\theta)$ with a total of n satellites distributed (perhaps non-uniformly) in a circular orbit, we denote by s_0 a servicing satellite of initial mass $m_0(t_0)$. The objective is to find the strategy for s_0 to visit all satellites in $\mathcal{S}_n(\theta)$ within a given amount of time t_f, so that the total mass of s_0 at t_f is maximized. For impulsive transfers the following expression holds between the mass of the satellite just before $(m_0(t_k^-))$ and just after $(m_0(t_k^+))$ an impulse at time $t = t_k$

$$m_0(t_k^+) = m_0(t_k^-)\,\mathrm{e}^{-\Delta V_k / c_0}, \tag{1}$$

where ΔV_k is the gain (or loss) of velocity due to the impulse at $t = t_k$ and $c_0 = I_{\rm sp} g_0$. For a sequence of $\ell \geq 2$ impulses at times $t_0 \leq t_1 < t_2 < \cdots < t_\ell \leq t_{\rm f}$, equation (1) yields (recall that $m_0(t_k^-) = m_0(t_{k-1}^+)$)

$$m_0(t_{\rm f}) = m_0(t_0)\, {\rm e}^{-\sum_{k=1}^{\ell} \Delta V_k / c_0}. \tag{2}$$

Subsequently, maximizing the total final mass of s_0 is equivalent to minimizing the total velocity increase incurred during all orbital transfers of s_0, as it visits all the satellites of $\mathcal{S}_n(\theta)$.

Let $\Delta V_{ij}(t_{ij}, \theta_{ij})$ denote the cost associated with the transfer of s_0 from satellite s_i to satellite s_j within a time interval t_{ij}. Note that the cost $\Delta V_{ij}(t_{ij}, \theta_{ij})$ includes both velocity changes due to the impulses at the initial and terminal points of the transfer orbit from s_i to s_j. The first velocity change will put s_0 to a transfer orbit that intersects the orbit of s_j. The second velocity change is necessary in order for s_0 to rendezvous with s_j and enter its orbit.

Let $\mathcal{P}_n$ denote the set of all permutations of the ordered sequence $(1, 2, \ldots, n)$. We denote $q \in \mathcal{P}_n$ by $q = (q_1, q_2, \ldots, q_n)$.

We seek $q^\star \in \mathcal{P}_n$ such that the sequence of transfers $s_{q_1^\star} \to s_{q_2^\star} \to \cdots \to s_{q_{n-1}^\star} \to s_{q_n^\star}$ solves the optimization problem

$$\min_{t_{q_i q_{i+1}}} \min_{q \in \mathcal{P}_n} \sum_{i=1}^{n-1} \Delta V_{q_i q_{i+1}}(t_{q_i q_{i+1}}, \theta_{q_i q_{i+1}}), \tag{3}$$

subject to the constraint

$$\sum_{i=1}^{n-1} t_{q_i q_{i+1}} \leq t_{\rm f}. \tag{4}$$

The optimization parameters are the rendezvous sequence $q \in \mathcal{P}_n$ and the corresponding time intervals $t_{q_i q_{i+1}}$ $(i = 1, 2, \ldots, n-1)$ for each rendezvous segment. Note that in the formulation of the optimization problem (3)-(4) we have assumed that the service satellite s_0 is already at the location of s_{q_1} at the beginning of the refueling process. In other words, the initial cost for s_0 to transfer to the first satellite in the sequence is not taken into account. This choice simplifies the analysis, and for uniform constellations a circular orbit can be made without loss of generality.

A solution to the previous single-vehicle refueling problem for a circular constellation has been proposed in Ref. 10, and involves the solution to the following two subproblems: (i) the *optimal time distribution problem*, and (ii) the *optimal rendezvous sequence problem*. In the optimal time distribution problem, it is assumed that the sequence q is given. Then we solve the problem

$$\min_{t_{q_i q_{i+1}}} \sum_{i=1}^{n-1} \Delta V_{\theta_{q_i q_{i+1}}}(t_{q_i q_{i+1}}) \tag{5}$$

subject to the inequality constraint (4), where in (5) we have written $\Delta V_{\theta_{q_i q_{i+1}}}$ instead of $\Delta V_{q_i q_{i+1}}$ to stress the fact that $\theta_{q_i q_{i+1}}$ is now a parameter in $\Delta V_{q_i q_{i+1}}(t_{q_i q_{i+1}})$.

The difficulty in solving the optimal time distribution problem lies in the fact that the functions $\Delta V_{\theta_{q_i q_{i+1}}}(t_{q_i q_{i+1}})$ in (5) are not differentiable with respect to $t_{q_i q_{i+1}}$. In fact, each function $\Delta V_{\theta_{q_i q_{i+1}}}(t_{q_i q_{i+1}})$ consists of several constant segments, due to the final coasting arcs, as seen in Fig. 1. This prevents the use of traditional gradient-based search methods.[23] The solution approach of Ref. 10 then proceeds as follows.

For ease of notation, and without loss of generality, in the remainder of this section we write $\Delta V_{\theta_i}(t_i)$ for the more cumbersome $\Delta V_{\theta_{q_i q_{i+1}}}(t_{q_i q_{i+1}})$. Inspection of Fig. 2 reveals that each cost function $\Delta V_{\theta_i}(t_i)$ is comprised of a series of constant segments connected by smooth, monotonically decreasing segments. For the $i^{\rm th}$ rendezvous segment, and following the notation of Fig. 2, let c_{ij}, $j = 1, 2, \ldots, j_{i,\max}$, denote the costs associated with the constant segments in the function $\Delta V_{\theta_i}(t_i)$. The upper limit for the index j, $j_{i,\max}$, depends on the maximum time of transfer $t_{i,\max}$ allowed to be distributed to the corresponding segment. A natural choice is to let $t_{i,\max} = t_{\rm f}$. Similarly, from Fig. 2, let β_{ij}, $j = 1, 2, \ldots, j_{i,\max}$ denote the times when a curve is followed by a step function, and A_{ij}, $j = 1, 2, \ldots, j_{i,\max}$ denote the time when a step function is followed by a curve. In addition, let A_{i0} denote the minimum time-of-flight allowed for the $i^{\rm th}$ segment. Since the cost approaches infinity when the time-of-flight approaches zero, it is wise to set A_{i0} to a positive value such that the cost required to complete the rendezvous does not become prohibitive.

Typically, $\beta_{ij} - A_{ij-1}$ is small compared to $A_{ij} - A_{ij-1}$ for any $j = 1, 2, \ldots, j_{i,\max}$, and their difference increases as the transfer time increases. Based on these observations, we may approximate the cost function

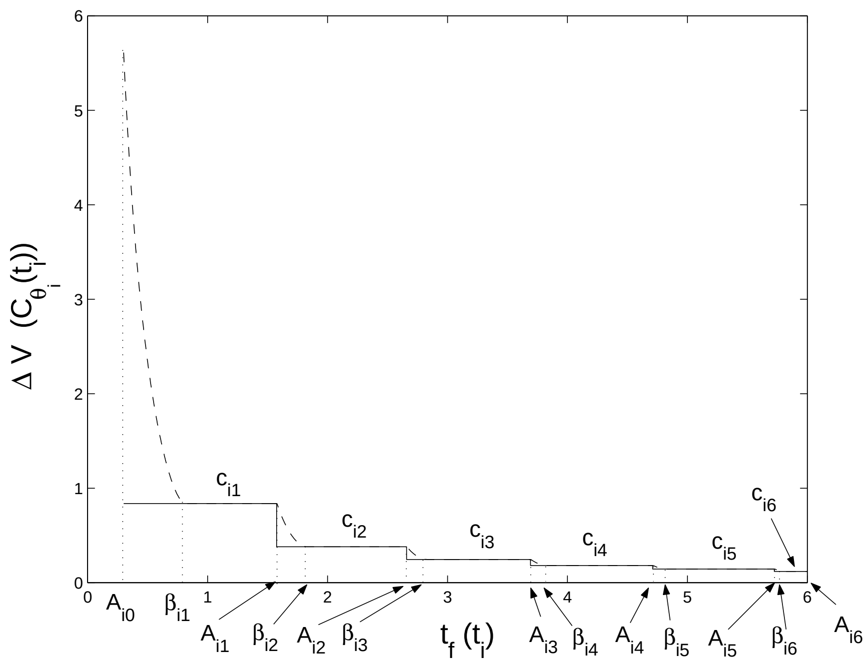


Figure 2. Step function approximation of the cost function.

for each rendezvous segment by a series of step functions. This is depicted in Fig. 2, where the original cost function is shown in dash lines and the step function approximation is shown in solid lines.

With this approximation, and for each rendezvous segment, the time interval $[A_{i0}, t_{i,\max}]$ is divided into $j_{i,\max}$ subintervals $[A_{ij-1}, A_{ij}]$ $(j = 1, 2, \ldots, j_{i,\max})$. The problem of optimal time distribution is now converted into a problem of determining the subinterval $[A_{ij-1}, A_{ij}]$ in which t_i should be assigned for each i^{th} rendezvous segment, where $1 \leq i \leq n-1$.

To solve this problem we introduce binary variables x_{ij}, where $i = 1, 2, \ldots, n-1$, $j = 1, 2, \ldots, j_{i,\max}$ such that

$$x_{ij} = \begin{cases} 1 & \text{if} \;\; t_i \in [A_{ij-1}, A_{ij}], \;\; j = 1, 2, \ldots, j_{i,\max}, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Then the integer program

$$\min_{x_{ij}} \quad \sum_{i=1}^{n-1} \sum_{j=1}^{j_{i,\max}} c_{ij} x_{ij}, \tag{7}$$

subject to the constraints

$$\sum_{j=1}^{j_{i,\max}} x_{ij} = 1, \quad i = 1, 2, \ldots, n-1, \tag{8a}$$

$$\sum_{i=1}^{n-1} \sum_{j=1}^{j_{i,\max}} A_{ij} x_{ij} \geq t_{\mathrm{f}}, \tag{8b}$$

$$\sum_{i=1}^{n-1} \sum_{j=0}^{j_{i,\max}-1} A_{ij} x_{ij} \leq t_{\mathrm{f}}, \tag{8c}$$

provides a solution to the optimal time distribution problem. This integer program can be solved using standard methods[24] (e.g., branch-and-bound, cutting planes, etc.). Once x_{ij} have been obtained the unknowns t_i $(i = 1, 2, \ldots, n-1)$ can be determined. For the details, the interested reader may peruse Ref. 10.

The optimal rendezvous sequence problem deals with the determination of the best rendezvous sequence for a given $t_{\rm f}$. In Ref. 10 it was shown that the optimal sequence is always one with the minimum *total sweep angle* (TSA), where for a given sequence $q \in \mathcal{P}_n$ the TSA is defined by

$$\text{TSA} = \sum_{i=1}^{n-1} |\theta_{q_i q_{i+1}}| \tag{9}$$

Extensive numerical simulations indicate that the optimal sequence is either sequential or bi-sequential.[13]

IV. The Peer-to-Peer Refueling Scenario

Assuming that each satellite in the constellation is capable of receiving or delivering fuel/propellant to each other satellite in the constellation, an alternative refueling scenario is possible, namely one in which each satellite can play the role of the servicing/refueling vehicle s_0. We call this the peer-to-peer (P2P) refueling scenario. Without loss of generality, in the P2P scenario we assume that the constellation $\mathcal{S}_n(\theta)$ has an even number of satellites, $n \geq 4$. Whenever two satellites meet to exchange fuel we say that these satellites are involved in a *fuel transaction*.

We assume that within a given time interval each satellite can be involved in a fuel transaction with at most one other satellite. We also assume that for each pair of satellites engaged in a fuel transaction, say s_i and s_j, only one is the active satellite which initiates the fuel transaction. For instance, if satellite s_i is active, it applies impulses to travel and rendezvous with satellite s_j; it then exchanges fuel with s_j, before traveling back to its originally designated orbital slot. During the whole process, satellite s_j remains at its pre-assigned orbital slot. Thus, only the active s_i satellite consumes fuel during the rendezvous maneuver.

Let f_i^- denote the amount of fuel stored onboard satellite s_i before the fuel transaction with satellite s_j, and let f_i^+ denote the amount of fuel stored onboard satellite s_i after this fuel transaction with s_j. Let also p_i^j denote the fuel expended by satellite s_i in order to rendezvous with satellite s_j and then return to its designated orbital slot. In order to achieve fuel equalization after one refueling period we further assume that whenever two satellites conduct a fuel transaction, the fuel is redistributed such that the two satellites have the same amount of fuel *after* the fuel exchange. That is, if satellites s_i and s_j conduct a fuel transaction, then we impose that

$$f_i^+ = f_j^+ = \frac{f_i^- + f_j^- - p_i^j}{2}. \tag{10}$$

The previous constraint implies that the amount of fuel g_i^j to be delivered by satellite s_i to satellite s_j in order to achieve fuel equalization must be equal to

$$g_i^j = \frac{f_i^- - f_j^- - p_i^j}{2}. \tag{11}$$

In the P2P scenario each fuel transaction requires two trips by the active satellite. The fuel consumption for the outbound trip is given by

$$p_{io}^j = (m_{si} + f_i^-)(1 - \mathrm{e}^{-\Delta V_{ij}/c_0}), \tag{12}$$

where m_{si} is the structural mass of satellite s_i and f_i^- is the amount of fuel of satellite s_i before the outbound trip. The fuel consumption for the return trip can be calculated as

$$p_{ir}^j = (2m_{si} + f_i^- + f_j^- - p_{io}^j)\frac{(1 - \mathrm{e}^{-\Delta V_{ji}/c_0})}{(1 + \mathrm{e}^{-\Delta V_{ji}/c_0})} \tag{13}$$

The total fuel burn for satellite s_i to meet with satellite s_j (when s_i is the active satellite) is therefore given by $p_i^j = p_{io}^j + p_{ir}^j$. A similar calculation can be performed for the case when s_j is the active satellite in the pair (s_i, s_j). Let p_j^i denote the fuel consumed by satellite s_j if it is the active satellite. Note that, in general, $p_i^j \neq p_j^i$. Thus, the fuel consumption for a fuel transaction between satellites i and j, denoted by p_{ij}, is the smaller of p_i^j and p_j^i. That is, if only satellite i can be active, then $p_{ij} = p_i^j$, whereas if only satellite j can be active, then $p_{ij} = p_j^i$. In case both satellites i and j can be active, then $p_{ij} = \min\{p_i^j, p_j^i\}$.

The P2P problem is simplified by the introduction of the *constellation graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices and $\mathcal{E}$ is the set of edges. Each vertex $\mathrm{v}_i \in \mathcal{V}$ $(i = 1, 2, \ldots, n)$ corresponds to a satellite in the constellation. An edge $\langle \mathrm{v}_i, \mathrm{v}_j \rangle \in \mathcal{E}$ denotes a possible/allowable fuel transaction between satellites s_i and s_j. Let $\mathcal{L} = \{(i, j) : \langle \mathrm{v}_i, \mathrm{v}_j \rangle \in \mathcal{E}\}$ and let $\mathcal{Q}(\mathrm{v}_i)$ denote the set of edges that are incident to the vertex v_i, that is, $\mathcal{Q}(\mathrm{v}_i) = \{ \langle \mathrm{v}_i, \mathrm{w} \rangle : \mathrm{w} \in \mathcal{V}, \text{ s.t. } \langle \mathrm{v}_i, \mathrm{w} \rangle \in \mathcal{E} \}$.

To each edge we assign a weight, which is equal to the fuel consumed by the corresponding fuel transaction. A simple calculation leads to the following weight for the edge $\langle \mathrm{v}_i, \mathrm{v}_j \rangle$ for all $(i, j) \in \mathcal{L}$

$$\pi_{ij} = \left| f_i^- - \bar{f}^- \right| + \left| f_j^- - \bar{f}^- \right| - \left| f_i^- + f_j^- - p_{ij} - 2\bar{f}^- \right|,$$

where $\bar{f}^-$ is the *initial* average amount of fuel in the constellation.

To each edge, we associate a binary variable $x_{ij} \in \{0, 1\}$ such that

$$x_{ij} = \begin{cases} 1 & \text{if edge } \langle \mathrm{v}_i, \mathrm{v}_j \rangle \text{ is in the matching,} \\ 0 & \text{otherwise.} \end{cases}$$

The P2P refueling problem can be formulated as the following maximum matching problem in terms of the a zero-one integer program:

$$\text{Maximize} \quad \sum_{(i,j) \in \mathcal{L}} \pi_{ij}\, x_{ij},$$

subject to,

$$\sum\nolimits_{\langle \mathrm{v}_i, \mathrm{v}_j \rangle \in \mathcal{Q}(\mathrm{v}_i)} x_{ij} \leq 1, \quad \forall\ 1 \leq i \neq j \leq n,$$
$$x_{ij} \in \{0,\ 1\}, \quad \forall\ (i, j) \in \mathcal{L}.$$

The solution to this problem can be obtained using standard methods.[25]

V. Comparison Between the Single-Vehicle and P2P Scenarios

We consider the case of a satellite constellation with an even number of satellites. For the sake of simplicity we also assume that initially all satellites have no fuel. We wish to refuel all of the satellites in the constellation such that at the end of the refueling period t_f all satellites have the same amount of fuel. We investigate two alternatives to achieve this objective. In the first alternative, a single servicing satellite s_0 refuels all satellites in the constellation. In the second alternative the satellite s_0 delivers fuel only to half of the satellites in the constellation. Subsequently, these satellites share the fuel with the remaining $n/2$ satellites in the constellation. We want to minimize the total fuel consumption during the ensuing orbital transfers. Equivalently, we want to maximize the total amount of fuel delivered to the constellation.

The fuel consumption for satellite s_i to rendezvous with satellite s_j can be computed from (12) and (13). The refueling spacecraft s_0 is assumed to begin the refueling process with an initial amount of fuel $f_0(t_0^-) = 500$ units. At the end of the whole refueling process satellite s_0 will be left with $f_0(t_\mathrm{f}^+) = 10$. Subsequently, 490 units of fuel are to be delivered to the constellation in the most efficient manner.

In the following, nondimensionalized units have been used for all quantities. Specifically, the unit of time is the period on the constellation orbit, which is assumed to have unit radius. Thus, the unit velocity is the velocity of the circular orbit divided by 2π. In the following examples, $m_{s_i} = 60$, and $c_0 = 2943$ for all satellites. The total maximum time allowed is $t_\mathrm{f} = 20$.

Spacecraft s_0 is initially at a higher circular orbit than the constellation orbit. It is required to return to the same orbit after completing the refueling process. The initial $\Delta V = 0.2$ and final ΔV (of the same value) required for s_0 to enter and leave the constellation orbit (these values correspond to a fuel consumption of $p_0 = 44.2619$ and $p_\mathrm{f} = 6.0094$, respectively) are the same for both refueling strategies, and thus are not part of the optimization process.

A. Example 1

In this example, we consider a constellation with $n = 6$ evenly distributed satellites in a circular orbit. Two possible refueling scenarios are compared. These scenarios are depicted in Fig. 3.

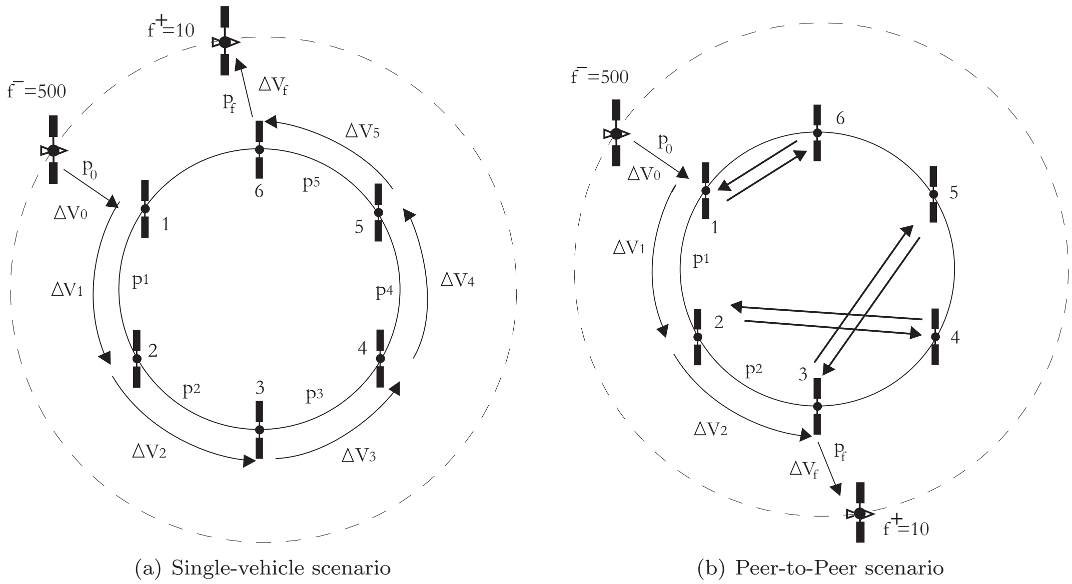


Figure 3. Two possible refueling strategies. Constellation with 6 evenly distributed satellites.

In the first scenario a single-servicing vehicle visits all satellites and distributes the fuel equally among all satellites in the constellation. This is shown in Fig. 3(a). There are five rendezvous segments. Due to the symmetry of the problem s_0 visits the satellites $s_1, s_2, \ldots, s_6$ in a sequential manner. The maximum time allowed for each rendezvous segment is set to 6 time units. The optimal time allotted for each rendezvous segment is calculated by solving the optimization problem outlined in the previous section. One then obtains the values of the ΔVs and the corresponding amounts of fuel consumption for each of the five rendezvous segments ($1 \leq i \leq 5$) shown in Table 1.

Seg. No.	ΔV_i	p_i
1	0.1676	30.6329
2	0.1676	24.8359
3	0.1676	19.4254
4	0.1676	14.3757
5	0.2205	12.5769

Table 1. Optimal ΔVs and fuel consumption for refueling with a single-spacecraft.

At the end of this process each of the six satellites ends up with an equal amount of fuel $f_i(t_{\mathrm{f}}^+) = 56.31$ ($i = 1, \ldots, 6$). The total amount of fuel used during all the transfers is thus $490 - 6 \times 56.31 = 152.14$.

Note that although the amount of ΔVs in Table 1 is the same for the first four rendezvous segments, the amount of fuel used for each transfer progressively decreases. This is because the mass of the service vehicle is reduced as it delivers fuel to the constellation. Hence, the transfers become increasingly more efficient during the refueling process.

The second refueling scenario involves two steps. During the first step, satellite s_0 refuels only three of the satellites in the constellation. The remaining three satellites are refueled by solving a P2P problem during the second step. The total time allowed to complete both steps is again taken as $t_{\mathrm{f}} = 20$. The time allowed for each step is not fixed but it is part of the optimization process. Note that the first three satellites to be refueled are not specified a priori but are also part of the optimization process.

The solution to this problem yields an optimal time to perform the first step of $T_1 = 6.66$ whereas the time to perform the second step is $T_2 = 20 - 6.66 = 13.34$. At the end of the first step satellites s_1, s_2 and s_3 have the same amount of fuel $f_1(T_1^-) = f_2(T_1^-) = f_3(T_1^-) = 129.95$, and satellites s_4, s_5 and s_6 are empty. There are two rendezvous segments. The required ΔVs and the fuel consumption for each of the two rendezvous segments for the first step of this refueling scenario are given in Table 2.

Seg. No.	ΔV_i	p_i
1	0.18212	27.8674
2	0.24635	21.9869

Table 2. Optimal ΔVs and fuel consumption during the first step of the mixed refueling strategy.

The refueling of the three remaining satellites is achieved by implementing a P2P strategy. The optimal pairings turn out to be $s_1 \leftrightarrow s_6$, $s_2 \leftrightarrow s_4$ and $s_3 \leftrightarrow s_5$. The corresponding ΔVs and the fuel consumptions for each rendezvous are given in Table 3.

Rendezvous	ΔV_{ij}	p_{ij}
$1 \rightarrow 6$	0.1133	8.6566
$6 \rightarrow 1$	0.1194	5.9313
$2 \rightarrow 4$	0.2093	15.6820
$4 \rightarrow 2$	0.2205	10.6258
$3 \rightarrow 5$	0.2093	15.6820
$5 \rightarrow 3$	0.2205	10.6258

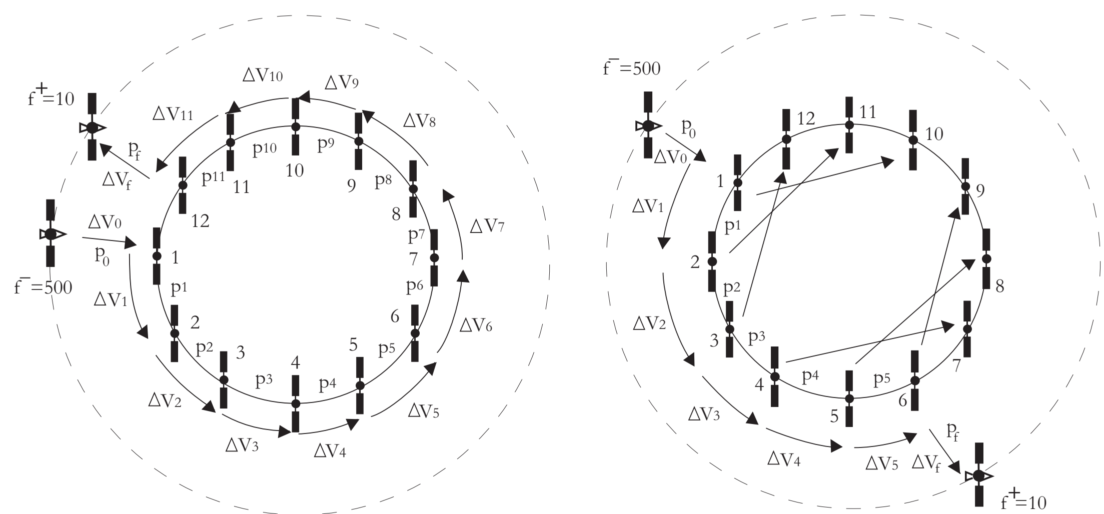
Table 3. Optimal pairs and corresponding ΔVs for P2P stage.

The final amount of fuel contained in each satellite at the end of this refueling process is given by $f_1(t_\mathrm{f}^+) = f_6(t_\mathrm{f}^+) = 57.68$, $f_2(t_\mathrm{f}^+) = f_3(t_\mathrm{f}^+) = f_4(t_\mathrm{f}^+) = f_5(t_\mathrm{f}^+) = 51.82$, leading to an average of $\bar{f} = 53.77$ for the whole constellation. The total amount of fuel used for the transfers is $490 - 6 \times 53.77 = 167.38$.

It follows that the pure single-vehicle strategy is more efficient in this case.

B. Example 2

Consider now a constellation of 12 satellites. The satellites are again evenly distributed along a circular orbit. The total time allowed for refueling is again $t_\mathrm{f} = 20$ time units. The single-vehicle and the mixed single-vehicle/P2P refueling alternatives to be compared are depicted schematically in Fig. 4.



(a) Single Scheduling Strategy

(b) Peer-to-Peer Strategy

Figure 4. Example 2. Constellation with 12 evenly distributed satellites in a circular orbit.

The singe-vehicle refueling strategy involves in this case 11 rendezvous segments. The solution of the single-spacecraft scheduling problem yields the results shown in Table 4. At the end of the refueling process each satellite will end up with an equal amount of fuel $f_i(t_\mathrm{f}^+) = 17.31$ $(1 \leq i \leq n)$. The total amount of fuel consumed during the refueling process is $490 - 12 \times 17.31 = 282.28$ in this case. Note again that although the ΔVs for the first 10 rendezvous segments are equal, the corresponding fuel consumed is progressively decreased, as the mass of the satellite is reduced after each fuel transfer.

Seg. No.	ΔV_i	p_i
1	0.182	35.9811
2	0.182	32.1340
3	0.182	28.5647
4	0.182	25.2531
5	0.182	22.1805
6	0.182	19.3297
7	0.182	16.6847
8	0.182	14.2307
9	0.182	11.9538
10	0.182	9.8412
11	0.380	15.8278

Table 4. Optimal ΔVs and fuel consumption for refueling with a single-spacecraft.

For the second refueling scenario, the first step is completed in $T_1 = 9.55$ units of time and the second step (P2P refueling) is completed in $T_2 = 20 - 9.55 = 10.45$ units of time. During the first step only satellites $s_1, s_2, \ldots, s_6$ are refueled. These satellites will end up with an equal amount of fuel $f_i(T_1^-) = 55.53$ $(1 \leq i \leq 6)$. The first step has five rendezvous segments. The corresponding ΔVs and the fuel used to perform the transfer during each of the five rendezvous segments of the first step are given in Table 5.

Seg. No.	ΔV_i	p_i
1	0.182	33.222
2	0.182	26.8151
3	0.182	20.8707
4	0.182	15.3554
5	0.182	10.2382

Table 5. Optimal ΔVs and fuel consumption during the first step of the mixed refueling strategy.

As before, it is seen that although the ΔVs are equal for each segment, the corresponding fuel consumption is reduced owing to the mass reduction of the service vehicle.

The remaining satellites $s_7, \ldots, s_{12}$ are refueled during the second step of this process using an optimal pairing with satellites $s_1, \ldots, s_6$. The optimal pairs, along with the corresponding values of p_{ij} and ΔV_{ij} are shown in Table 6.

After the second step is completed, the final amounts of fuel in each satellite are as follows: $f_1(t_\mathrm{f}^+) = f_2(t_\mathrm{f}^+) = f_3(t_\mathrm{f}^+) = f_{10}(t_\mathrm{f}^+) = f_{11}(t_\mathrm{f}^+) = f_{12}(t_\mathrm{f}^+) = 19.37, f_4(t_\mathrm{f}^+) = f_5(t_\mathrm{f}^+) = f_6(t_\mathrm{f}^+) = f_7(t_\mathrm{f}^+) = f_8(t_\mathrm{f}^+) = f_9(t_\mathrm{f}^+) = 19.30$, leading to an average fuel of $\bar{f} = 19.34$. The total fuel spent during the orbital transfers is $490 - 12 \times 19.34 = 257.92$. Thus, the mixed single-vehicle/P2P strategy is more efficient than the pure single-vehicle strategy in this case.

VI. Discussion and Extensions

The previous results indicate that a mixed strategy is better as the number of satellites to be refueled increases. As we keep the total refueling time constant ($t_\mathrm{f} = 20$) we can repeat the comparison for different number of satellites in the constellation. The results are shown in Fig. 5 and confirm the conclusion that the mixed strategy with a P2P stage leads to better results as the number os satellites increases. This is due to the fact that as the number of satellites increases, each rendezvous has to be performed in a shorter time interval. A mixed strategy is expected to be more efficient as the total time allowed for refueling is reduced. This is easy to verify by observing, for instance, Fig. 4. To simplify the discussion, let us assume that each

Rendezvous	ΔV_{ij}	p_{ij}
$1 \to 10$	0.2023	9.2317
$10 \to 1$	0.2208	7.5531
$2 \to 11$	0.2023	9.2317
$11 \to 2$	0.2208	7.5531
$3 \to 12$	0.2023	9.2317
$12 \to 3$	0.2208	7.5531
$4 \to 7$	0.2208	10.0386
$7 \to 4$	0.2023	6.8871
$5 \to 8$	0.2208	10.0386
$8 \to 5$	0.2023	6.8871
$6 \to 9$	0.2208	10.0386
$9 \to 6$	0.2023	6.8871

Table 6. Optimal pairs and corresponding ΔVs and fuel consumption for P2P stage.

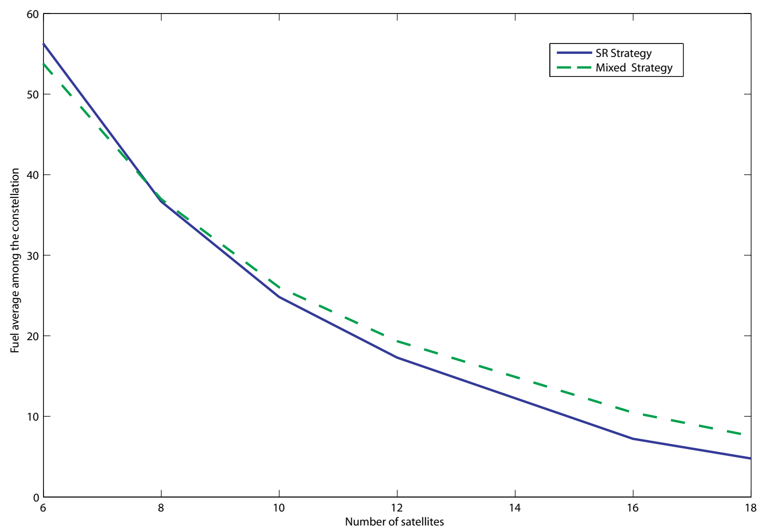


Figure 5. Comparison between the single-vehicle and mixed refueling strategies for various numbers of satellites in the constellation.

rendezvous segment between any satellite pair takes one unit of time. Figure 4(a) then indicates that for s_0 to complete the refueling of the whole constellation will need exactly 11 time units (discard momentarily the initial and final transfers to and from the constellation orbit). Using the mixed strategy shown in Fig. 4(b) it is possible to complete the refueling in approximately $5 + 2 = 7$ time units (5 units for the first step and 2 units for the P2P step). Alternatively, for the same total time, the mixed strategy is expected to be more efficient.

Finally, we note that one can take advantage of the flexibility offered by the P2P architecture to further reduce the cost of a mixed strategy. For instance, one may allow unequal time of travel for the outbound and return trips in the P2P step. Also, one can allow each satellite to complete its trip before all the satellites have been refueled. In other words, s_1 in Fig. 4(b) can initiate its trip to s_{10} without having to wait for s_0 to complete the refueling of s_6. Similarly with satellite s_2 and so forth. Both these extensions lead to an increased time for each rendezvous segment during the P2P step (while the total time still remains the same). Since the fuel consumption decreases monotonically as time increases (cf. Fig. 1), one expects that these extensions will lead to even more efficient results for the mixed strategy. Our preliminary results confirm these predictions.[26]

VII. Conclusions

In this paper we have compared two different refueling strategies of constellations in a circular orbit. The first strategy uses a single spacecraft to refuel the whole constellation. The second strategy distributes the fuel among half the satellites in the constellation which, in turn, refuel the remaining ones by solving a P2P optimization problem. Numerical examples indicate that the latter, mixed strategy, may lead to fuel savings when the number of satellites is large.

Needless to say, delivery/redistribution of fuel is only one case where a P2P and/or mixed rendezvous strategy can be beneficial. Other cases include resupply of consumables, service and repair missions, avionics upgrades, etc. In all these cases, the results of this paper indicate that a distributed delivery of consumables can lead to reduced delivery costs.

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