Control of Zero-Bias Magnetic Bearings Using Control Lyapunov Functions

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Abstract

Zero-bias or low-bias control of AMB's is essential for the design of magnetic bearings with low power losses. This paper investigates the use of *control Lyapunov functions* (clf's) to solve the problem of global asymptotic stabilization for a zero-bias Active Magnetic Bearing (AMB). We use the cascade structure of the system to derive a clf for the AMB model which is then used to derive a stabilizing control law. This control law is designed so that the closed-loop system is homogeneous of degree with respect to a certain dilation. Simulation results are provided to evaluate the proposed control design.

1 Introduction

Recently there has been a lot of interest in the use of actively controlled magnetic bearings to support high-speed flywheels for energy storage. Since the power losses at the bearings are proportional to the flux, it is imperative to minimize the flux at the bearings. Owing to the nonlinear relation between the flux through the coils and the force generated at the bearings, the control design for low- and zero-bias AMB's is more challenging than for the nonzero bias-current case. Traditional design of AMB's, for instance, uses a bias current of $I_0 \approx 0.5\, I_{\max}$ to linearize the system about I_0 followed by the use of linear control design techniques.

A comparison between linear and nonlinear operation for a 5-DOF AMB has been provided by Charara et al. [6], [7] and Smith and Weldon [8]. Specifically, disturbance rejection and power consumption issues are discussed in these references. The zero-bias nonlinear techniques developed in these studies include feedback linearization and sliding mode control. Lévine et al. [9] developed an alternative zero-bias control method by studying the system's differential flatness properties. Low-bias techniques for control design have also been studied in [10, 11] and [13]. In [10] de Queiroz et al. applied the integrator-backstepping method to a 2-DOF model and in [11] to a 6-DOF model. In [13] the authors developed a gain-scheduled H_∞ control scheme for the low bias control of an AMB.

2 AMB Model

Next, we briefly introduce the model of a zero-bias AMB used in this paper. Figure 1 shows a simple schematic of the AMB.

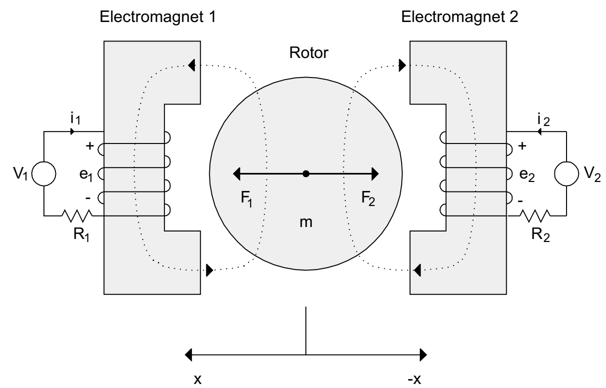

Figure 1: Schematic of an Active Magnetic Bearing.

The simplified AMB model consists of two electromagnets used to move a mass m in one dimension. To regulate the position x of the mass to zero, the control designer uses the voltage inputs, V_1 and V_2, to vary the forces acting on the rotor mass. In this study, the voltage inputs are chosen such that only one electromagnet is active at any given time[1]. In this manner, the two electromagnets do not produce forces that oppose each other, reducing the overall current and thus the ensuing power losses. The term "zero-bias" means that bias currents or voltages are avoided during operation. With this in mind, one can introduce a "generalized" flux to implement a complementarity flux condition as follows

$$\Phi \geq 0 \Rightarrow \Phi_1 = \Phi, \quad \Phi_2 = 0$$
$$\Phi < 0 \Rightarrow \Phi_1 = 0, \quad \Phi_2 = \Phi$$

One can then verify that the net force acting on the bar is given by [2]

$$F(\Phi) = F_1(\Phi_1) - F_2(\Phi_2) = \frac{1}{\mu_0 A_g}\Phi|\Phi| \qquad (1)$$

The variable Φ is the magnetic flux through each active coil, μ_0 is the permeability of free space ($= 1.25 \times 10^{-6} H/m$) and A_g is the area of each electromagnet pole.

[1] This condition is often referred to in the literature as the *complementary current condition*. Here we use a slightly different approach as we impose a *complementary flux condition*. [2].

From Newton's law the mechanical dynamics are

$$m\ddot{x} = \frac{1}{\mu_0 A_g}\Phi|\Phi|$$

Faraday's law gives the electrical dynamics as simply (coil resistance neglected)

$$\dot{\Phi} = \frac{e}{N} \tag{2}$$

where N is the number of turns of the coil of each electromagnet.

A non-dimensionalized state model is easily obtained if the following assignments are made. Let, $\kappa = m\mu_0 A_g$ and define the following state and control variables $x_1 = \kappa x$, $x_2 = \kappa\dot{x}$, $x_3 = \Phi$, and $u = e/N$. Then the state equations are

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= x_3^{[2]} \\ \dot{x_3} &= u \end{aligned} \tag{3}$$

where the $x_3^{[2]} = \mathrm{sgn}(x_3)x_3^2 = x_3|x_3|$. A detailed discussion of the previous AMB model as well as a controllability analysis of (3) is given in [2].

Equation (3) represents a nonlinear system, affine in the control input, in the standard form $\dot{x} = f(x) + g(x)u$. The vector fields f and g are given by

$$f(x) = \begin{bmatrix} x_2 \\ x_3^{[2]} \\ 0 \end{bmatrix}, \qquad g(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

The goal of this study is to find a control law such that (i) the closed-loop system has an isolated equilibrium at the origin, and (ii) the origin is (globally) asymptotically stable.

In this paper we use the theory of control Lyapunov function's (clf's) (see, for instance, [12]) to stabilize the AMB system described by (3). Generally speaking, if a system has a clf, then there exists a control law (with certain smoothness properties) that renders the system asymptotically stable. Typically, clf's for general nonlinear systems are difficult to find. However, techniques for finding clf's for cascaded systems, such as the one in (3), are available [12, 1]. Once the clf is known, the control law can be constructed using known formulas [4, 1]. Therefore, the stabilization problem can be cast as a problem of finding a clf.

The notation used in this work is standard. $\mathbb{R}^n$ denotes the n-dimensional vector space with Euclidean norm $|x| = (\sum^n x_i^2)^{1/2}$. A symmetric matrix P is positive definite if all its (real) eigenvalues are positive. This fact is denoted by $P > 0$.

3 Control Lyapunov Functions

Definition 1 *A function $V : \mathbb{R}^n \to \mathbb{R}$ is a control Lyapunov function (clf) for the system $\dot{x} = f(x) + g(x)u$ if it satisfies the following properties:*

(i) $V(x) > 0$ for all $x \in \mathbb{R}^n\backslash\{0\}$ and $V(0) = 0$ (i.e. V is positive definite)

(ii) $V \in \mathcal{C}^1$

(iii) $V(x) \to \infty$ as $|x| \to \infty$ (i.e. V is radially unbounded)

(iv) $L_f V(x) < 0$ for all $x \neq 0$ such that $L_g V(x) = 0$

Artstein in [3] has shown that the existence of a clf for a system is equivalent to globally asymptotic stablizability by a control law which is everywhere smooth except, perhaps, the origin[2]. In fact, Sontag in [4] has given an explicit expression for such a control law that stabilizes a system using its clf. Sontag's formula is given by

$$u = \begin{cases} 0 & L_g V(x) = 0 \\ -\dfrac{L_f V + \sqrt{L_f V^2 + L_g V^4}}{L_g V} & \text{otherwise} \end{cases} \tag{5}$$

This control law is smooth in $\mathbb{R}\backslash\{0\}$. Sontag's control law is continuous at the point $x = 0$ if and only if the clf satisfies the *small control property* (scp) [5], i.e.,

For every $\varepsilon > 0$, there exists $\delta > 0$ such that for all $x \neq 0$, $|x| < \delta$, there exists u, such that $|u| < \varepsilon$ and satisfies $L_f V(x) + L_g V(x)u < 0$.

A continuous control law that is smooth in $\mathbb{R}^n\backslash\{0\}$ is called *almost smooth*. Hence the results of Artstein and Sontag show that the existence of a clf with the scp is necessary and sufficient for the existence of an almost smooth stabilizing control law.

In summary, if it can be determined that a system possesses a clf, then Sontag's formula can be used to design a control law – smooth everywhere except, perhaps, the origin – that renders the origin globally asymptotically stable. This control law is continuous at the origin if the clf satisfies the scp. The main drawback of the clf approach is that, generally, it is difficult to determine if a system possesses a clf. However, for systems that have a cascaded structure, there exist constructive algorithms to find clf's.

3.1 Cascaded Systems and clf's

In [1] Praly, et al. demonstrate how to construct clf's for a class of cascaded systems the form

$$\dot{z} = f_0(z, y) \tag{6a}$$
$$\dot{y} = g_0(y)u \tag{6b}$$

where $z \in \mathbb{R}^{n-1}$ and $y \in \mathbb{R}$ and where $f(0, 0) = 0$ and $g(0) \neq 0$. Letting $z = [z_1, z_2]^T = [x_1, x_2]^T$ and $y = x_3$ it is easily seen that (4) has a cascaded structure, with $f_0(z, y) = [z_2, y^{[2]}]^T$, $g_0(y) = 1$.

A common control strategy used with cascaded systems is backstepping [12, 10]. In this scheme, one allows the y variable that appears in the z dynamics to be a "virtual" control input to the z subsystem. A control law $y = u_0(z)$ and a Lyapunov function (hence a clf) $V_0(z)$ are constructed to asymptotically stabilize (6a)

[2] That is, the control law is a $\mathcal{C}^1$ function away from the origin.

about $z = 0$. Once the virtual control law $u_0(z)$ and the clf $V_0(z)$ for the $n-1$ dimensional z dynamics is found, a clf for the entire n dimensional system is constructed.

To this end, let V be a candidate clf for system (3) and consider the expression for the derivative of V along the trajectories of (6)

$$\dot{V} = \frac{\partial V(z,y)}{\partial z} f_0(z,y) + \frac{\partial V(z,y)}{\partial y} g_0(y) u \qquad (7)$$

From equation (7), two sufficient conditions for a V to be a clf for equation (6) can be identified: (i) if $y = u_0(z)$, then $\frac{\partial V(z,y)}{\partial z} f_0(z,y) = L_{f_0(z,u_0(z))} V_0(z)$ and (ii) if $\frac{\partial V(z,y)}{\partial y} = 0$, then $y = u_0(z)$. Whenever these two conditions hold, then $\frac{\partial V(z,y)}{\partial y} = 0$ implies that $\dot{V}(x) = L_{f_0(z,u_0(z))} V_0(z) < 0$ and V will be a clf for (6).

Both the previous conditions can be satisfied by the following clf

$$V(z,y) = \frac{1}{2}(y - u_0(z))^2 + V_0(z) \qquad (8)$$

Thus, given V_0 and u_0 for (6a) one can construct a clf for the whole system (6). The problem with this approach is that it may be difficult to find a control law u_0 with the required smoothness properties to make $V(z,y)$ smooth enough to satisfy the requirement that $V \in \mathcal{C}^1$; see Property (ii) of Definition 1.

To remedy this difficulty, Praly *et al.* in [1] introduce a "desingularizing" function $\psi(z,y)$ so that V has the required smoothness properties. The function $\psi(z,y) \in \mathcal{C}^0$ and is chosen such that $\psi(z,y) = 0$ implies that $y = u_0(z)$. Related to the function $\psi(z,y)$ is the $\mathcal{C}^1$ function

$$\Psi(z,y) = \int_0^y \psi(z,q)\,dq \qquad (9)$$

where, for all $z \in \mathbb{R}^{n-1}$, $\Psi(z,y) \to \infty$ as $|y| \to \infty$. The form of the clf is then given by

$$V(z,y) = \Psi(z,y) - \Psi(z,u_0(z)) + \beta V_0(z)^{\alpha}, \quad \beta > 0 \qquad (10)$$

where α is such that $V_0(z)^{\alpha} \in \mathcal{C}^1$.

Assuming a Lyapunov function $V_0(z)$ for the z-subsystem in (6a) is known, the problem of finding a clf for (6) is then reduced into finding a desingularizing function ψ. Once the clf is known, one may use Sontag's formula (5) to construct a controller. Alternatively, one may use the control given in the following lemma.

Lemma 1 ([1]) *Given the system (6) assume that $u_0(z)$ is a control law that asymptotically stabilizes (6a) and $V_0(z)$ is the corresponding Lyapunov function for the closed-loop system. Consider the positive definite function $V(z,y)$ as in equation (10). Then the following choice of the control law will asymptotically stabilize the cascaded system described by (6)*

$$\begin{aligned} u(z,y) = \left(\frac{\partial \Psi}{\partial y}(z,y)\right)^{-1} \Big\{ &\left(\frac{\partial \Psi}{\partial z}(z,u_o(z)) - \frac{\partial \Psi}{\partial z}(z,y)\right) f_0(z,y) \\ &+ \alpha\beta V_0(z)^{\alpha-1}\Big(L_{f_0(z,u_0(z))}V_0(z) - L_{f_0(z,y)}V_0(z)\Big)\Big\} \\ &- \Theta(z,y) \end{aligned} \qquad (11)$$

where $\Theta(z,y) \in \mathcal{C}^0$ and has the same sign as $\frac{\partial \Psi}{\partial y}(z,y) = \psi(z,y)$.

The control law in (11) can be used whenever a continuous extension at the zeros of $\frac{\partial \Psi}{\partial y}$ exist [1]. In this case the control law in (11) is continuous.

Remark 1 *It can be easily shown [1] that the Lyapunov function candidate V in (10) is positive definite and radially unbounded. With the control law in (11) the time derivative of V along the trajectories of the closed-loop system is*

$$\dot{V} = \alpha\beta V_0(z)^{\alpha-1} L_{f_0(z,u_0(z))} V_0(z) - \psi(z,y)\Theta(z,y)$$

Clearly, $\dot{V} < 0$ for all $(z,y) \neq (0,0)$ and hence the control law (11) is globally asymptotically stabilizing.

Next, we will apply Lemma 1 to the AMB model (3).

4 Zero-bias control of an AMB

To begin, notice that if

$$y^{[2]} = \sigma(z) = -k_1 z_1 - k_2 z_2 \qquad (12)$$

in (4) then $f_0(z,y)|_{y=u_0(z)} = Az$, where the matrix A is

$$A = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \qquad (13)$$

The function $\sigma(z)$ is called the *stabilizing function* and the constants k_1 and k_2 are selected to make A Hurwitz. If $u_0(z)$ is selected as the following continuous function, which is actually $\mathcal{C}^1$ away from the origin,

$$u_0(z) = \mathrm{sgn}(\sigma(z))|\sigma(z)|^{\frac{1}{2}} \qquad (14)$$

then one may check that $y = u_0(z)$ implies that $y^{[2]} = \sigma(z)$ and $f_0(z,u_0(z)) = Az$.

Since the closed-loop z-subsystem with $u_0(z)$ as in (14) is linear, an obvious choice for this subsystem Lyapunov function $V_0(z)$ is given by

$$V_0(z) = z^T P z \qquad (15)$$

where $P > 0$ that satisfies the Lyapunov inequality $A^T P + PA < 0$.

With $u_0(z)$ in hand, one next determines the desingularizing function. Since $u_0(z)^{[2p]} \in \mathcal{C}^1$ for $p \geq 1$, we let $s(z) = 0$ and thus, $\psi(z,y) = y^{[2p]} - u_0(z)^{[2p]}$, $p \geq 1$ is used as the desingularizing function[3].

The function $\psi(z,y)$ is now integrated with respect to y to obtain $\Psi(z,y)$ and $\Psi(z,u_0(z))$. A simple calculation shows that

$$\Psi(z,y) = \int_0^y \psi(z,q)\,dq = \frac{y^{[2p+1]}}{2p+1} - y u_0(z)^{[2p]} \qquad (16)$$

[3] The function $x^{[q]}$ and some of its properties are given in the Appendix.

and

$$\Psi(z,u_0(z)) = -\frac{2p}{2p+1}u_0(z)^{[2p+1]} \qquad (17)$$

Inserting equations (16) and (17) into (10), one finds that

$$\begin{aligned} V(z,y) &= \frac{y^{[2p+1]}}{2p+1} - yu_0(z)^{[2p]} \\ &\quad + \frac{2p}{2p+1}u_0(z)^{[2p+1]} + \beta V_0(z)^{\alpha} \qquad (18) \end{aligned}$$

with $p \geq 1$, $\beta > 0$, and $\alpha > \frac{1}{2}$, is an appropriate clf for the system (4). The value of $\alpha > \frac{1}{2}$ ensures that $V_0(z)^{\alpha} \in \mathcal{C}^1$.

Given the clf in equation (18), one can now apply Lemma 1 to obtain a control law.

Proposition 1 *Let constants k_1 and k_2 such that the matrix A in (13) is Hurwitz and let P be a positive definite matrix such that*

$$A^T P + PA < 0$$

Let $V_0 = x^T P x$ and consider the control law

$$\begin{aligned} u = (x_3^{[2p]} - u_0^{[2p]})^{-1}\Big\{ p\, u_0^{2p-2}(u_0 - y)(k_1 x_2 + k_2 x_3^{[2]}) \\ + \alpha\beta V_0^{\alpha-1}\frac{\partial V_0}{\partial x_2}(u_0^{[2]} - x_3^{[2]})\Big\} - \Theta(x) \qquad (19) \end{aligned}$$

where $p \geq 1$, $\beta > 0$, $\alpha > \frac{1}{2}$, and where $\Theta(x)$ has the same sign as $x_3^{[2p]} - u_0^{[2p]}$ with

$$u_0 = -\mathrm{sgn}(k_1 x_1 + k_2 x_2)|k_1 x_1 + k_2 x_2|^{\frac{1}{2}} \qquad (20)$$

This control law globally asymptotically stabilizes system (3).

Proof: The proposition follows from Lemma 1 by noticing first that $\frac{\partial \Psi(z,y)}{\partial y} = \psi(z,y)$. Next, using the definitions of $\Psi(z,y)$ and $\Psi(z,u_0(z))$ in equations (16) and (17) respectively, one obtains,

$$\begin{aligned} \frac{\partial \Psi(z,y)}{\partial z} &= -y\frac{\partial u_0(z)^{[2p]}}{\partial z} \\ \frac{\partial \Phi(z,u_0(z))}{\partial z} &= -\frac{2p}{2p+1}\frac{\partial u_0(z)^{[2p+1]}}{\partial z} \end{aligned}$$

Since,

$$\begin{aligned} \frac{\partial u_0(z)^{[2p]}}{\partial z} &= p\,\mathrm{sgn}(\sigma)^{p-1}\sigma^{p-1}\frac{\partial \sigma}{\partial z} \\ \frac{\partial u_0(z)^{[2p+1]}}{\partial z} &= \frac{2p+1}{2}\,\mathrm{sgn}(\sigma)|\sigma|^{\frac{2p-1}{2}}\frac{\partial \sigma}{\partial z} \end{aligned}$$

and recalling that $\mathrm{sgn}\, u_0 = \mathrm{sgn}\,\sigma$, we get

$$\begin{aligned} \frac{\partial \Psi(z,u_0(z))}{\partial z} - \frac{\partial \Phi(z,y)}{\partial z} &= -p\,|\sigma|^{p-1}(u_0(z) - y)\frac{\partial \sigma}{\partial z} \\ &= -p\,u_0(z)^{2p-2}(u_0(z) - y)\frac{\partial \sigma}{\partial z} \end{aligned}$$

Furthermore, the difference between the Lie derivative terms in (11) can be written as

$$L_{f_0(z,u_0(z))}V_0(z) - L_{f_0(z,y)}V_0(z) = \frac{\partial V_0(z)}{\partial z_2}(u_0(z)^{[2]} - y^{[2]})$$

Inserting the last two equations into (11) obtains (19). ■

A simple choice for Θ that satisfies the requirements of the previous lemma is

$$\Theta(z,y) = \gamma(y - u_0(z)), \qquad \gamma > 0 \qquad (21)$$

4.1 Homogeneity Properties of the Control Law

We remind the reader that once the clf (18) is known, one can also use the (5) to construct a stabilizing control law. The added benefit of using (19) instead, is that one can ensure that the closed-loop system is homogeneous of degree zero with respect to a certain dilation.

Given a vector $r = (r_1, \ldots, r_n) \in \mathbb{N}^n$ of positive integers and $\lambda > 0$, a *dilation* is a mapping $\Delta_\lambda^r : \mathbb{R}^n \to \mathbb{R}^n$ given by $\Delta_\lambda^r(x) = (\lambda^{r_1}x_1, \lambda^{r_2}x_2, ..., \lambda^{r_n}x_n)$. A function $f : \mathbb{R}^n \to \mathbb{R}$ is *homogeneous of degree $\ell \geq 0$ with respect to Δ_λ^r* if $f(\Delta_\lambda^r(x)) = \lambda^\ell f(x)$. A vector field f is *homogeneous of degree $\ell \leq \max(r_i)$ with respect to* Δ_λ^r if f_i is a homogeneous function of degree $r_i - \ell$ for $i = 1, \ldots, n$.

Generally speaking, a homogeneous function is one that a (nonlinear) scaling of coordinates produces a proportional scaling of the value of the function itself. Homogeneous systems of degree zero, in particular, have certain appealing properties. For example, a homogeneous degree-zero vector field is invariant with respect to the chosen dilation. Thus, solutions scale according to the dilation. As a result, local asymptotic stability for homogeneous degree-zero vector fields implies global exponential stability with respect to the homogeneous norm associated with this dilation [14].

Notice that the drift vector field f in system (4) is homogeneous of degree zero with respect to the dilation $\Delta_\lambda(x) = (\lambda^2 x_1, \lambda^2 x_2, \lambda x_3)$. With this dilation, $V_0(z)$ is homogeneous of degree four and $u_0(z)$ is homogeneous of degree one. A simple calculation shows that the clf (18) is homogeneous of degree $2p+1$ when $\alpha = \frac{2p+1}{4}$. Since the clf is homogeneous of degree greater equal to one, it satisfies the scp [1]. Using similar arguments one can also show that the control law (19) is homogeneous of degree one for all $p \geq 1$, hence continuous[4]. Furthermore, since the vector fields f and g are homogeneous of degree zero and one respectively, the closed loop system is homogeneous of degree zero. Moreover, the larger the p, the smoother the control law on $\mathbb{R}^n \setminus \{0\}$. Thus, p can be used as a "tuning" parameter to smooth the control law away from the origin.

5 Numerical Example

We apply the control law in (11) to a specific AMB with zero bias. The specifications for this AMB are shown in Table 1.

[4]The continuity of the control law also follows from the scp of the clf according to the discussion in Section 3.

Table 1: AMB specifications

Symbols	Meaning
$N = 400$	$\sharp$ of turns in coil
$A_g = 1531.79\ mm^2$	cross sectional area of airgap
$m = 14.16\ kg$	mass of bar
$\mu_o = 4\pi \times 10^{-7}\ H/m$	permeability of free space
$g_0 = 0.55\ mm$	nominal width of airgap ($x = 0$)

The constant g_0 is the distance from each electromagnet to the rotor when the rotor is centered at $x = 0$; see Figure 1. The simulations were conducted for several values of the parameters p, γ and β. The value of α was always chosen to satisfy the homogeneity requirement. The gains k_1 and k_2 were selected as $k_1 = 10000$ and $k_2 = 200$. This choice, places both eigenvalues of the matrix A at -100. Figure 2 shows the states and control voltage when $p = 1$, $\alpha = 3/4$, and $\beta = 1$. The several plots show the dependence of the control on the gain γ. The gain γ controls the rate of convergence of the "outer" loop controller x_3 to the "inner" loop control law u_0. Figure 3 shows the states and control

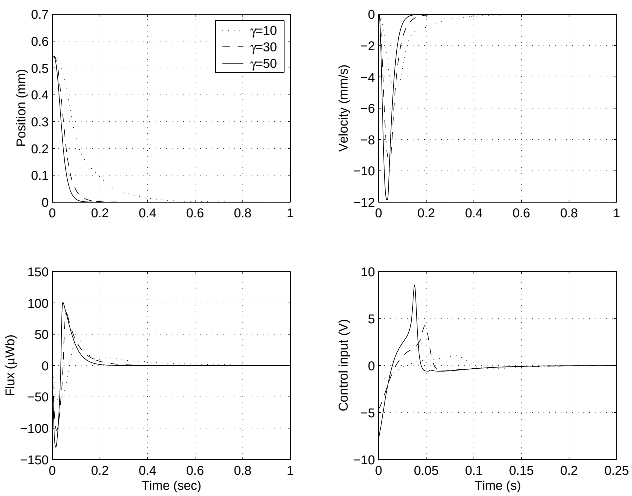

Figure 2: States and control input with $p = 1$, $\alpha = 3/4$, $\beta = 1$, $k_1 = 10000$, $k_2 = 200$.

voltage with p increased to $p = 5$. To retain the homogeneity of the control law, α is increased to $\alpha = 11/4$. One can see that if $p = 1$, larger values of the γ parameter lead to smaller settling times for the system. The same trend in γ can be seen when $p = 5$. Figure 4 shows the states and control input with $p = 1$, $\alpha = 3/4$, and $k_1 = 50, k_2 = 15$ for various values of γ. In all cases, the controller in (19) renders the point $(x, \dot{x}, \Phi) = (0, 0, 0)$ asymptotically stable. One can also infer from the plots that p acts as a "smoothness" parameter. As p increases, the settling time for x increases and the overall behavior of the states and the control is smoother. This is to be expected, since a larger value of p corresponds to higher smoothness of the control law u_0. Finally, for comparison, Fig. 5 shows the response of Sontag's controller (5) using the clf in (18) for different values of the gains k_1 and k_2 and for $\beta = 1$ and $p = 1$. The effect of varying p in Sontag's formula is shown in Fig. 6

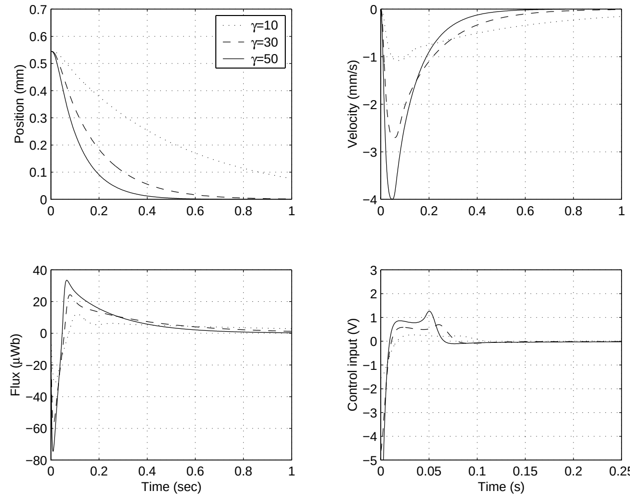

Figure 3: States and control input with $p = 5$, $\alpha = 11/4$, $\beta = 1$, $k_1 = 10000$, $k_2 = 200$.

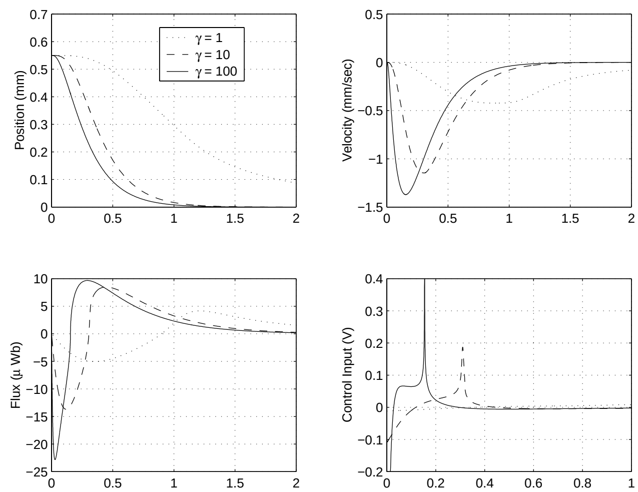

Figure 4: States and control input with $p = 1$, $\alpha = 3/4$, $\gamma = 30$, $k_1 = 50$, $k_2 = 15$.

6 Conclusions

This work addresses the problem of zero-bias control of an AMB. Zero- or low-bias control is important for designing low-loss AMB's. A simplified model of an AMB is used to construct nonlinear control laws that stabilize the system from any initial conditions. These control laws take into consideration the homogeneity properties of the AMB model. The control laws can be constructed to be as smooth as desired. Smoother control laws typically imply lower control signals, at the expense of the speed of response.

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Appendix

Let the function $x^{[q]} := \mathrm{sgn}(x)x^q$. It is easy to verify that the above function has the following properties.

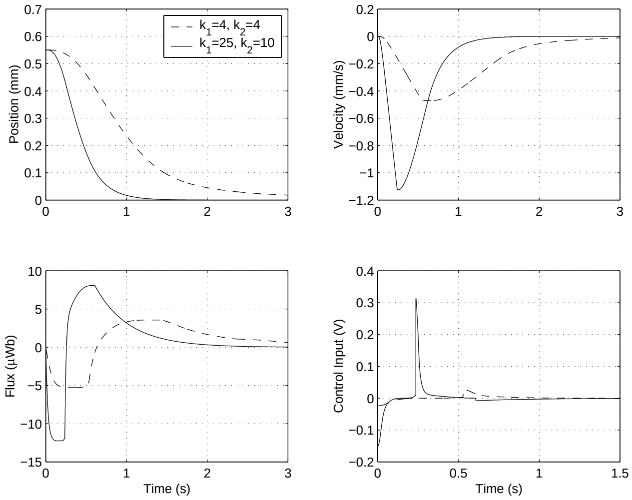

Figure 5: States and control input for Sontag's formula with $p = 1, \beta = 1$.

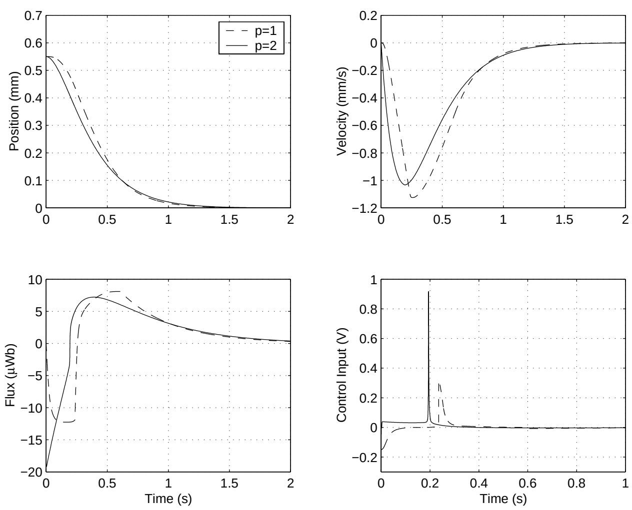

Figure 6: States and control input for Sontag's formula with $\beta = 1$ and $k_1 = 25, k_2 = 10$.

1. $x^{[q]}x^{[p]} = x^{p+q}, \qquad x^{[q]}x^{p} = x^{[p+q]}$
2. $\frac{x^{[p]}}{x^{[q]}} = x^{p-q}, \qquad \frac{x^{[p]}}{x^{q}} = x^{[p-q]}$
3. $\frac{dx^{[p]}}{dx} = px^{[p-1]}, \qquad \int x^{[p]} = \frac{x^{[p+1]}}{p+1}$
4. If the function $f(x)$ is homogeneous of degree p then $f(x)^{[q]}$ is homogeneous of degree pq.
5. $x^{[q]} \in \mathcal{C}^0$ for $q > 0$.
6. $x^{[q]} \in \mathcal{C}^1$ for $q \geq 2$.
7. If q is an odd integer, then $x^{[q]}$ is an even function.
8. If q is an even integer, then $x^{[q]}$ is an odd function.

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