A Passivity Approach to Attitude Stabilization Using Nonredundant Kinematic Parameterizations

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Abstract

In a recent paper we showed that there exist linear controllers which globally asymptotically stabilize the attitude motion of a rigid body using a nonredundant, three-dimensional set of kinematic parameters. In this paper we show, using the inherent passivity properties of the system, that these results can be extended to stabilizing control laws without any angular velocity measurements. A numerical example demonstrates the theoretical results.

1. Introduction

In a recent paper [14] we have shown that there exist *linear* globally asymptotically stabilizing control laws for the attitude motion of a rigid body using minimal, three-dimensional parameterizations for the kinematics. In particular, in [14] we derived linear control laws in terms of the classical Cayley-Rodrigues parameters [4, 11] and the newly developed Modified Rodrigues parameters [7, 10, 11, 13, 14]. The approach in [14] uses a Lyapunov function which is the sum of a quadratic term in the angular velocities and a logarithmic term in the kinematic parameters.

In the present paper we use the inherent passivity properties of the system, as well as the structural properties of the kinematic equations in terms of the Rodrigues and the Modified Rodrigues parameters, in order to derive control laws which do not use angular velocity measurements. We therefore provide a limited solution to the general output feedback problem for the attitude motion, i.e., when only orientation information is available. The methodology follows very closely the one in [6], where the authors use a quaternion description for the kinematics in order to derive velocity-free, globally asymptotically stabilizing control laws for the attitude motion.

Control laws which do not use angular velocity measurements, apart of theoretical interest, can be very important in applications as well, especially in cases when one wants to avoid the use of angular velocity sensors (e.g., for economical reasons) or when such a choice is not available (e.g., due to equipment failure). In aerospace applications (e.g., spacecraft) the latter case is probably more relevant, since attitude orientation sensors (e.g., sun or star sensors, horizon scanners or gyroscopes) are relatively bulky and expensive as compared to angular velocity sensors. In robotics, on the other hand, elimination of tachometers is highly desirable [1, 8].

2. Equations and Preliminary Results

The angular velocity for the attitude motion of a rigid body obeys the differential equation

$$J\dot{\omega} = S(\omega)J\omega + u, \qquad \omega(0) = \omega_0 \tag{1}$$

where $\omega := (\omega_1, \omega_2, \omega_3)^T$ denotes the angular velocity vector in a body-fixed frame, $u := (u_1, u_2, u_3)^T$ is the acting torque vector, and J is the inertia matrix. The matrix $S(\cdot)$ denotes the skew-symmetric matrix

$$S(\omega) := \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

In this paper, the orientation of the body with respect to the inertial frame will be described either in terms of the Rodrigues parameters [4, 11], or in terms of the recently developed Modified Rodrigues parameters [7, 10, 11, 13, 14]. The kinematic equations in terms of the Rodrigues parameters take the form

$$\dot{\rho} = H(\rho)\omega, \qquad \rho(0) = \rho_0 \tag{3}$$

where

$$H(\rho) := \tfrac{1}{2}(I - S(\rho) + \rho\rho^T) \tag{4}$$

and I denotes the 3×3 identity matrix.

The kinematic equations in terms of the Modified Rodrigues parameters take the form

$$\dot{\sigma} = G(\sigma)\omega, \qquad \sigma(0) = \sigma_0 \tag{5}$$

where

$$G(\sigma) = \tfrac{1}{2}\left(\frac{1 - \sigma^T\sigma}{2} I - S(\sigma) + \sigma\sigma^T\right) \tag{6}$$

Although the kinematic description using the Modified Rodrigues parameters looks similar to the classical Cayley-Rodrigues parameters, it has the distinct advantage that it remains valid for eigenaxis rotations up to 360 deg, whereas the Cayley-Rodrigues parameters cannot describe motions which correspond to eigenaxis rotations of more than 180 deg [10, 13, 14].

It can be easily shown that the matrices $H(\rho)$ and $G(\sigma)$ in (4) and (6) satisfy the following important identities:

(H1) $$\rho^T H(\rho)\omega = \left(\frac{1 + \rho^T\rho}{2}\right)\rho^T\omega,$$

(H2) $$H^T(\rho)(I + \rho\rho^T)^{-1}H(\rho) = \left(\frac{1 + \rho^T\rho}{4}\right)I$$

for all $(\omega, \rho) \in \mathbb{R}^3 \times \mathbb{R}^3$, and

$$\textbf{(G1)} \qquad \sigma^T G(\sigma)\omega = \left(\frac{1+\sigma^T\sigma}{4}\right)\sigma^T\omega$$

$$\textbf{(G2)} \qquad G^T(\sigma)G(\sigma) = \left(\frac{1+\sigma^T\sigma}{4}\right)^2 I$$

for all $(\omega, \sigma) \in \mathbb{R}^3 \times \mathbb{R}^3$, respectively.

The equations (1)-(3), equivalently (1)-(5), describe a system in *cascade interconnection.* Thus, if one initially considers only the subsystem (3) with ω acting as a control variable, and then one considers the complete system (1)-(3) with u as the control variable, one can show the following result.

Proposition 2.1 ([14]) *(i) Consider the system (3) with ω considered as a control variable. The choice of the linear feedback control law*

$$\omega = -k_1\rho \tag{7}$$

with $k_1 > 0$, globally exponentially stabilizes (3) at the origin with rate of decay $\frac{k_1}{2}$.

(ii) The linear control law

$$u = -k_1\rho - k_2\omega \tag{8}$$

with $k_1 > 0$ and $k_2 > 0$, globally asymptotically stabilizes the system (1)-(3) at the origin.

Proof. (i) The positive definite function

$$U(\rho) = \ln(1 + \rho^T\rho) \tag{9}$$

where $\ln(\cdot)$ denotes the natural logarithm, is a Lyapunov function for the closed-loop system (3)-(7).

(ii) The positive definite function

$$V(\omega, \rho) = \tfrac{1}{2}\omega^T J\omega + k_1 U(\rho) \tag{10}$$

is a Lyapunov function for the closed-loop system (1), (3)-(8).

■

Similarly, for the Modified Rodrigues parameters we have

Proposition 2.2 ([14]) *(i) Consider the system (5) with ω considered as a control variable. The choice of the linear feedback control law*

$$\omega = -k_1\sigma \tag{11}$$

with $k_1 > 0$, globally exponentially stabilizes (5) at the origin with rate of decay $\frac{k_1}{2}$.

(ii) The linear control law

$$u = -k_1\sigma - k_2\omega \tag{12}$$

with $k_1 > 0$ and $k_2 > 0$, globally asymptotically stabilizes the system (1)-(5) at the origin.

Proof. (i) The positive definite function

$$W(\sigma) = \ln(1 + \sigma^T\sigma) \tag{13}$$

is a Lyapunov function for the closed-loop system (5)-(11).

(ii) The positive definite function

$$V(\omega, \rho) = \tfrac{1}{2}\omega^T J\omega + k_1 W(\sigma) \tag{14}$$

is a Lyapunov function for the closed-loop system (1), (5)-(12).

■

Remark 2.1 One word of caution should be mentioned at this point, as far as our use of the term "global" stabilization is concerned. Strictly speaking, the attitude motion of a rigid body cannot be globally continuously stabilized since the configuration space of the motion (the rotation group $SO(3)$) is non-contractible. Thus, by "global asymptotic stabilization" we mean here that the system of the corresponding *kinematic parameters* is globally asymptotically stable, i.e., asymptotic stability is guaranteed for all initial orientations not corresponding to singular configurations. From a practical point of view this slight abuse of the terminology should not cause any concern, since the domain of definition of the kinematic parameters is a dense subset of the configuration space (actually in the case of the Modified Rodrigues parameters is the whole space minus a single, isolated point) [14].

3. A Passivity Approach

Propositions 2.1 and 2.2 state that there exist linear controllers which globally asymptotically stabilize the attitude motion of a rigid body using *nonredundant* sets of kinematic parameters. In this section we show that these results can be extended to stabilizing control laws without any angular velocity measurements. In particular, we show that the linear control laws (8) and (12) can be implemented without angular velocity feedback and thus, one only needs orientation measurements.

The methodology used in this section follows very closely the one in [6], where the authors use a quaternion description for the kinematics in order to derive velocity-free, globally asymptotically stabilizing control laws for the attitude motion of a rigid body. This approach is similar to the recent results of [1] and [9] on output stabilization of Euler-Lagrange systems, where it is shown that asymptotic stabilization for such systems may be possible without velocity measurements via the inclusion of a dynamic extension (lead filter) to the system. The so-called "dirty derivative" controllers of [9] provide the necessary damping for the global stabilization of the closed-loop system. There is an important complication when dealing with the attitude stabilization problem, however, since one is not able to reconstruct the attitude parameters by integrating the angular velocity measurements.

Our approach, like the one in [6] takes advantage of the inherent passivity properties of the attitude equations and some structural properties of the kinematic equations. Passive systems are very appealing in practice because of their very attractive robustness and stability characteristics. In particular, passivity is invariant under feedback interconnection, and the Passivity Theorem [2] states that the feedback interconnection of a passive and a strictly passive system is globally asymptotically stable. The results of this section depend in an essential way on the properties **(H2)** and **(G2)** of the matrices $H(\rho)$ and $G(\sigma)$ respectively.

Preliminaries

According to [2], a well-defined system

$$\dot{x} = f(x,u), \qquad x(0) = x_0 \tag{15a}$$
$$y = h(x,u) \tag{15b}$$

with input $u \in \mathbb{R}^m$ and output $y \in \mathbb{R}^m$ is called *passive* if there exists a constant $\beta = \beta(x_0)$ such that

$$\int_0^T y^T u\, dt + \beta \geq 0 \tag{16}$$

for all $T \geq 0$.

It is called *strictly passive* (better, input-strictly passive) if there exist constants $\delta > 0$ and $\beta = \beta(x_0)$ such that

$$\int_0^T y^T u\, dt + \beta \geq \delta \int_0^T \|u\|^2\, dt \tag{17}$$

for all $T \geq 0$.

The next Proposition shows that the equations (1)-(3) and (1)-(5) have some inherent passivity properties.

Proposition 3.1 *(i) The system (1) with input u and output ω is passive.*

(ii) The system (5) with input ω and output σ is passive.

(iii) The system (3) with input ω and output ρ is passive.

Proof. (i) Let the function $V_1(\omega) = \frac{1}{2}\omega^T J \omega$. Differentiation along the trajectories of (1) yields that $\dot{V}_1(\omega) = \omega^T u$, therefore

$$\int_0^T \omega^T u\, dt = V_1(\omega(T)) - V_1(\omega_0) \tag{18}$$

and since $V_1(\omega) \geq 0$ for all $\omega \in \mathbb{R}^3$ we have that

$$\int_0^T \omega^T u\, dt + V_1(\omega_0) \geq 0 \tag{19}$$

and (16) is satisfied with $\beta = V_1(\omega_0)$.

(ii) Let the function $V_2(\sigma) = 2\ln(1+\sigma^T\sigma)$. Differentiation along the trajectories of (5) and use of **(G1)** yields that $\dot{V}_2(\sigma) = \sigma^T \omega$, therefore

$$\int_0^T \sigma^T \omega\, dt = V_2(\sigma(T)) - V_2(\sigma_0) \tag{20}$$

and since $V_2(\sigma) \geq 0$ for all $\sigma \in \mathbb{R}^3$ we have that

$$\int_0^T \sigma^T \omega\, dt + V_2(\sigma_0) \geq 0 \tag{21}$$

and passivity is satisfied with $\beta = V_2(\sigma_0)$.

(iii) The proof is identical to the case (ii), where we now use the positive definite function $V_3(\rho) = \ln(1 + \rho^T\rho)$ and property **(H1)**. ∎

The previous Proposition shows that the attitude equations can be considered as a cascade interconnection of two passive systems. The passivity of system (1) is a well-known fact and has been used repeatedly in the past. The passivity of system (5) or of the system (3), is neither as a well-known nor as a frequently used result. Although we will not use the passivity of the kinematic equations in this paper, we mentioned this property here for completeness. It would be interesting though to investigate if the passivity of (5) or (3) can be used in some other framework.

In Proposition 2.2 we have shown that the linear control law

$$u = -k_1\sigma - k_2\omega$$

globally asymptotically stabilizes the system (1)-(5). Consider now the more general control law

$$u = -k_1\sigma + v \tag{22}$$

with $k_1 > 0$, where v is the new input. The following Theorem shows that the passivity between the new input v and the output ω is preserved for the system (1)-(5); see also Fig. 1.

Proposition 3.2 *The system (1),(5)-(22) with input v and output ω is passive.*

Proof. Let the function $V(\omega,\sigma) = V_1(\omega) + k_1 V_2(\sigma)$ where V_1 and V_2 as in Proposition 3.1. Differentiation along the trajectories of (1) yields that $\dot{V}(\omega,\sigma) = \omega^T u + k_1\sigma^T\omega$. Using (22) we get that $\dot{V}(\omega,\sigma) = \omega^T v$. The rest of the proof follows as in Proposition 3.1. ∎

One can show a similar result for the control law

$$u = -k_1\rho + v \tag{23}$$

where $k_1 > 0$ and v is the new input.

Proposition 3.3 *The system (1),(3)-(23) with input v and output ω is passive.*

Properties **(G2)** and **(H2)** imply an "orthogonality" condition for the matrices $G(\sigma)$ and $H(\rho)$; in particular, the inverses of these matrices can be written as a function of their transposes. One can easily verify, for instance, that

$$H^{-1}(\rho) = \left(\frac{4}{1+\rho^T\rho}\right)(I + \rho\rho^T)^{-1} H^T(\rho) \tag{24}$$

and

$$G^{-1}(\sigma) = \left(\frac{4}{1+\sigma^T\sigma}\right)^2 G^T(\sigma) \tag{25}$$

One can use this result to establish "orthogonal" input/output transformations for the systems (1),(5)-(22) and (1),(3)-(23) which preserve passivity.

Proposition 3.4 *The system (1)-(5) with input* $y = \left(\frac{4}{1+\sigma^T\sigma}\right)^2 G(\sigma)v$ *and output* $w = G(\sigma)\omega = \dot{\sigma}$ *is passive.*

Proof. Using **(G2)** we have that

$$\begin{aligned}\int_0^T w^T y\,dt &= \int_0^T \left(\frac{4}{1+\sigma^T\sigma}\right)^2 \omega^T G^T(\sigma)G(\sigma)v\,dt \\ &= \int_0^T \left(\frac{1+\sigma^T\sigma}{4}\right)^2 \left(\frac{4}{1+\sigma^T\sigma}\right)^2 \omega^T v\,dt \\ &= \int_0^T \omega^T v\,dt \qquad (26)\end{aligned}$$

Using now Proposition 3.2 we establish the desired result. ■

Notice that if y is the new input as defined by Proposition 3.4 then v is given by

$$v = G^T(\sigma)y \qquad (27)$$

Similarly, for the case of the Rodrigues parameters one has that

Proposition 3.5 *The system (1)-(3) with input* $y = \left(\frac{4}{1+\rho^T\rho}\right)(I+\rho\rho^T)^{-1}H(\rho)v$ *and output* $w = H(\rho)\omega = \dot{\rho}$ *is passive.*

Proof. The proof is similar to the one of Proposition 3.4 and thus, omitted. ■

Notice that if y is the new input as defined by Proposition 3.5 then v is given by

$$v = H^T(\rho)y \qquad (28)$$

Main Results

Since by Propositions 3.4 and 3.5 the map from y to w is passive, one may explore the possibility of globally asymptotically stabilizing the system by choosing a feedback such that the map from w to y is strictly passive [2, 3]. Although one can choose any strictly passive, possibly nonlinear, map between w and y, the easiest approach is to use a linear, time invariant, strictly passive system instead. Recall that a linear time-invariant system is strictly passive if and only if it is strictly positive real [12]. Moreover, the Kalman-Popov-Yakubovic Lemma provides testable conditions on the state space realization of a linear system so that it is strictly positive real [2, 5, 12]. Based on these preliminary observations, we are now ready to show the main Theorem on the implementation of the linear control laws (8) and (12) without angular velocity feedback. We first present the result in terms of the Modified Rodrigues parameters, followed by the corresponding result in terms of the Cayley-Rodrigues parameters.

To this end, let A be any stability matrix, B any full column rank matrix, with the pair (A, B) controllable, and Q any positive definite matrix. Let also the matrix P be the solution of the Lyapunov equation

$$A^T P + PA = -Q \qquad (29)$$

Clearly then P is positive definite.

Theorem 3.1 *Consider the system (1)-(5) and let the control law*

$$u = -k_1\sigma - k_2 G^T(\sigma)y \qquad (30)$$

with $k_1 > 0$, $k_2 > 0$, *and where* y *is the output of the linear, time-invariant system*

$$\begin{aligned}\dot{x} &= Ax + B\sigma \qquad (31a)\\ y &= B^T PAx + B^T PB\sigma \qquad (31b)\end{aligned}$$

Then $\lim_{t\to\infty}(\omega(t), \sigma(t)) = 0$, *for all initial conditions* $(\omega_0, \sigma_0) \in \mathbb{R}^3 \times \mathbb{R}^3$.

Proof. Consider the positive definite function

$$V(\omega, \sigma, \dot{x}) = \tfrac{1}{2}\omega^T J\omega + 2k_1 \ln(1+\sigma^T\sigma) + \frac{k_2}{2}\dot{x}^T P\dot{x} \qquad (32)$$

The time derivative of V along the trajectories of the closed-loop system is then

$$\begin{aligned}\dot{V} &= \omega^T J\dot{\omega} + k_1\left(\frac{4}{1+\sigma^T\sigma}\right)\sigma^T G(\sigma)\omega + k_2\dot{x}^T P\ddot{x} \\ &= \omega^T(-k_1\sigma - k_2 G^T(\sigma)y) + k_1\sigma^T\omega + k_2\dot{x}^T PA\dot{x} \\ &\quad + k_2\dot{x}^T PBG(\sigma)\omega \\ &= \frac{k_2}{2}\dot{x}^T(PA + A^T P)\dot{x} = -\frac{k_2}{2}\dot{x}^T Q\dot{x} \le 0 \qquad (33)\end{aligned}$$

First observe that since V is radially unbounded, all solutions are bounded. Consider now the set $\mathcal{E} = \{(\omega, \sigma, x) : \dot{V} = 0\}$. Trajectories in $\mathcal{E}$ then satisfy $\dot{x} = 0$ and hence $x(t) = x_0$ for all $t \ge 0$ and from (31a) also $\sigma(t) = \sigma_0$ for all $t \ge 0$. Then $\dot{\sigma} = 0$ and from (5) also $\omega(t) = 0$ for all $t \ge 0$. Since $y = B^T P\dot{x}$ one has also that $y = 0$, and using (1) and (30) we have that $\omega = \dot{\omega} = 0$ and $y = 0$ implies that $\sigma = 0$. The largest invariant set in $\mathcal{E}$ is therefore the set $\mathcal{M} = \{(\omega, \sigma, x) \in \mathcal{E} : \omega = 0, \sigma = 0, x = x_0\}$.

By LaSalle's Invariance Principle [5] all trajectories of the system asymptotically approach $\mathcal{M}$, thus $\lim_{t\to\infty}(\omega(t), \sigma(t)) = 0$, as claimed. ■

Remark 3.1 The motivation for choosing the linear system (31) stems from the following fact. If we realize (31) as

$$\begin{aligned}\dot{x} &= z \qquad (34a)\\ \dot{z} &= Az + B\dot{\sigma} \qquad (34b)\\ y &= B^T Pz \qquad (34c)\end{aligned}$$

and use the Kalman-Yakubovic-Popov Lemma [5], we see that equations (34b)-(34c) define a minimal realization of a strictly positive real system with input $\dot{\sigma}$ and output y. From Proposition 3.4 the system (1)-(5) defining a map from y to $w = \dot{\sigma}$ is passive. Some technical assumptions aside, the asymptotic stability of the feedback system then follows as a result of a feedback interconnection of a passive with a strictly passive system. By imposing the additional condition that the linear system from $\dot{\sigma}$ to y is strictly proper, we can realize the map from σ to y with a proper system as in (31).

Similarly, for the Cayley-Rodrigues parameters one obtains the following result.

Theorem 3.2 *Consider the system (1)-(3) and let the control law*

$$u = -k_1\rho - k_2 H^T(\rho)y \quad (35)$$

with $k_1 > 0$, $k_2 > 0$, and where y is the output of the linear, time-invariant system

$$\dot{x} \;=\; Ax + B\rho \quad (36a)$$

$$y \;=\; B^T PAx + B^T PB\rho \quad (36b)$$

Then $\lim_{t\to\infty}(\omega(t), \rho(t)) = 0$, for all initial conditions $(\omega_0, \rho_0) \in \mathbb{R}^3 \times \mathbb{R}^3$.

The proof of this Theorem is similar to the one of Theorem 3.1 and it will not be repeated here. Just notice that by choosing the positive definite function

$$V(\omega, \rho, \dot{x}) = \tfrac{1}{2}\omega^T J\omega + k_1 \ln(1+\rho^T\rho) + \frac{k_2}{2}\dot{x}^T P\dot{x} \quad (37)$$

we have for the closed-loop trajectories of the system (1)-(3) with control law (35) that

$$\begin{aligned} \dot{V} &= -k_2\omega^T H^T(\rho)y + k_2\dot{x}^T P\ddot{x} \\ &= -k_2\omega^T H^T(\rho)y + k_2\dot{x}^T PA\dot{x} + k_2\dot{x}^T PB\dot{\rho} \\ &= k_2\dot{x}^T PA\dot{x} \le -\frac{k_2}{2}\dot{x}^T Q\dot{x} \le 0 \end{aligned} \quad (38)$$

and the rest of the proof follows as in Theorem 3.1.

4. Numerical Example

In this section we demonstrate the previous theoretical results by means of a numerical simulation. In particular, we provide an example comparing the linear control laws (8) and (12) with their velocity-free implementations (35) and (30).

We consider a rigid body with inertia parameters

$$J = diag(10, 6.3, 8.5) \quad (kg \cdot m^2)$$

and subject to zero initial angular velocity and initial orientation given, in terms of the Rodrigues parameters, by

$$\rho(0) = (0.7625, 0.3165, 1.3207)^T$$

or, in terms of the Modified Rodrigues parameters, by

$$\sigma(0) = (0.2675, 0.1110, 0.4633)^T$$

The following values were chosen for the gains and the filter parameters

$$k_1 = 2, \quad k_2 = 1$$

$$A = -10I, \quad B = 10I, \quad Q = 20I, \quad P = I \quad (39)$$

The simulation results for the Rodrigues parameters are shown in Figures 2-3. The solid lines represent the trajectories with the linear control law (8) and the dashed lines represent the trajectories with the velocity-free control law (35). Only the first components of the vectors are depicted here, since the other components exhibit similar behavior. Similarly, Figures 4-5 depict the results for the Modified Rodrigues parameters.

Clearly, the velocity-free controllers stabilize the system about the zero orientation as predicted by Theorems 3.1 and 3.2.

5. Concluding Remarks

We have derived linear globally asymptotically stabilizing control laws for the attitude motion of a rigid body in terms of nonredundant kinematic parameters. Based on the inherent passivity properties of the equations of the attitude motion, we have shown how to implement these linear control laws when angular velocity measurements are not available. The proposed controllers use filtered measurements of the kinematic parameters in order to estimate the angular velocities from attitude measurements. These results are in line with the recent developments on the stabilization of Euler-Lagrange mechanical systems [9], where it was shown that, under some mild assumptions, there is a rigorous theoretical justification for the commonly used practice of "approximate" differentiation using lead filters when velocity measurements are not available.

Finally, the results are given in terms both of the classical Cayley-Rodrigues parameters, and the recently developed Modified Rodrigues parameters.

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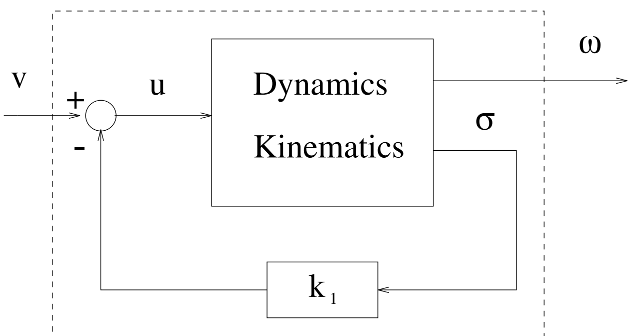

Figure 1: Passive connection with control $u = k_1\sigma + v$.

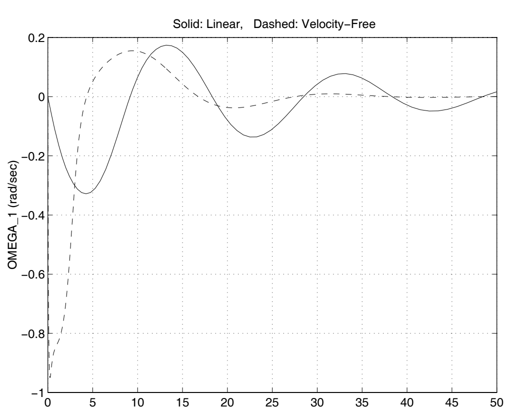

Figure 2: Angular velocity history for Rodrigues parameters.

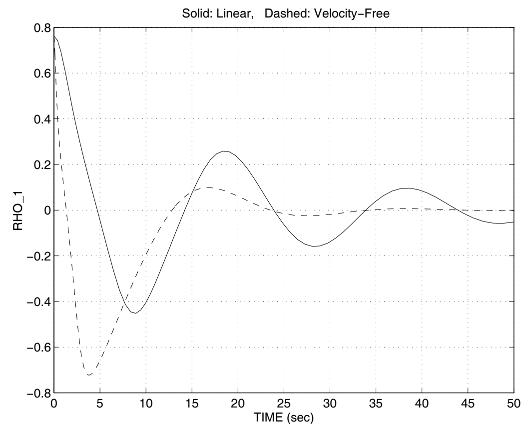

Figure 3: Rodrigues parameters history.

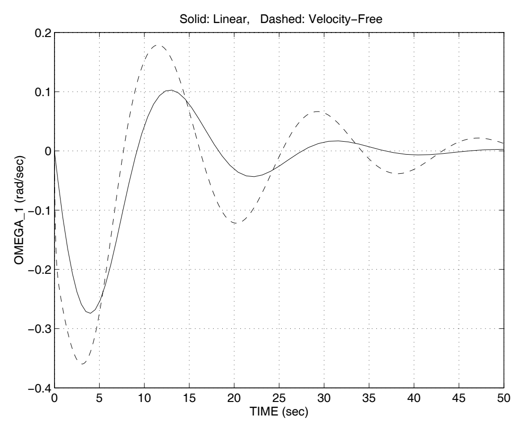

Figure 4: Angular velocity history for Modified Rodrigues parameters.

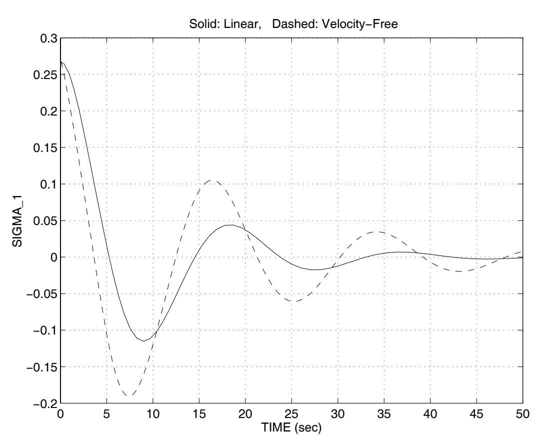

Figure 5: Modified Rodrigues parameters history.