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Control Design for Systems in Chained Form with Bounded Inputs*

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Abstract

Discontinuous, time-invariant controllers have been recently proposed in the literature as an alternative method to stabilize nonholonomic systems. These control laws are not Lipschitz continuous at the origin and hence they may use significant amount of control effort, especially if the initial conditions are close to an equilibrium manifold. We seek to remedy this situation by constructing *bounded* convergent controllers (with exponential convergence rates) for nonholonomic systems in chained form.

1 Introduction

In this paper we focus on the problem of designing feedback control laws for a nonholonomic system in chained form using inputs bounded by an *a priori* specified upper bound. It is well known that nonholonomic systems may not satisfy Brockett's necessary condition [3], thus no time-invariant smooth, static stabilizing controller exists. One possible avenue to deal with the difficulties implied by Brockett's theorem is to use time-varying controllers. This approach has been extensively investigated during the last few years with great success [12, 9, 11, 4, 2, 14]. It can be shown that time-varying smooth control laws for driftless systems have necessarily algebraic (not exponential) convergence rates [10].

More recently, another group of researchers concentrated on the design of time-invariant discontinuous controllers which achieve exponential convergence rates. Based on a nonlinear transformation, an exponentially convergent controller (which, however, may not necessarily achieve stability in the sense of Lyapunov) is constructed in [1] for chained form systems. A non-smooth controller for attitude stabilization of an underactuated spacecraft was proposed in [16]. This idea was later expanded upon and used to construct exponentially stabilizing control laws for a 3-dimensional system in power form [15]. Recently, time-invariant discontinuous controllers for n-dimensional power form systems was reported in [7] using an iterative algorithm.

A common characteristic of all these discontinuous controllers is that the control input may become excessively large, especially for initial conditions close to a certain singular manifold which includes the origin. In [8] the non-smooth controller proposed in [16] was modified, to remedy the problem of large control inputs. In this paper we generalize this idea to general nonholonomic systems in chained form. The construction of the proposed controller was inspired in part by the recent developments on input saturation for linear systems [6, 5]. These results (either in terms of global or semi-global stability) cannot be used directly, however, since the transformed linear system is not asymptotically null controllable.

The paper is organized as follows. In Section 2, we introduce a nonlinear coordinate transformation (called the σ-process) presented in [1], which transforms an n-dimensional chained form system to a linear system. Although this linear system has open loop positive eigenvalues, these eigenvalues can be made arbitrary small by selecting control gain small enough. This observation, is used in Section 4 to construct a bounded controller, which guarantees exponential stability of the linear system with bounded inputs. In addition, the domain of attraction of the closed-loop system contains an a priori given set, corresponding to the so-called "good" region. In Section 5 we complete the controller design by constructing a bounded controller such that for all initial conditions outside the "good" region, the trajectories of the closed-loop system converge to this region in finite time. A numerical example is provided in Section 6 to illustrate the theory.

The notation used in the paper is standard. For a vector $x \in \mathbb{R}^n$, $|x|$ denotes the euclidean norm, for a square matrix A, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote its maximum and minimum eigenvalues respectively, $sp(A)$ denotes its spectrum, and A^T denotes its transpose. I denotes the identity matrix. Finally, the notation $f \in \mathcal{L}_2$ implies that $\int_0^\infty |f(t)|^2\, dt < \infty$.

2 The σ-process

Several nonholonomic systems, after appropriate state and input transformations [9], can be put to the so-called chained canonical form. The 1-chain single generator system with two inputs is given by

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_i &= x_{i-1}u_1 \qquad i = 3, \dots n \end{aligned} \tag{1}$$

The following transformation, valid for all $x_1 \neq 0$,

$$\begin{aligned} \xi_1 &= x_1 \\ \xi_2 &= x_2 \\ \xi_i &= \frac{x_i}{x_1^{i-2}} \qquad i = 3, \dots n \end{aligned} \tag{2}$$

applied to Eq. (1) yields

$$\begin{aligned} \dot{\xi}_1 &= u_1 \\ \dot{\xi}_2 &= u_2 \\ \dot{\xi}_i &= (\xi_{i-1} - (i-2)\xi_i)\,\frac{u_1}{\xi_1} \qquad i = 3, \dots n \end{aligned} \tag{3}$$

If we let $u_1 = -k\,\xi_1$ the ξ-system becomes

$$\dot{\xi} = \begin{bmatrix} -k & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & -k & k & \dots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \dots & (n-2)k \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_2 \tag{4}$$

This is a linear system with u_2 as the new input. For more detail on this transformation, please refer to [1]. Since

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the system in Eq. (4) is stabilizable, one can choose a linear control law [1]

$$u_2 = p_2\,\xi_2 + p_3\,\xi_3 + \ldots + p_n\,\xi_n \tag{5}$$

to place the eigenvalues in the left-half of the complex plane, and make the closed-loop system (in ξ-coordinates) globally exponentially stable. The previous linear control law is not defined in the set

$$\mathcal{S} = \{x \in \mathbb{R}^n : x_1 = 0 \text{ and } x_i \neq 0,\ i = 2, 3, \ldots, n\ \} \tag{6}$$

Moreover, one cannot conclude that the original closed-loop system in Eq. (1) is asymptotically stable, since the transformation in Eq. (2) is not a diffeomorphism.

Although the control law in Eq. (5) is well defined for all initial conditions such that $x_1(0) \neq 0$, it is clear from Eq. (2) that the control input u_2 may take excessively large values when the initial conditions are close to the singular manifold $\mathcal{S}$. Similar problems are encountered with the discontinuous control laws proposed in [16, 15, 7].

3 Statement of the problem and approach

We wish to derive a globally valid control law for the system in Eq. (1) such that the following two properties hold.

1. For all initial conditions $x(0) \in \mathcal{D} = \mathbb{R}^n \backslash \mathcal{S}$, we have that $\lim_{t\to\infty} x(t) = 0$.
2. The control law u is bounded as $|u_i| \leq \bar{u}, (i = 1, 2)$, where $\bar{u}$ is any *a priori* given positive number.

We only impose convergence of the closed-loop trajectories of the system in Eq. (1) to the origin. Attractivity to the origin for the system in Eq. (1) can be easily deduced if the linear system in Eq. (4) is asymptotically stable or even convergent [1, 7]. Moreover, since $\xi_1 = x_1$ and $\xi_2 = x_2$, the control inputs u_1 and u_2 are the same for both systems. If the system in Eq. (4) is asymptotically stable (or even convergent) with input bounded by $\bar{u}$, then the trajectories of the system in Eq. (1) will converge to the origin and the control will also be bounded by $\bar{u}$.

Recently, numerous results have appeared in the literature dealing with the problem of global or semi-global stabilization of linear systems with bounded inputs [5, 6]. Unfortunately, the open-loop system in Eq. (4) has positive eigenvalues, so it is not asymptotically null-controllable [5]. Thus, we cannot use directly these results to derive bounded controllers for (4). However, the eigenvalues of the uncontrolled linear system in Eq. (4) can be moved arbitrarily close to the imaginary axis by appropriate choice of the control gain k. This allows the construction of exponentially stabilizing controllers for the system in Eq. (4) which are bounded by an arbitrarily small upper bound.

4 A semi-global controller

In this section we design a controller such that, if the initial conditions are in a given set, the trajectories of the system in Eq. (1) tend asymptotically to the origin and the control input is bounded by $\bar{u}$. In addition, this set can be chosen arbitrarily large.

To proceed with our analysis, we first decompose the system in Eq. (4) as

$$\dot{\xi}_1 = -k\,\xi_1 \tag{7a}$$

$$\dot{\tilde{\xi}} = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ -k & k & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & -k & (n-2)k \end{bmatrix} \tilde{\xi} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_2 \tag{7b}$$

with $\tilde{\xi} = [\xi_2,\ \xi_3,\ \ldots,\ \xi_n]^T$. Define the constant matrices A and B as follows

$$A = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ -1 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & -1 & (n-2) \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{8}$$

Then the $\tilde{\xi}$-subsystem can be rewritten as

$$\dot{\tilde{\xi}} = k\,A\,\tilde{\xi} + B\,u_2 \tag{9}$$

The construction of the controller requires a certain class of functions which increase no slower than linear.

Definition 4.1 A continuous function $\phi(x)$ will be called a *linear dominant function* (l.d.f for short) if it satisfies the following three properties:

1. It is monotonically increasing for $x \geq 0$ and $\phi(0) = 1$.
2. It is an even function, i.e., $\phi(x) = \phi(-x) \quad \forall x \in \mathbb{R}^n$.
3. $|x| \leq \phi(x)$ for all $x \in \mathbb{R}^n$.

From the definition it follows immediately that $\lim_{|x|\to\infty} \phi(x) = \infty$. For example, the functions $\phi(x) = 1 + |x|$, $\phi(x) = \sqrt{1 + x^2}$ and $\phi(x) = 1 + x^2$ are all l.d.f. In particular, any function of the from $\phi(x) = (1 + x^{2p})^{\frac{1+\epsilon}{2p}}$ with $\epsilon > 0$ and $p = 1, 2, \ldots$ is l.d.f.

The following theorem provides a controller for the system in Eq. (3) which is bounded by $\bar{u}$.

Theorem 4.1 *Consider the system described by Eq. (3) and the region given by $\mathcal{D}_\delta^g = \{\,\xi \in \mathcal{D} : |\tilde{\xi}| \leq \delta\,\}$. Let $\bar{u}$ be a given positive number and let P be the positive definite symmetric matrix which solves the equation*

$$(A + I)P + P(A + I)^T = BB^T \tag{10}$$

Define the matrix $A_c = A - BB^T P^{-1}$ and let $k = \min\{\bar{u}, \bar{u}\mu/\delta\}$ where $\mu = \lambda_{\min}(P)$. Then, the control law

$$u_1 = -k\,\xi_1/\phi(\xi_1) \tag{11a}$$

$$u_2 = -k\,B^T P^{-1}\tilde{\xi}/\phi(\xi_1) \tag{11b}$$

with $\phi(\cdot)$ as in Definition 4.1, renders the system in Eq. (3) asymptotically stable. In addition, for all initial conditions $\xi(0) \in \mathcal{D}_\delta^g$, both u_1 and u_2 are bounded by $\bar{u}$.

Proof. The equation for ξ_1 is given by $\dot{\xi}_1 = -k\,\xi_1/\phi(\xi_1)$. All solutions of this system converge exponentially to the origin and the control law u_1 is bounded by $|u_1| = k\,|\xi_1|/\phi(\xi_1) \leq \bar{u}$.

Define a new independent variable,

$$\tau = \int_0^t \frac{d\sigma}{\phi(\xi_1(\sigma))} \tag{12}$$

Note that τ is monotonically increasing and $\lim_{t\to\infty} \tau = \infty$. Denoting differentiation with respect to τ by ($'$), one obtains that,

$$\tilde{\xi}' = kA\,\tilde{\xi} + B\tilde{u}_2 \tag{13}$$

where $\tilde{u}_2 = u_2\,\phi(\xi_1)$. Since the pair (A, B) in Eq. (13) is controllable, it can be easily shown that the pair $((A+I), B)$ is also controllable. Moreover, all the eigenvalues of $-(A+I)$ are negative. Therefore, there exists a unique $P > 0$ which satisfies Eq. (10). From Eq. (10) we have,

$$(A + I)kQ + Qk(A + I)^T = BB^T \tag{14}$$

where $Q = P/k$. It is now easy to check that,

$$(Ak + Ik - BB^TQ^{-1})Q + Q(Ak + Ik - BB^TQ^{-1})^T = (A + I)kQ + Qk(A + I)^T - 2BB^T = -BB^T \quad (15)$$

Since the pair $((Ak+Ik-BB^TQ^{-1}), B)$ is controllable and Q is positive definite, the matrix $Ak+Ik-BB^TQ^{-1}$ is Hurwitz. In particular, the matrix $Ak - BB^TQ^{-1} = k\,A_c$ is Hurwitz and $Re(\lambda) < -k$ for all $\lambda \in sp(kA_c)$. With u_2 as in Eq. (11b) one obtains $\tilde{\xi}' = k\,A_c\,\tilde{\xi}$ and the $\tilde{\xi}$-subsystem is exponentially stable (in τ) with rate k. Since ξ_1 decreases monotonically to zero one obtains that $\phi(\xi_1(t)) \leq \phi(\xi_1(0))$ for all $t \geq 0$. Hence

$$\tau = \int_0^t \frac{d\sigma}{\phi(\xi_1(\sigma))} \geq \int_0^t \frac{d\sigma}{\phi(\xi_1(0))} = \frac{t}{\phi(\xi_1(0))} \quad (16)$$

From the exponential stability of the system $\tilde{\xi}' = k\,A_c\,\tilde{\xi}$ one obtains that

$$\begin{aligned} |\tilde{\xi}(t)| = |\tilde{\xi}(\tau(t))| &\leq c_0\,|\tilde{\xi}(0)|\,\exp(-k\tau) \\ &\leq c_0\,|\tilde{\xi}(0)|\exp(-\bar{k}t) \end{aligned} \quad (17)$$

where $\bar{k} = k/\phi(\xi_1(0))$. Hence, the $\tilde{\xi}$-subsystem with control (11) is exponentially stable (in t) with asymptotic rate of convergence $\bar{k}$.

Note that A_c is Hurwitz and satisfies the matrix inequality $A_cP + PA_c^T < 0$. Therefore, if $|\tilde{\xi}(0)| \leq \delta$ one obtains that,

$$\begin{aligned} \tilde{\xi}^T(t)P^{-1}\tilde{\xi}(t) &\leq \tilde{\xi}^T(0)P^{-1}\tilde{\xi}(0) \\ &\leq \lambda_{\max}(P^{-1})|\tilde{\xi}(0)|^2 \\ &\leq \lambda_{\max}(P^{-1})\,\delta^2 \quad \forall t \geq 0 \end{aligned} \quad (18)$$

A straightforward calculation shows that,

$$\max_{\tilde{\xi}^T P^{-1}\tilde{\xi} \leq \sigma^2} |P^{-1}\tilde{\xi}| = \sigma\,\lambda_{\max}^{\frac{1}{2}}(P^{-1}) \quad (19)$$

For all initial conditions $\xi(0) \in \mathcal{D}_\delta^g$ we finally have that,

$$\begin{aligned} |u_2| &= k\,|B^TP^{-1}\tilde{\xi}/\phi(\xi_1)| \leq k\,|P^{-1}\tilde{\xi}| \\ &\leq k\,\delta\,\lambda_{\max}(P^{-1}) = k\,\frac{\delta}{\mu} \leq \bar{u} \end{aligned} \quad (20)$$

■

The previous theorem shows that for all initial conditions in the "good" region $\mathcal{D}_\delta^g$ the trajectories of the closed-loop system with control law as in Eq. (11) tend exponentially to zero. The set $\mathcal{D}_\delta^g$ can be made arbitrarily large by appropriate choice of the parameter δ. As $\delta \to \infty$ then the region $\mathcal{D}_\delta^g$ increases and tends to the region $\mathcal{D}$.

Remark 4.1 Theorem 4.1 makes no claim that the trajectories have to stay in $\mathcal{D}_\delta^g$. Nonetheless, from the proof of Theorem 4.1 one immediately obtains that for all initial conditions in the set $\tilde{\mathcal{D}}_\delta^g = \{\,\xi \in \mathcal{D} : \tilde{\xi}^T P^{-1}\tilde{\xi} \leq \delta^2\,\}$, the trajectories of the closed-loop system remain in $\tilde{\mathcal{D}}_\delta^g$ (i.e., $\tilde{\mathcal{D}}_\delta^g$ is a positively invariant set) and they tend exponentially to the origin.

It is worth noticing that the matrix P in Eq. (10) is independent of k and thus the set $\tilde{\mathcal{D}}_\delta^g$ does not depend on the choice of k. Moreover, from (19) we have the bound $|P^{-1}\tilde{\xi}| \leq \lambda_{\max}^{\frac{1}{2}}(P^{-1})\,\delta$ for all $\tilde{\xi} \in \tilde{\mathcal{D}}_\delta^g$.

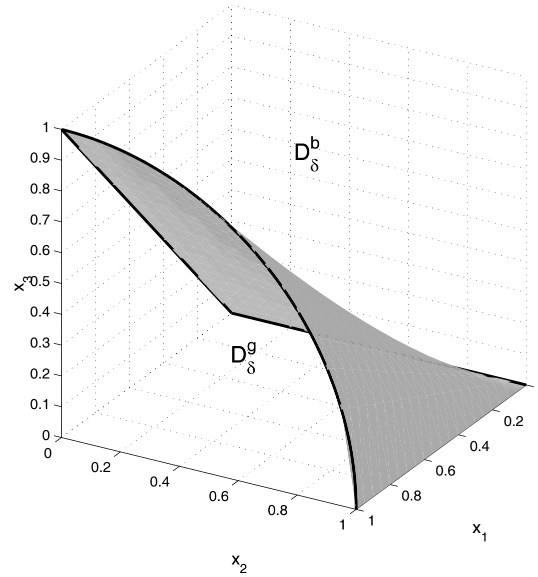

Figure 1: The regions $\mathcal{D}_\delta^g$ and $\mathcal{D}_\delta^b = \mathcal{D}\backslash\mathcal{D}_\delta^g$ for $\delta = 1$.

From Theorem 4.1 and the Remark 4.1 we have immediately the following corollary.

Corollary 4.1 *Consider the system in Eq. (1) with the control*

$$u_1 = -k\,x_1/\phi(x_1) \quad (21a)$$

$$u_2 = -k\,B^TP^{-1}\tilde{\xi}/\phi(x_1) \quad (21b)$$

where $\mu = \lambda_{\min}^{\frac{1}{2}}(P)$ and k, B, P, $\tilde{\xi}$ as in Theorem 4.1 and $\phi(\cdot)$ as in Definition 4.1. Then, for all initial conditions $\xi(0) \in \tilde{\mathcal{D}}_\delta^g$, the trajectories remain in $\tilde{\mathcal{D}}_\delta^g$ for all $t \geq 0$ and satisfy the property $\lim_{t\to\infty} x(t) = 0$. In addition, the control law is bounded by $|u_i| \leq \bar{u}$ $(i = 1, 2)$.

5 A global controller

To complete the construction of the controller, we need to force all trajectories starting in the "bad" region $\tilde{\mathcal{D}}_\delta^b = \mathcal{D}\backslash\tilde{\mathcal{D}}_\delta^g$ to enter the region $\tilde{\mathcal{D}}_\delta^g$ in finite time.

Proposition 5.1 *Consider the system in Eq. (3) and the control law*

$$u_1 = k\,\xi_1/\phi(\xi_1) \quad (22a)$$

$$u_2 = -k\,\frac{\xi_2}{\phi(\xi_1)\phi(\xi_2)} \quad (22b)$$

with $k > 0$. Then, for every $\gamma > 0$ and $\xi_1(0) \neq 0$, there exists a time $t^* > 0$ such that $|\tilde{\xi}(t)| \leq \gamma$ for all $t \geq t^*$. Moreover, if $k < \bar{u}$ then $|u_i| \leq \bar{u}$ $(i = 1, 2)$.

Proof. The differential equation for ξ_1 is given by

$$\dot{\xi}_1 = k\,\xi_1/\phi(\xi_1) \quad (23)$$

Clearly, $\lim_{t\to\infty} \xi_1(t) = \infty$ for $\xi_1(0) \neq 0$. Consider again the change of independent variable introduced in Eq. (12). Since

$$\tau = \int_0^t \frac{d\sigma}{\phi(\xi_1(\sigma))} = \frac{1}{k}\int_{\xi_1(0)}^{\xi_1(t)} \frac{d\xi}{\xi} \quad (24)$$

one obtains that τ is monotonically increasing and $\lim_{t\to\infty} \tau = \infty$.

With the control law as in Eq. (22) the closed-loop system in Eq. (7b) can be written in the form

$$\frac{d\zeta}{d\tau} = \begin{bmatrix} -k & 0 & \dots & 0 & 0 \\ k & -2k & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & k & -(n-2)k \end{bmatrix} \zeta + \begin{bmatrix} k \\ 0 \\ \vdots \\ 0 \end{bmatrix} \xi_2 \tag{25}$$

where $\zeta = [\xi_3, \dots, \xi_n]^T$ and where ξ_2 satisfies the equation $\xi_2' = -k\,\xi_2/\phi(\xi_2)$. The last equation implies that $\xi_2 \in \mathcal{L}_2$. Since the matrix in Eq. (25) is Hurwitz, $\zeta \in \mathcal{L}_2$ [13]. Moreover, $\lim_{t\to\infty} \zeta(t) = 0$ and thus $\lim_{t\to\infty} \tilde{\xi}(t) = 0$. Therefore, there exists a time $t^* > 0$ such that $|\tilde{\xi}(t)| \le \gamma$ for all $t \ge t^*$.

Note that if $k < \bar{u}$ then $|u_1| = k\,|\xi_1|/\phi(\xi_1) < \bar{u}$, and similarly,

$$|u_2| = \frac{k}{\phi(\xi_1)} \frac{|\xi_2|}{\phi(\xi_2)} < \frac{k}{\phi(\xi_1)} < \bar{u} \tag{26}$$

This completes the proof of the proposition. ∎

Letting $\gamma = \delta/\lambda_{\max}^{\frac{1}{2}}(P^{-1})$ in Proposition 5.1 one obtains that $\tilde{\xi}^T(t)P^{-1}\tilde{\xi}(t) \le \lambda_{\max}(P^{-1})|\tilde{\xi}(t)|^2 \le \delta^2$ for all $t \ge t^*$. In other words, the control law in Eq. (22) will force all trajectories enter the region $\tilde{\mathcal{D}}_\delta^g$ in finite time.

The following Theorem combines the results of Theorem 4.1 and Proposition 5.1 to obtain a global convergent controller bounded by a specified upper bound.

Theorem 5.1 *Let the system in Eq. (1) and consider the following control law*

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{cases} \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \text{if } x(0) = 0 \\ \begin{pmatrix} \bar{u} \\ 0 \end{pmatrix} & \text{if } x(0) \in \mathcal{S} \\ \begin{pmatrix} \text{Eq. (21a)} \\ \text{Eq. (21b)} \end{pmatrix} & \text{if } \xi \in \tilde{\mathcal{D}}_\delta^g \\ \begin{pmatrix} \text{Eq. (22a)} \\ \text{Eq. (22b)} \end{pmatrix} & \text{if } \xi \in \tilde{\mathcal{D}}_\delta^b \end{cases} \tag{27}$$

Then, for all initial conditions $x(0) \in \mathbb{R}^n$, the closed-loop system trajectories satisfy the property $\lim_{t\to\infty} x(t) = 0$ and the control law is bounded as $|u_i| \le \bar{u}$ $(i = 1, 2)$.

Proof. Note that if $x(0) \notin \mathcal{S}$ then $x(t) \notin \mathcal{S}$ for all $t > 0$ and the control law in Eq. (27) is well defined for all $t \ge 0$. The rest of the proof follows as a direct consequence of Corollary 4.1 and Proposition 5.1. ∎

From the previous discussion , it should be clear that the asymptotic convergence to the origin with the control law in Eq. (27) is exponential.

6 Numerical example

We consider a 5-dimensional chained form system. We assume $\bar{u} = 10$ and we choose $\delta = 0.37$. Because the minimum eigenvalue of the matrix P in Eq. (10) is typically small, the convergence in the $\mathcal{D}_\delta^g$ may be slow. To keep the rates of convergence in both regions the same for the simulations we have chosen $k = 10$ both in $\mathcal{D}_\delta^g$ and $\mathcal{D}_\delta^b$. Our simulations showed that this value gives a good compromise between the maximum control input and the speed of response.

The simulations for an initial condition $x_0 = [1, 1, -2, 1, 3]$ and $\phi(x) = \sqrt{1 + x^2}$ are shown in Fig. 2. The upper plot shows the states and the lower plot in Fig. 2 shows the control inputs. The control inputs are bounded by $\bar{u}$ as required.

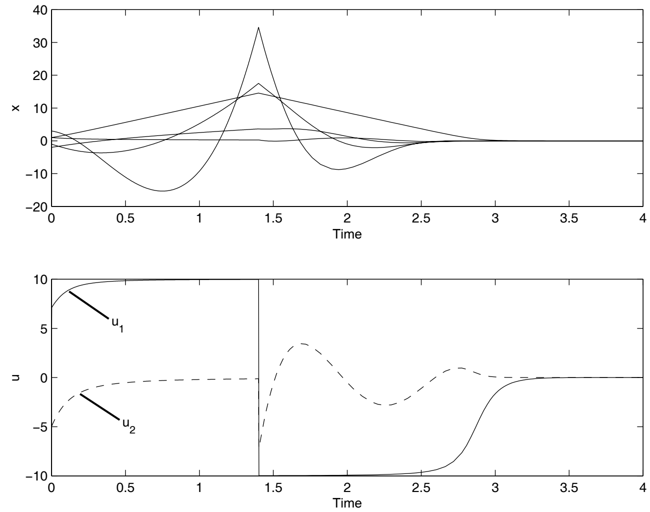

Figure 2: History of states and control inputs with constraints.

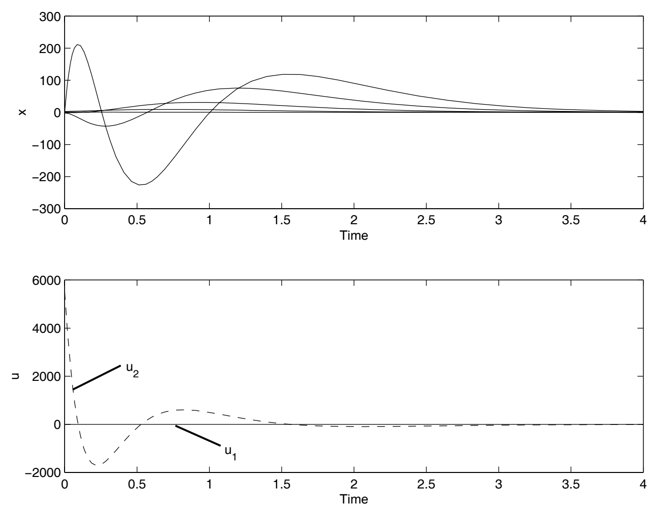

Figure 3: History of states and control inputs without constraints.

For comparison, Fig. 3 shows the state and control histories for the corresponding control law without input constraints. The gains where chosen such that the convergence rates are approximately the same as for the bounded input case.

7 Conclusion

In this paper, we describe an approach to address a common problem associated with a class of discontinuous controllers for nonholonomic systems proposed recently in the literature. Namely, these feedback controllers may require very large control inputs if the initial conditions are close to a singular manifold. The proposed methodology decomposes the state space into two separate regions and the controller forces all trajectories into a region where all control inputs are typically small. The control law guarantees exponential convergence of the closed-loop trajectories to the origin using bounded control inputs.

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