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Inverse Optimality Results for the Attitude Motion of a Rigid Spacecraft*

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Abstract

We present an approach for constructing optimal feedback control laws for optimal regulation of a rotating rigid spacecraft. We employ the inverse optimal control approach which circumvents the task of solving a Hamilton-Jacobi equation and results in a controller optimal with respect to a meaningful cost functional. The design reported in the paper is the first optimal control design for attitude regulation of the complete, nonlinear system, which includes a penalty on the angular velocity, orientation *and* the control torque.

1. Introduction

Optimal control of rigid bodies has a long history stemming from interest in the control of rigid spacecraft and aircraft [1, 2, 3]. The main thrust of this research has been directed, however, towards the time-optimal and fuel-optimal control problems [4, 5, 6]. The optimal regulation problem over a finite or infinite horizon has been treated in the past mainly for the angular velocity subsystem and for special quadratic costs [7, 8, 9]. The case of general quadratic costs has also been addressed in [10]. Optimal control for the complete attitude problem, i.e., including the orientation equations is more difficult and has been addressed in terms of trajectory planning [11, 12], or in semi-feedback form [13]. The main obstruction in constructing feedback control laws in this case stems from the difficulty in solving the Hamilton-Jacobi equation, especially when the cost includes a penalty term on the control effort. In [14] the authors obtain closed-form optimal solutions for special cases of quadratic costs without penalty on the control effort. Suboptimal results can be obtained by minimizing an upper bound of the cost. Alternatively, one can penalize only the high-gain portion of the control input [15].

In this paper we follow an alternative approach in order to derive optimal feedback control laws for the complete rigid body system. We employ the *inverse optimal* control approach which circumvents the task of solving a Hamilton-Jacobi equation and results in a controller optimal with respect to a meaningful cost functional. This approach has been long dormant in the area of nonlinear control and was recently revived in [16] to develop a methodology for design of *robust* nonlinear controllers. The inverse optimality approach requires the knowledge of a control Lyapunov function and a stabilizing control law of a particular form. For the spacecraft problem, we construct them both using the method of integrator backstepping [17]. The penalty on the control depends on the current state and decreases for states away from the origin. This allows for the necessary increased control action for initial conditions away from the equilibrium, while for states close to the origin the controller reduces to an LQR-type of control law.

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2. Inverse Optimal Control Approach

We consider nonlinear systems affine in the control variable

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are smooth, vector- and matrix-valued functions respectively, with $f(0) = 0$. Moreover, $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ denote the state and control vectors, respectively. Let us now assume that the static, state-feedback control law

$$u = \kappa(x) := -R^{-1}(x)\, g^T(x) V_x^T \tag{2}$$

where $R : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is a positive definite matrix-valued function (i.e., $R(x) = R^T(x) > 0$ for all $x \in \mathbb{R}^n$), stabilizes the system in Eq. (1) with respect to the Lyapunov function $V(x)$. Here V_x denotes the gradient of V (*row* vector).

Since $\kappa(x)$ is an asymptotically stabilizing control law with corresponding (strict) Lyapunov function V, the following inequality holds along trajectories of the closed-loop system

$$\frac{dV}{dt} = V_x\,(f(x) + g(x)\kappa(x)) < 0, \qquad \forall\, x \neq 0 \tag{3}$$

The next proposition shows that the control law in Eq. (2) is closely related to an optimal control for the system in Eq. (1) with respect to a specific cost.

Proposition 2.1 *Consider the system in Eq. (1) and the stabilizing control law $\kappa(x)$ in Eq. (2). Then the control law*

$$u = \kappa^*(x) := \beta\, \kappa(x), \qquad (\beta \geq 2) \tag{4}$$

is optimal with respect to the cost

$$\mathcal{J} = \int_0^\infty \{\ell(x) + u^T R(x) u\}\, dt \tag{5}$$

where

$$\begin{aligned} \ell(x) &= -2\beta\, V_x\,(f(x) + g(x)\kappa(x)) \\ &\quad + \beta(\beta - 2)\, V_x\, g(x) R^{-1}(x) g^T(x)\, V_x^T \end{aligned} \tag{6}$$

Notice that because of Eq. (3) we have $\ell(x) > 0$ for all $x \neq 0$ and the performance index in Eq. (6) represents a meaningful cost, in the sense that it includes a positive penalty on the state and a positive penalty on the control for each x. In fact, the function $r(x, u) = u^T R(x) u$ is continuous, nonnegative, convex in u and has a unique global minimum at $u = 0$ for each fixed $x \in \mathbb{R}^n$.

Proof:[of Proposition 2.1] The proof of the proposition can be established very easily by showing that the positive definite function $W(x) := 2\beta V(x)$ is a solution to the corresponding Hamilton-Jacobi-Bellman equation of the optimization problem in Eqs. (1)-(6), i.e., $W(x)$ solves the equation

$$0 = \min_u \left\{ \ell(x) + u^T R(x) u + W_x \left(f(x) + g(x) u \right) \right\} \tag{7}$$

and the optimal control is given by

$$u^*(x) = -\frac{1}{2} R^{-1}(x)\, g^T(x)\, W_x \tag{8}$$

■

Remark 2.1 The previous proposition provides a general solution to the inverse optimal control problem for control-affine nonlinear systems. In particular, if a stabilizing control law, along with the associated Lyapunov function, is known then a scalar multiple of this control law is optimal with respect to the cost in Eq. (5). Notice that this cost depends on the Lyapunov function V of the original stabilizing feedback as well as on the particular system dynamics. This is understandable, since by requiring *closed-form solutions* to a nonlinear optimal feedback problem it is sensible to choose costs which are compliant with the system dynamics. In other words, the cost should reflect somehow, and take into account, the form of the nonlinearity of the system. This restricts of course the choice of performance indices. On the other hand, one avoids solving the often formidable Hamilton-Jacobi equation.

Remark 2.2 The assumption of a strict Lyapunov function is not restrictive. In fact, if Eq. (3) is not a strict inequality then $\ell(x)$ is only positive semi-definite. Proposition 2.1 then holds by imposing an observability or a detectability assumption on the pair of vector fields (f, ℓ).

The following corollary follows immediately from Proposition 2.1.

Corollary 2.1 *Consider the control-affine nonlinear system in Eq. (1) and assume that the control law in Eq. (2) is globally asymptotically stabilizing, with $V(x)$ the corresponding Lyapunov function and $R(x)$ some positive definite, matrix-valued function. Then the control law*

$$u^* = -2\, R^{-1}(x)\, g^T(x) V_x^T \tag{9}$$

minimizes the performance index

$$\mathcal{J} = \int_0^\infty \{-4\, V_x\, f(x) + 2\, V_x\, g(x) R^{-1}(x)\, g^T(x) V_x^T + u^T R(x) u\}\, dt$$

3. The Rigid Spacecraft

In this section we use the inverse optimal results of Proposition 2.1 in order to derive control laws which are optimal with respect to a cost which includes a penalty on the control input as well as the angular position and velocity of a rigid rotating spacecraft. The complete attitude motion of a rigid spacecraft can be described by the state equations [15]

$$\dot{\omega} = J^{-1} S(\omega) J \omega + J^{-1} u \tag{10a}$$
$$\dot{\rho} = H(\rho) \omega \tag{10b}$$

where $\omega \in \mathbb{R}^3$ is the angular velocity vector in a body-fixed frame, $\rho \in \mathbb{R}^3$ is the Cayley-Rodrigues parameters vector describing the body orientation, $u \in \mathbb{R}^3$ is the acting control torque, and J is the (positive definite) inertia matrix. The symbol $S(\cdot)$ denotes a 3×3 skew-symmetric matrix such that $S(v)w = -v \times w$, and the matrix-valued function $H : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ denotes the kinematics Jacobian matrix for the Cayley-Rodrigues parameters, given by

$$H(\rho) := \frac{1}{2}(I - S(\rho) + \rho\rho^T) \tag{11}$$

where I denotes the 3×3 identity matrix. In the sequel, $\|\cdot\|$ denotes the euclidean norm, i.e., $\|x\|^2 = x^T x$, for $x \in \mathbb{R}^n$.

Observe that the system in Eqs. (10) is in cascade interconnection, that is, the kinematics subsystem (10b) is controlled only indirectly, through the angular velocity vector ω. Stabilizing control laws for systems in this hierarchical form can be efficiently designed using the method of *backstepping*[17]. According to this approach, one thinks of ω as the *virtual control* in Eq. (10b) and designs a control law, say $\omega_d(\rho)$, which stabilizes this system. Subsequently, one designs the actual control input u so as to stabilize the system in Eq. (10a) without destabilizing the system in Eq. (10b) by forcing, for example, $\omega \to \omega_d$. The main benefits of this methodology is that it is flexible, and lends itself to a systematic construction of stabilizing control laws along with the corresponding Lyapunov functions.

3.1. Backstepping

The first step for applying the results of Proposition 2.1 is to construct a control-Lyapunov function for the system in Eq. (10). Here we use backstepping in order to derive a control-Lyapunov function, along with a stabilizing controller of a particular form for the system in Eq. (10).

Consider the kinematics subsystem in Eq. (10b) with ω promoted to a control input and let the control law

$$\omega_d = -k_1 \rho, \qquad k_1 > 0 \tag{12}$$

With this control law the closed-loop system becomes

$$\dot{\rho} = -k_1 H(\rho) \rho \tag{13}$$

Proposition 3.1 *The system in Eq. (13) is globally exponentially stable.*

Proof: Consider the following Lyapunov function

$$V_1(\rho) = \frac{1}{2}\|\rho\|^2 \tag{14}$$

The derivative of V_1 along the trajectories of Eq. (13) is given by

$$\dot{V}_1 = -\frac{k_1}{2}(1 + \|\rho\|^2)\,\|\rho\|^2 \leq -k_1\, V_1 < 0, \qquad \forall \rho \neq 0 \tag{15}$$

Global exponential stability with rate of decay $k_1/2$ follows. ■

Consider now the error variable

$$z = \omega - \omega_d = \omega + k_1 \rho \tag{16}$$

The differential equation for the kinematics is written as

$$\dot{\rho} = -k_1 H(\rho) \rho + H(\rho) z \tag{17}$$

Notice that for $z = 0$ the system in Eq. (17) is globally exponentially stable by virtue of Proposition 3.1. The differential equation for z is

$$\begin{aligned} \dot{z} &= \left(J^{-1}S(\omega)J + k_1 H(\rho)\right) z \\ &- k_1 \left(J^{-1}S(\omega)J + k_1 H(\rho)\right) \rho + J^{-1} u \end{aligned} \quad (18)$$

We want to find $u = u(\rho, z)$ such that the system of Eqs. (17)-(18) is globally asymptotically stable. To this end, consider the following candidate Lyapunov function

$$V(\rho, z) = k_1^2 V_1(\rho) + \frac{1}{2}\|z\|^2 = \frac{k_1^2}{2}\|\rho\|^2 + \frac{1}{2}\|z\|^2 \quad (19)$$

Taking the derivative of V along the trajectories of Eqs. (17)-(18) one obtains

$$\begin{aligned} \dot{V} &= -\frac{k_1^3}{2}(1 + \|\rho\|^2)\|\rho\|^2 - k_1 z^T J^{-1} S(\omega) J\rho \\ &+ z^T J^{-1} S(\omega) J z + z^T \left(\frac{k_1}{2}(I + \rho\rho^T) z + J^{-1} u\right) \end{aligned} \quad (20)$$

and upon completion of squares,

$$\begin{aligned} \dot{V} &= -\frac{k_1^3}{4}(1 + 2\|\rho\|^2)\|\rho\|^2 - \frac{k_1^3}{4}\left\|\rho - \frac{2}{k_1^2} J S(\omega) J^{-1} z\right\|^2 \\ &- \frac{k_1}{4}\left\|\left(I + \frac{2}{k_1} J S(\omega) J^{-1}\right) z\right\|^2 \\ &+ z^T \left[\frac{k_1}{2}\left(\frac{3}{2} I + \rho\rho^T\right) - \frac{2}{k_1} J^{-1} S(\omega) J^2 S(\omega) J^{-1}\right] z \\ &+ z^T J^{-1} u \end{aligned} \quad (21)$$

With the choice of the feedback control law

$$u = -J \left[\left(k_2 + \frac{3}{4}k_1\right) I + \frac{k_1}{2}\rho\rho^T - \frac{2}{k_1} J^{-1} S(\omega) J^2 S(\omega) J^{-1}\right] z \quad (22)$$

where $k_2 > 0$, Eq. (21) yields

$$\begin{aligned} \dot{V} &= -\frac{k_1^3}{4}(1 + 2\|\rho\|^2)\|\rho\|^2 - \frac{k_1^3}{4}\left\|\rho - \frac{2}{k_1^2} J S(\omega) J^{-1} z\right\|^2 \\ &- \frac{k_1}{4}\left\|\left(I + \frac{2}{k_1} J S(\omega) J^{-1}\right) z\right\|^2 - k_2\|z\|^2 \end{aligned} \quad (23)$$

and the control law in Eq. (22) renders the closed-loop system globally asymptotically stable since $\dot{V}(\rho, z) < 0$ for all $(\rho, z) \neq (0, 0)$.

3.2. Optimal Control Law

The method of backstepping has been used in the previous section to construct an asymptotically stabilizing control for the system in Eqs. (10) along with the corresponding Lyapunov function. In order to use the results of Proposition 2.1 we need a stabilizing control law of the form in Eq. (2). Noticing that with V as in Eq. (19) one has $V_x g(x) = V_z J^{-1} = z^T J^{-1}$, one can rewrite the control law in Eq. (22) as

$$u = -R^{-1}(\rho, \omega) J^{-1} z \quad (24)$$

$$\begin{aligned} R(\rho, \omega) &= J^{-1}\left[\left(k_2 + \frac{3}{4}k_1\right) I + \frac{k_1}{2}\rho\rho^T \right. \\ &\left. - \frac{2}{k_1} J^{-1} S(\omega) J^2 S(\omega) J^{-1}\right]^{-1} J^{-1} \end{aligned} \quad (25)$$

Note that, since $S(\omega) = -S(\omega)^T$ and $J = J^T$, we have $R(\rho, \omega) > 0$, $\forall \rho, \omega \in \mathbb{R}^n$. From Corollary 2.1 we finally have that the control law

$$u^* = -J \left[\left(2k_2 + \frac{3}{2}k_1\right) I + k_1 \rho\rho^T - \frac{4}{k_1} J^{-1} S(\omega) J^2 S(\omega) J^{-1}\right] z \quad (26)$$

minimizes the cost functional

$$\mathcal{J} = \int_0^\infty \{\ell(\rho, \omega) + u^T R(\rho, \omega) u\}\, dt \quad (27)$$

$$\begin{aligned} \ell(\rho, \omega) &= k_1^3(1 + 2\|\rho\|^2)\|\rho\|^2 + 4k_2\|\omega + k_1\rho\|^2 \\ &+ k_1^3 \left\|\rho - \frac{2}{k_1^2} J S(\omega) J^{-1}(\omega + k_1\rho)\right\|^2 \\ &+ k_1 \left\|\left(I + \frac{2}{k_1} J S(\omega) J^{-1}\right)(\omega + k_1\rho)\right\|^2 \end{aligned} \quad (28)$$

and $R(\rho, \omega)$ as in Eq. (25). The performance index in Eq. (27) represents a meaningful cost since $\ell(\rho, \omega) > 0$ and $R(\rho, \omega) > 0$ for all $(\rho, \omega) \neq (0, 0)$, therefore it penalizes both the states ρ and ω, as well as the control effort u. As ρ and ω increase, the penalty on the control decreases. This is a desirable feature of the optimal control law, since it implies more aggressive control action far away from the equilibrium. Indeed, as the system state starts deviating from the intended operating point the controller allows for increasingly corrective action. At the same time, for ρ and ω small we have that

$$\begin{aligned} \ell(\rho, \omega) &\sim 2k_1^3\|\rho\|^2 + (4k_2 + k_1)\|\omega + k_1\rho\|^2 + \mathcal{O}(\|(\rho, \omega)\|^4) \\ R(\rho, \omega) &\sim J^{-1}\left[(k_2 + \frac{3}{4}k_1) I\right]^{-1} J^{-1} + \mathcal{O}(\|(\rho, \omega)\|^2) \end{aligned}$$

so, close to the origin, the control law reduces to an LQR-type linear control law. The control law in this case minimizes the LQR cost

$$\mathcal{J} = \int_0^\infty \{[\omega^T \rho^T] Q \begin{bmatrix} \omega \\ \rho \end{bmatrix} + u^T R u\}\, dt \quad (29)$$

$$Q = \begin{bmatrix} 4k_2 + k_1 & k_1(4k_2 + k_1) \\ k_1(4k_2 + k_1) & k_1^2(3k_1 + 4k_2) \end{bmatrix}, \quad R = \left(\frac{4}{4k_2 + 3k_1}\right) J^{-2}$$

The previous equations be used as a guideline for choosing the positive definite matrices Q and R such that LQR-performance is guaranteed close to the equilibrium point.

Remark 3.1 It is important to realize that the optimal control law in Eq. (26) avoids the cancellation of the nonlinearities. Notice, for example, that from Eq. (20) one can globally asymptotically stabilize the system by choosing the control law

$$u = -k_2 J z - \frac{k_1}{2} J (I + \rho\rho^T) z - S(\omega) J\omega \quad (30)$$

which renders

$$\dot{V} = -\frac{k_1^3}{2}(1 + \|\rho\|^2)\|\rho\|^2 - k_2\|z\|^2 < 0, \quad \forall (\rho, z) \neq (0, 0)$$

There are no obvious optimality characteristics associated with this control law. In fact, as was pointed out in [18] controllers which cancel nonlinearities are, in general, *non*optimal since the nonlinearity may be actually beneficial in meeting the stabilization and/or performance objectives.

An undesirable feature of the optimal control law in Eq. (26) is that it depends on the moment of inertia matrix J, which may not be always accurately known. The robustness properties of the optimal control law will be addressed in the future.

3.3. The symmetric case

When the rigid body is symmetric, its inertia matrix is a multiple of the identity matrix and $S(\omega)J\omega \equiv 0$ for all $\omega \in \mathbb{R}^3$. In this case the optimal control law simplifies to

$$u^* = -J\left[(2k_2 + k_1)I + k_1\,\rho\rho^T\right] z \tag{31}$$

which minimizes the cost in Eq. (5) where

$$\begin{aligned} \ell(\omega,\rho) &= 2\,k_1^3(1+\|\rho\|^2)\|\rho\|^2 + 4\,k_2\|z\|^2 \\ R(\omega,\rho) &= J^{-1}\left[(k_2+\frac{k_1}{2})I + \frac{k_1}{2}\rho\rho^T\right]^{-1} J^{-1} \end{aligned}$$

This control law reduces to an LQR-type feedback control law close to the origin, with

$$Q = \begin{bmatrix} 4k_2 & 4k_1k_2 \\ 4k_1k_2 & 2k_1^3 + 4k_2k_1^2 \end{bmatrix}, \qquad R = \left(\frac{2}{2k_2+k_1}\right) J^{-2}$$

We note that the symmetric case has been previously addressed by Wie *et al.* [19], where an Euler parameter description for the kinematics was used.

4. Numerical Example

Numerical simulations were performed to establish the validity of the theory. We assume a rigid spacecraft with inertia matrix $J = diag(10, 15, 20)\ kg\,m$. A rest-to-rest maneuver is considered, thus $\omega(0) = 0$. First, we consider the kinematics subsystem in Eq. (10b) with ω regarded as the control input. Let the initial conditions $\rho(0) = (1.4735, 0.6115, 2.5521)$ in terms of the Cayley-Rodrigues parameters. The trajectories of the system with the control law in Eq. (12) with $k_1 = 1$ are shown in Figs. (1) and (2). The exponential stability of the closed-loop system is evident from these figures. At this step the choice of k_1 is basically dictated by the required speed for the completion of the rest-to-rest maneuver.

For the stabilization of the complete system we use the control law in Eq. (26). The state trajectories for different values of the gain k_2 are depicted in Figs. (3) and (4). The optimal trajectories have a very uniform behavior which is essentially independent of the value of k_2 and they follow very closely the corresponding trajectories for the kinematics subsystem. The control action varies a great deal, however, with k_2. The initial control action consists, essentially, in making $\omega \longrightarrow -k_1\rho$. This is clearly shown in Fig. (3).

Finally, Fig. (6) shows the time history of the Frobenious norm of the control penalty matrix $R(\omega,\rho)$. The control penalty is decreased rapidly at the initial portion of the trajectory when increased control action is necessary in order to "match" ω with ω_d within a short period of time.

5. Conclusions

Due to the difficulty in obtaining closed-form solutions to the Hamilton-Jacobi-Bellman equation, the *direct* optimal control problem for nonlinear systems remains open. However, the knowledge of a control Lyapunov function allows us to solve the *inverse* optimal control problem, i.e., find a controller which is optimal with respect to a meaningful cost. The inverse optimal stabilization design for a rigid spacecraft in this paper is, to the authors' knowledge, the first optimal *feedback* control law that minimizes a cost for the general – nonsymmetric case – that incorporates penalty on both the state (angular velocity and orientation) *and* the control effort (torque).

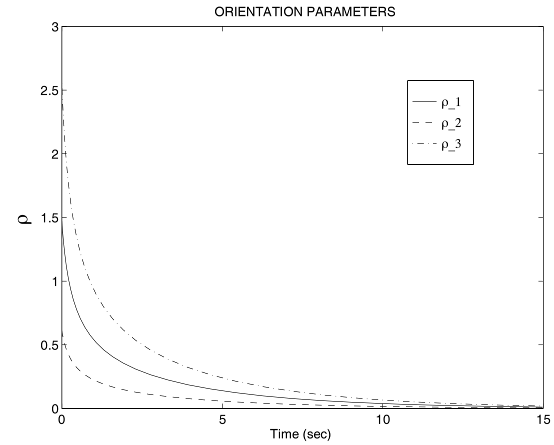

Figure 1: Orientation parameters for the kinematics.

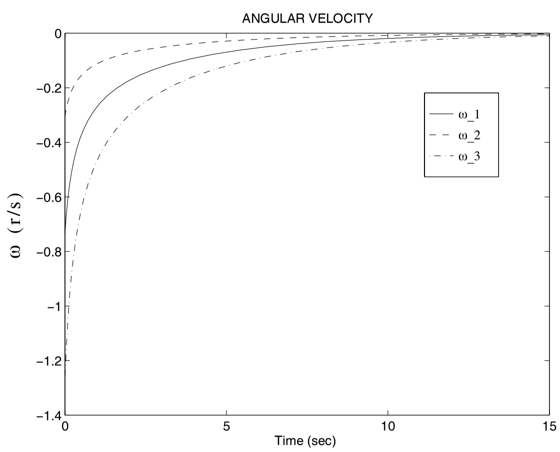

Figure 2: Angular velocity for the kinematics.

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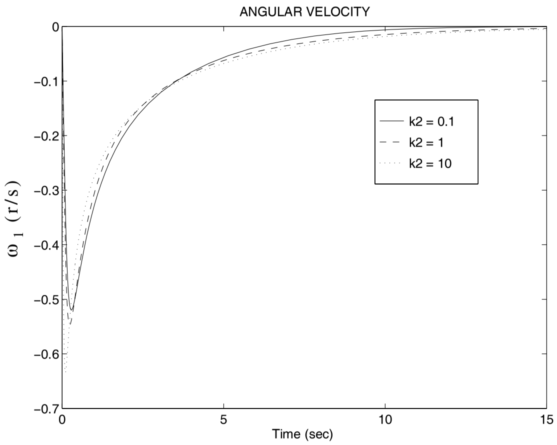

Figure 3: Angular velocity ω_1.

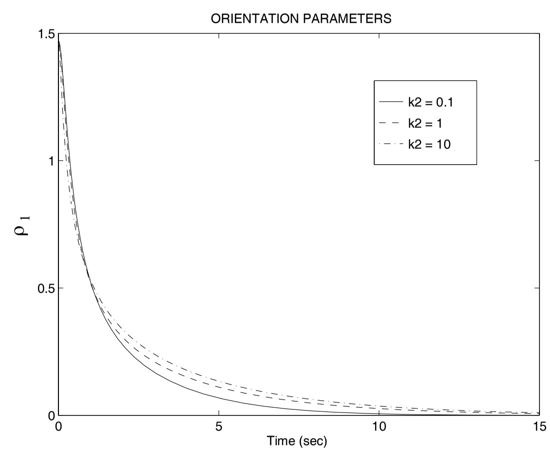

Figure 4: Orientation parameter ρ_1.

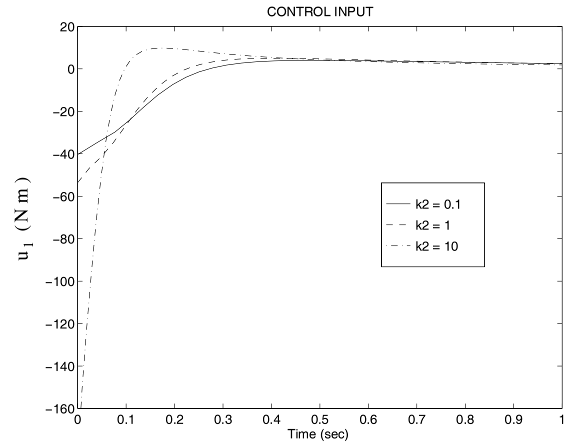

Figure 5: Control input u_1.

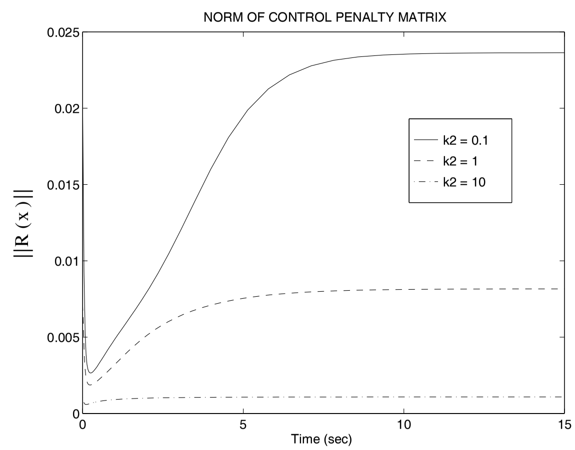

Figure 6: Norm of $R(\omega, \rho)$.

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