NEW CONTROL LAWS FOR THE ATTITUDE STABILIZATION OF RIGID BODIES

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Abstract. This paper introduces a new class of globally asymptotically stabilizing feedback control laws for the complete (i.e., dynamics and kinematics) attitude motion of a rotating rigid body. Control laws are given in terms of two new parameterizations of the rotation group derived using stereographic projection. The stabilizing properties of the proposed controllers are proved using Lyapunov functions which involve a quadratic plus a logarithmic term. An interesting feature of the proposed Lyapunov functions is that they often lead to controllers that are linear with respect to the kinematic parameters.

Key Words. Attitude control; stereographic projection; stability; Lyapunov methods.

1. INTRODUCTION

A recent paper (Tsiotras and Longuski, 1994a) introduced a new method for parameterizing the set of rotational matrices, i.e. the kinematics of the rotational motion. The proposed new kinematic formulation involves three parameters for describing the kinematics of the motion, that is, it falls within the same category as the Euler angles and the Cayley-Rodrigues parameters. It is derived by *stereographically projecting* an appropriate set of coordinates (direction cosines) onto their respective projective plane. In this paper one more non-standard, three-dimensional kinematic description is introduced, the derivation of which also uses the idea of the stereographic projection. In this case the stereographic projection is applied to the Euler parameters, instead. The resulting parameters are closely related to the Cayley-Rodrigues parameters but they are superior to them, since they are not limited to eigenaxis rotations of only up to 180 deg.

The scope of this paper is twofold: First to discuss the potential of the new kinematic formulations in control applications, and second to introduce a new type of Lyapunov function which can facilitate control design for attitude control problems. In particular, the problem of the complete, global asymptotic stabilization of the attitude motion, using feedback control is of interest. By *complete* we mean that not only angular velocity stabilization is required, but stabilization of the orientation as well. By *global asymptotic stabilization* we mean that all of the trajectories of the closed loop system remain bounded and tend to zero for arbitrary initial conditions. (It is tacitly assumed that all initial conditions are in the domain of definition of the corresponding differential equations, such that standard arguments about the existence and uniqueness of solutions hold). By *feedback* we mean that we have complete knowledge about the state of the system, which is thus available for use in the control loop. The question of incomplete information about the state (output feedback) is not addressed.

Another implicit assumption made is that the feedback control history can be implemented through gas jet actuators. This can be achieved in practice using, for example, a Pulse-Width Pulse-Frequency Modulator (PWPF) (Wie and Barba, 1985). Since most current gas jets are of the on-off type, a PWPF modulator can be used to produce the continuously varying control profile. This is accomplished by producing a pulse command sequence to the thruster valve by adjusting the pulse width and pulse frequency. The average torque produced by the thruster equals the demanded torque input.

Section 2 of the paper introduces the equations of motion. The main results are given in Section 3. The first two theorems in Section 3 present two known results, but are repeated here the sake of completeness and for comparison. The first result (Theorem 1) gives a linear stabilizing control law in terms of Euler parameters and is due to Mortensen (1968). The second result (Theorem 2) has appeared in several papers (Junkins *et al.*, 1991; Li and Bainum, 1992) and gives a globally stabilizing control law in terms of the Cayley-Rodrigues parameters. The stabilizing control law is based on a quadratic Lyapunov function and is nonlinear. Theorem 3 gives a *linear* feedback control law in terms of the Cayley-Rodrigues parameters, using a new type of Lyapunov function which is not quadratic. Instead, the Lyapunov function is the sum of a quadratic term for the dynamics and a logarithmic term for the kinematics. This is the first new result of the paper. Theorems 4 through 7 are also all new and present stabilizing control laws in terms of the new kinematic parameters. Theorem 8 is an extension of Theorem 4.4 in Tsiotras and Longuski (1994b) for the case of a nonsymmetric rigid body.

2. EQUATIONS OF MOTION

The dynamics of the rotational motion of a rigid body is described by the following set of differential equations

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 + u_1 & \text{(1a)} \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 + u_2 & \text{(1b)} \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 + u_3 & \text{(1c)} \end{aligned}$$

where $\omega_1, \omega_2, \omega_3$ denote the components of the body angular velocity vector $\hat{\omega}$ with respect to the body principal axes, where u_1, u_2, u_3 are the acting torques, and where the positive scalars I_1, I_2, I_3 are the principal moments of inertia of the body with respect to its mass center.

In addition to the dynamics, which provides the time history of the angular velocity vector, the orientation of a rigid body is given by the kinematic equations. If R is the rotation matrix which relates the inertial and the body reference frames, then it obeys the differential equation

$$\dot{R} = S(\hat{\omega})R \tag{2}$$

where $S(\hat{\omega})$ is the skew-symmetric matrix

$$S(\hat{\omega}) := \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} \tag{3}$$

The introduction of the matrix R is necessary because direct integration of $\omega_1, \omega_2, \omega_3$ does not provide any useful information about the orientation of the body. Because of the redundancy of system (2) (R is an orthogonal matrix) one can also determine the orientation if certain parameters are known (Euler parameters, Cayley-Rodrigues parameters, Euler angles). In Tsiotras and Longuski (1994a) a new three-parameter set for describing the kinematics is introduced, which is shown to have some significant advantages over the other classical three-dimensional parameterizations. In Subsection 3.3 another kinematic formulation is introduced, which is also derived using stereographic projection. For a complete discussion on the attitude representations see Stuelpnagel (1964) and the recent survey paper by Shuster (1993).

The main results of the paper are presented in the next section.

3. STABILIZING CONTROLLERS

The main interest is in designing feedback control laws for the attitude motion of a rigid body. A Lyapunov function approach is used to prove global asymptotic stability of the associated closed-loop system. An insightful choice of a certain type of Lyapunov function, involving the sum of a quadratic and a logarithmic term, often allows the design of *linear* feedback control laws. Linear feedback control laws in terms of the four-dimensional parameterization of Euler parameters have been already proposed in the literature (Mortensen, 1968). However, in the case when one uses a non-redundant set of parameters, no *linear* globally stabilizing control law is known thus far. Theorem 3 and Theorem 4 give such linear control laws for the case of *three* kinematic parameters, using the proposed Lyapunov functions.

3.1. *Euler Parameters*

Euler's Principal Rotation Theorem (Kane *et al.*, 1983) states that a completely general angular displacement of a rigid body can be accomplished by a single rotation through an angle Φ (principal angle) about a unit vector $\hat{\mathrm{e}}$ (principal vector), which is fixed in both the body and the inertial frames. If one defines

$$q_0 := \cos\frac{\Phi}{2}, \quad q_i := e_i \sin\frac{\Phi}{2}, \quad (i = 1, 2, 3) \tag{4}$$

where $\hat{\mathrm{e}} = (e_1, e_2, e_3)$ is the principal vector, then the parameters q_0, q_1, q_2, q_3 (Euler parameters) satisfy the following set of linear, ordinary differential equations

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_1 & -\omega_2 & -\omega_3 \\ \omega_1 & 0 & \omega_3 & -\omega_2 \\ \omega_2 & -\omega_3 & 0 & \omega_1 \\ \omega_3 & \omega_2 & -\omega_1 & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{5}$$

From (4) one easily establishes the following Euler parameter constraint

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \tag{6}$$

The four-dimensional parameterization of (4) introduces a redundant parameter in order to achieve a nonsingular description of the motion. It is well-known, for example, that although three is the least number of parameters required to describe the kinematics of a rotating rigid body, every such three-dimensional parameterization of the motion is nevertheless singular (Stuelpnagel, 1964; Shuster 1993). This is the case with the classical three-dimensional parameterizations using Eulerian angles and the Cayley-Rodrigues parameters. Since quaternions provide a global, singularity-free description of the rotational motion, their use has become very popular in control problems (Wie and Barba, 1985; Wie *et al.*, 1989; Vadali *et al.*, 1984).

Mortensen (1968) showed the following result about the attitude stabilization of a rigid body in terms of Euler parameters.

Theorem 1 *The linear feedback control law*

$$u_i = -\omega_i - q_i, \qquad (i = 1, 2, 3) \tag{7}$$

globally asymptotically stabilizes the system of equations (1) and (5).

The proof of this theorem is based on the following Lyapunov function for the system (1) and (5)

$$V = \tfrac{1}{2}\sum_{i=1}^{3} I_i\omega_i^2 + (q_0 - 1)^2 + q_1^2 + q_2^2 + q_3^2 \tag{8}$$

Note that the Lyapunov function in (8) is the sum of two quadratic terms, the first of which involves only the angular velocity and represents the kinetic energy of the rotational motion. The second term involves the kinematic parameters and can be thought of as a "potential" energy-like quantity. With this choice of a Lyapunov function the control law is linear. As will be shown in the sequel, however, a Lyapunov function of the this type (quadratic plus quadratic) leads to nonlinear feedback control laws for other kinematic parameter sets.

3.2. *Cayley-Rodrigues Parameters*

Rodrigues parameters can be used to eliminate the constraint equation (6) associated with the Euler parameter set, thus reducing the number of coordinates necessary to describe the kinematics from four to three. This is achieved by defining

$$\rho_i := \frac{q_i}{q_0}, \qquad (i = 1, 2, 3) \tag{9}$$

The associated kinematic equations then take the form (Kane *et al.*, 1983)

$$\dot\rho_1 = \frac{1}{2}(\omega_1 - \omega_2\rho_3 + \omega_3\rho_2 + \rho_1\sum_{i=1}^{3}\rho_i\omega_i) \tag{10a}$$

$$\dot\rho_2 = \frac{1}{2}(\omega_2 - \omega_3\rho_1 + \omega_1\rho_3 + \rho_2\sum_{i=1}^{3}\rho_i\omega_i) \tag{10b}$$

$$\dot\rho_3 = \frac{1}{2}(\omega_3 - \omega_1\rho_2 + \omega_2\rho_1 + \rho_3\sum_{i=1}^{3}\rho_i\omega_i) \tag{10c}$$

The Rodrigues parameters are also the components of the Gibbs vector defined by

$$\hat\rho = \hat{\mathrm{e}}\tan\frac{\Phi}{2} \tag{11}$$

Clearly, from (11) one can easily establish that the Cayley-Rodrigues parameters cannot be used to describe eigenaxis rotations of more than 180 deg.

The Cayley-Rodrigues parameters were, for the most part, ignored in the literature of attitude dynamics; recently, however, these parameters were found to have an advantage over the Euler parameters for certain control applications (Dwyer, 1985). This explains the revived interest in these kinematic parameters. Control laws based on the Cayley-Rodrigues parameters have been derived for example in (Dwyer, 1985; Junkins *et al.*, 1991; Li and Bainum, 1992; Slotine and Di Benedetto, 1990). The main result concerning global stabilization using the Rodrigues parameters is given in the following theorem (Junkins *et al.*, 1991; Li and Bainum, 1992).

Theorem 2 *The choice of the feedback control*

$$u_i = -\omega_i - \rho_i(1 + \rho_1^2 + \rho_2^2 + \rho_3^2) \tag{12}$$

$(i = 1, 2, 3)$ *globally asymptotically stabilizes the system (1) and (10).*

The proof is based on the use of the following *quadratic* Lyapunov function

$$V = \tfrac{1}{2}\sum_{i=1}^{3} I_i\omega_i^2 + \rho_1^2 + \rho_2^2 + \rho_3^2 \tag{13}$$

Note that the choice of V is closely related to the choice of the feedback control which achieves stability *for this specific* Lyapunov function. Clearly, the nonlinearity associated with the control law (12) is a consequence of the choice of the Lyapunov function (13) used to prove stability. If one chooses a different Lyapunov function, one easily establishes the fact that a *linear* control law suffices to provide global asymptotic stability for the system (1) and (10), as the following theorem shows.

Theorem 3 *The choice of the linear feedback control law*

$$u_i = -\omega_i - \rho_i, \qquad (i = 1, 2, 3) \tag{14}$$

globally asymptotically stabilizes the system (1) and (10).

The proof is based on the following "quadratic plus logarithmic" Lyapunov function

$$V = \tfrac{1}{2}\sum_{i=1}^{3} I_i\omega_i^2 + \ln(1 + \rho_1^2 + \rho_2^2 + \rho_3^2) \tag{15}$$

Here $\ln(\cdot)$ denotes, as usual, the natural logarithm.

From equations (12) and (13) and equations (14) and (15) it is clear that one has, in effect, traded the complexity of the Lyapunov function with the complexity of the resulting control law. Lyapunov functions of the "quadratic plus logarithmic" type were first used in connection with the attitude stabilization problem of an axi-symmetric spacecraft using only two control torques (Tsiotras and Longuski, 1994b).

3.3. *Stereographic Coordinates from Euler Parameters*

In the previous section it was shown that the Cayley-Rodrigues parameters eliminate the redundancy associated with the Euler parameters, but they have the disadvantage of becoming unbounded when $q_0 = 0$. If instead of (9) one eliminates the constraint (6) by introducing the parameters

$$\sigma_i := \frac{q_i}{1+q_0}, \qquad (i = 1,2,3) \tag{16}$$

one obtains the following set of differential equations in terms of σ_i, $(i = 1,2,3)$

$$\dot{\sigma}_1 = \frac{1}{2}(\frac{\omega_1\tilde{\sigma}}{2} + \omega_3\sigma_2 - \omega_2\sigma_3 + \sigma_1\sum_{i=1}^{3}\omega_i\sigma_i) \tag{17a}$$

$$\dot{\sigma}_2 = \frac{1}{2}(\frac{\omega_2\tilde{\sigma}}{2} + \omega_1\sigma_3 - \omega_3\sigma_1 + \sigma_2\sum_{i=1}^{3}\omega_i\sigma_i) \tag{17b}$$

$$\dot{\sigma}_3 = \frac{1}{2}(\frac{\omega_3\tilde{\sigma}}{2} + \omega_2\sigma_1 - \omega_1\sigma_2 + \sigma_3\sum_{i=1}^{3}\omega_i\sigma_i) \tag{17c}$$

where $\tilde{\sigma} := 1 - \sigma_1^2 - \sigma_2^2 - \sigma_3^2$. Definition (16) amounts to a stereographic projection of the unit length vector (q_0, q_1, q_2, q_3), which lies on a four-dimensional unit sphere onto the three-dimensional Euclidean space, using $(-1,0,0,0)$ as the base point (South pole) of the projection (Conway, 1978). The coordinates in (16) do not have the disadvantage of the Rodrigues parameters which do not allow eigenaxis rotations greater than 180 deg. Indeed, from (4) and (16) one easily sees that the σ_i can be viewed as components of the vector

$$\hat{\sigma} = \hat{\mathbf{e}}\tan\frac{\Phi}{4} \tag{18}$$

which is well-defined for all eigenaxis rotations in the range $0 \leq \Phi < 360$ deg.

As Shuster (1993) recently pointed out, these stereographic coordinates of the Euler parameters have been previously derived also by Marandi and Modi (1987), where they were called the *Modified Rodrigues Parameters* and they were used for libration control of spacecraft in orbit. It is surprising, however, that in spite of their obvious advantages over the Rodrigues parameters, their use in attitude determination and control has been, for the most part, ignored. It is hoped that the results of this paper will revive interest in these kinematic parameters.

If one uses these coordinates for the kinematics, one can easily establish the following result.

Theorem 4 *The linear feedback control law*

$$u_i = -\omega_i - \sigma_i, \qquad (i = 1,2,3) \tag{19}$$

globally asymptotically stabilizes the system of equations (1) and (17).

The Lyapunov function for the system (1) and (17). is given by

$$V = \tfrac{1}{2}\sum_{i=1}^{3} I_i\omega_i^2 + \ln(1+\sigma_1^2+\sigma_2^2+\sigma_3^2) \tag{20}$$

It should be clear at this point that the linearity of the control law (19) is a consequence of the choice of the Lyapunov function in (20). As in (15) this is a "quadratic plus logarithmic" type of Lyapunov function. If, instead, one uses the "quadratic plus quadratic" type of Lyapunov function

$$V = \tfrac{1}{2}\sum_{i=1}^{3} I_i\omega_i^2 + 2(\sigma_1^2+\sigma_2^2+\sigma_3^2) \tag{21}$$

one obtains a *nonlinear* feedback control, as the following theorem states.

Theorem 5 *The choice of feedback control*

$$u_i = -\omega_i - \sigma_i(1+\sigma_1^2+\sigma_2^2+\sigma_3^2) \tag{22}$$

$(i = 1,2,3)$ *globally asymptotically stabilizes the system of equations (1) and (17).*

3.4. *Stereographic Coordinates from Direction Cosines*

Along the same lines with the previous section, one can stereographically project one of the columns of the rotation (direction cosine) matrix onto the complex plane, to eliminate the redundancy of the associated kinematic equations. (Recall that the rotation matrix is orthogonal.) If one applies the stereographic projection to the third column of the rotation matrix using

$$\mathrm{w} := \frac{b - i\,a}{1+c} \tag{23}$$

one obtains the kinematic equation (Tsiotras and Longuski, 1994a,c)

$$\dot{\mathrm{w}} = -i\,\omega_3\mathrm{w} + \frac{\omega}{2} + \frac{\bar{\omega}}{2}\mathrm{w}^2 \tag{24}$$

where $\omega := \omega_1 + i\,\omega_2$, where $\mathrm{w} := \mathrm{w}_1 + i\,\mathrm{w}_2$, and where the bar denotes complex conjugate. In equation (23) a, b, c are the direction cosines of the body 3-axis of the rigid body with respect to the inertial axes, and the base point of the projection is chosen to be the South pole $(-1,0,0)$ of the unit three-dimensional sphere. The reason for choosing the South pole as the base point of the stereographic projection in equations (23) and (16) is to move the inherent singularity associated with the kinematic parameterization ($\mathrm{w} = \infty$ in (23) and $\sigma_i = \infty$ in (16)) as far away from the respective equilibrium points ($\mathrm{w} = 0$ and $\sigma_i = 0$) as possible. In this way, global asymptotic stability of the respective kinematic equations corresponds to asymptotic stability from the *largest* possible set of physical configurations, which does not include the (isolated) singular point.

The third coordinate necessary to complement (24) and complete the kinematics obeys the differential equation

$$\dot{z} = \omega_3 + Im(\omega \bar{\mathrm{w}}) \tag{25}$$

It can be shown that the (w, z) kinematic parameterization of the rotational motion implies that the body orientation can be described by means of two successive rotations. The first is a rotation through an angle z about the 3-axis, and the second is a rotation through an angle $\arccos c$ about the unit vector $(b/\sqrt(a^2+b^2), -a/\sqrt(a^2+b^2), 0)$.

Using these coordinates one has the following stabilizing control law.

Theorem 6 *The choice of feedback control*

$$u = -\omega - \mathrm{w}(1 + i\,z), \quad u_3 = -\omega_3 - z \tag{26}$$

where $u = u_1 + i\,u_2$ and $\omega = \omega_1 + i\,\omega_2$, globally asymptotically stabilizes the system (1), (24) and (25).

Here the following Lyapunov function for the associated closed-loop system is used

$$V = \tfrac{1}{2}\sum_{i=1}^{3} I_i \omega_i^2 + \ln(1 + |\mathrm{w}|^2) + \tfrac{1}{2}z^2 \tag{27}$$

were $|\cdot|$ denotes the absolute value of a complex number, i.e., $\mathrm{w}\bar{\mathrm{w}} = |\mathrm{w}|^2$, $\mathrm{w} \in \mathbb{C}$.

Using the quadratic Lyapunov function

$$V = \tfrac{1}{2}\sum_{i=1}^{3} I_i \omega_i^2 + |\mathrm{w}|^2 + \tfrac{1}{2}z^2 \tag{28}$$

instead, one can show the following theorem.

Theorem 7 *The choice of feedback control*

$$u = -\omega - \mathrm{w}(1 + |\mathrm{w}|^2) - i\,z\mathrm{w} \tag{29a}$$

$$u_3 = -\omega_3 - z \tag{29b}$$

were $u = u_1 + i\,u_2$ and $\omega = \omega_1 + i\,\omega_2$, globally asymptotically stabilizes the system (1), (24) and (25).

For $\mathrm{w} = 0$ one obtains stabilization of the rigid body about one of its body-axes. If w was defined using the stereographic projection of the third column of the rotation matrix, as in (23), then $\mathrm{w} = 0$ implies stabilization of the body 3-axis about the inertial 3-axis. In such a case, the final motion is simply a stable rotation of the rigid body about its body 3-axis. If one is interested only in stabilization about one of the body axes, then equation (24) alone suffices to describe the kinematics, since (25) plays no role because it describes (roughly) the relative rotation of the body about this body axis. Since in all such cases the motion of the body naturally decomposes into the motion of one of the body axes and, essentially, a rotation about this axis, it appears that the (w, z) coordinates are preferable for designing feedback control laws. A feedback control law which globally asymptotically stabilizes a rigid body about its body 3-axis can be constructed, for example, as follows.

Theorem 8 *The linear feedback control*

$$u = -\omega - \mathrm{w} \tag{30}$$

globally asymptotically stabilizes the sub-system (1a), (1b) and (24). That is, for arbitrary initial conditions, $\lim_{t\to\infty}(\omega(t), \mathrm{w}(t)) = (0, 0)$.

The Lyapunov function

$$V = \tfrac{1}{2}\sum_{i=1}^{3} I_i \omega_i^2 + \ln(1 + |\mathrm{w}|^2) \tag{31}$$

is used to show this result.

The linear control law (30) is used in Tsiotras and Longuski (1994b) to stabilize the (ω, w) system in the case of an axially symmetric rigid body. Theorem 8 states that the same control law will also work for a *nonsymmetric body*, if one is interested only in stabilization to a simple rotation about the body 3-axis. Another interesting feature of the control law (30) is that it can be implemented using only *two* control actuators about principal axes. In this case, however, (30) cannot be used to control the final value of ω_3.

4. CONCLUDING REMARKS

A new approach for designing linear and nonlinear feedback control laws for the attitude stabilization of a rigid body has been presented. In particular, a new type of Lyapunov function for this class of problems is proposed, which often leads to linear control laws. The proposed Lyapunov functions include a quadratic term for the angular velocities (kinetic energy) and a logarithmic term for the kinematic parameters. Control laws are given both in terms of traditional kinematic parameterizations (Euler parameters and Cayley-Rodrigues parameters) and in terms of two new kinematic parameterizations derived by stereographic projection. All control laws are given in their most natural form, i.e. taking all control gains to be unity. Specific applications will require tuning the control gains to meet time and/or energy requirements. All of the control laws proposed in this paper have the property that they do not require information about the body principal moments of inertia and they are therefore *robust* with respect to system parametric uncertainty.

As a final remark, note that the issue of singularities associated with the three-dimensional parameterizations has not been explicitly addressed. The global stability of the closed loop-system, however, will *a posteriori* guarantee that the state variables remain bounded and thus, the body will avoid these singular orientations.

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