Leader-Follower Cooperative Attitude Control of Multiple Rigid Bodies

Dimos V. Dimarogonas, Panagiotis Tsiotras and Kostas J. Kyriakopoulos

***Abstract*— In this paper we extend our previous results on coordinated control of rotating rigid bodies to the case of teams with heterogenous agents. We assume that only a certain subgroup of the agents (the leaders) are vested with the main control objective, that is, maintain constant relative orientation amongst themselves. The rest of the team must meet relaxed control specifications, namely maintain their respective orientations within certain limits dictated by the orientation of the leaders. The proposed control laws respect the limited information each rigid body has with respect to the rest of its peers (leaders or followers) as well as with the rest of the team. Each rigid body is equipped with a control law that utilizes the Laplacian matrix of the associated communication graph, which encodes the communication links between the team members. Similarly to the linear case, the convergence of the multi-agent system relies on the connectivity of the communication graph.**

I. INTRODUCTION

Cooperative distributed control strategies for multiple vehicles have gained increased attention in recent years in the control community, owing to the fact that such strategies provide attractive solutions to large-scale multi-agent problems, both in terms of complexity and computational load. A typical control objective for a team of agents is the state-agreement or consensus problem. This design objective has been extensively pursued in the recent years. Several results are based on treating the vehicle as a single integrator [11],[1],[5],[14] or double integrator [16],[10],[6]. A common analysis tool that is frequently used to model these distributed systems is algebraic graph theory [4].

Extending the previous results to systems whose dynamics are nonlinear is a nontrivial task. A large and important class (in terms of applications) of systems whose dynamics are nonlinear are systems of rotating rigid bodies. Motivated by the fact that – despite the nonlinear dynamics – linear controllers can stabilize a single rigid body [19], in this paper we propose a control strategy that exploits graph theoretic tools for cooperative control of multiple rigid bodies. We extend our previous work [2] in this area to address cases of teams with heterogenous agents. For some applications (i.e., Earth monitoring or stellar observation using a satellite cluster with a large baseline) it may be necessary for some satellites to acquire and maintain a certain (perhaps nonzero) relative orientation among themselves. One of the primary control objective is therefore to stabilize a subgroup of the agent team (leaders) to certain relative orientations. The orientations of the rest of the team (followers) are to remain within a certain orientation boundary, determined by the convex hull of the leaders' orientations. At the same time, each agent is allowed to communicate its state (orientation and angular velocity) only with certain members of the team. These constraints limit the information exchange between the agents. The proposed control law for each agent respects the limited information each rigid body has with respect to the rest of the team.

Cooperative control of multiple rigid bodies has been addressed recently by many authors, notably [7], [20], [8],[9]. While these papers use distributed consensus algorithms to achieve the desired objective, the specific algebraic graph theoretic framework (that is, the use of graph Laplacians) encountered in this work has not been considered in these papers. Moreover, in [12],[13], the author uses a different parameterization for the rigid body dynamics and does not include the type of leader-follower structure, used in this paper. Thus, the stability analysis is different than the current paper.

The rest of the paper is organized as follows: Section II describes the system and the problem treated in this paper. Section III presents the control law used for the followers to converge to the convex hull of the leaders' orientations, while in Section IV, the relative orientation controller for the leaders is provided. The case of lack of global objective is treated in Section V. Simulations that support the theoretical results are contained in Section VI, while Section VII summarizes the results of this work and indicates possible extensions.

II. SYSTEM AND PROBLEM DEFINITION

We consider a team of N rigid bodies (henceforth called agents) indexed by $\mathcal{N} = \{1, \ldots, N\}$. The dynamics of agent i are given by [19]:

$$J_i \dot{\omega}_i = S(\omega_i) J_i \omega_i + u_i, \quad i \in \mathcal{N}, \tag{1}$$

where $\omega_i \in \mathbb{R}^3$ is the angular velocity vector, $u_i \in \mathbb{R}^3$ is the acting torque vector, and J_i is the symmetric inertia matrix of agent i, all expressed in the ith agent's body fixed frame. The matrix $S(\cdot)$ denotes a skew-symmetric matrix representing the cross product between two vectors, i.e. $S(v_1)v_2 = -v_1 \times v_2$.

In this paper, the orientation of the rigid bodies with respect to the inertial frame will be described in terms of the Modified Rodriguez Parameters (MRPs)[15], [17]. We hasten to point out that this choice can be done without loss of generality. If necessary, the analysis in terms of quaternions

D. V. Dimarogonas is with the KTH ACCESS Linnaeus Center, School of Electrical Engineering, Royal Institute of Technology (KTH), SE-100 44,Stockholm, Sweden, e-mail: dimos@ee.kth.se. Corresponding author.

P. Tsiotras is with the School of Aerospace, Georgia Institute of Technology, Atlanta, GA, 30332 USA e-mail: tsiotras@gatech.edu

K. J. Kyriakopoulos is with the Control Systems Laboratory, National Technical University of Athens, Greece, email: kkyria@central.ntua.gr

can be carried out by the interested reader *mutatis mutandis* following the developments below. The use of the MRPs, nonetheless, simplifies the analysis and the ensuing formulas. Another advantage of the MRPs is the fact that it can parameterize the attitude for eigenaxis rotations up to 360 deg. In contrast, other three-dimensional parameterizations are limited to eigenaxis rotations of less than 180 deg; see [18], [15] for more details.

Using the MRPs, the kinematics of agent i are given by:

$$\dot{\sigma}_i = G(\sigma_i)\,\omega_i, \quad i \in \mathcal{N}, \tag{2}$$

where $G : \mathbb{R}^3 \mapsto \mathbb{R}^{3\times 3}$ is given by

$$G(\sigma_i) = \frac{1}{2}\left(\frac{1-\sigma_i^{\intercal}\sigma_i}{2} I_3 - S(\sigma_i) + \sigma_i\sigma_i^{\intercal}\right).$$

The matrix $G(\sigma_i)$ has the following properties [19]:

$$\sigma_i^{\intercal} G(\sigma_i)\,\omega_i = \left(\frac{1+\sigma_i^{\intercal}\sigma_i}{4}\right)\sigma_i^{\intercal}\omega_i, \tag{3}$$

$$G(\sigma_i)\,G^{\intercal}(\sigma_i) = \left(\frac{1+\sigma_i^{\intercal}\sigma_i}{4}\right)^2 I_3. \tag{4}$$

We assume that the agents belong to either one of the two subsets, namely, the subset of leaders $\mathcal{N}^l$, or to the subset of followers $\mathcal{N}^f$, i.e. $\mathcal{N}^l \bigcap \mathcal{N}_f = \emptyset$ and $\mathcal{N}^l \bigcup \mathcal{N}^f = \mathcal{N}$.

A first objective of each leader is to converge to a desired *relative* orientation with respect to the rest of the leaders. We assume that each leader is assigned to a specific subset $N_i^l \subseteq \mathcal{N}^l$ of the rest of the leaders, called the ith's leader agent *leader communication set*. This is the set of leaders the ith leader can communicate in order to achieve the desired objective. The control objective of each leader $i \in \mathcal{N}^l$ is to be stabilized in a desired relative orientation σ_{ij}^d with respect to each member $j \in N_i^l$. Moreover, it is assumed that the communication topology is bidirectional in the sense that $j \in N_i^l$ if and only if $i \in N_j^l$ for all $i, j \in \mathcal{N}^l$, $i \neq j$.

A second objective is for the leaders to "drag" the followers along so that, at the final leader configuration, the latter are "contained" within the convex hull of the leader orientations. This is a sub-case of the containment control problem dealt with in multi-agent systems. This problem has also been encountered in [3]. The reader is referred to that reference for a discussion on specific applications of this problem. For this objective, both the leaders and the followers are assigned to a specific subset $N_i \subseteq \mathcal{N}$ of the rest of the team called ith agent *leader-follower communication set*. This is the set of other agents the ith agent can communicate with in order to achieve the desired objective (that is, containment of the followers' final orientations in the convex hull of the leaders' orientations). For this case we assume that for each leader $i \in \mathcal{N}^l$, the sets N_i, N_i^l are disjoint, i.e. $N_i \cap N_i^l = \emptyset$, for all $i \in \mathcal{N}^l$. Hence, for this objective the leader-follower communication set of each leader contains only followers.

The previous two control objectives can be encoded by two different communication graphs, that are defined with respect to the limited communication of the agents as follows:

1) The *leader communication graph* $\mathcal{G}^l = \{V^l, E^l, C\}$ is an undirected graph that consists of: (i) a set of vertices $V^l = \mathcal{N}^l$ indexed by the leaders of the multi-agent team, (ii) a set of edges, $E^l = \{(i,j) \in V^l \times V^l | i \in N_j^l\}$ containing pairs of nodes that represent inter-leader formation specifications, and (ii) a set of labels $C = \{c_{ij}\}$, where $(i,j) \in E^l$, that specify the desired inter-agent relative positions in the leader formation configuration.
2) The *leader-follower communication graph* $\mathcal{G} = \{V, E\}$ is an undirected graph that consists of: (i) a set of vertices $V = \{1, ..., N\}$ indexed by the team members and (ii) a set of edges, $E = \{(i,j) \in V \times V | i \in N_j\}$ containing pairs of nodes that represent inter-agent communication specifications.

III. Control Design and Stability Analysis

A. Tools from Algebraic Graph Theory

In this section we first review some tools from algebraic graph theory [4] that we use in the sequel.

For an undirected graph $\mathcal{G}$ with n vertices, the *adjacency matrix* $A = A(\mathcal{G}) = (a_{ij})$ is the $n \times n$ symmetric matrix given by $a_{ij} = 1$, if $(i,j) \in E$ and $a_{ij} = 0$, otherwise. If there is an edge connecting two vertices i, j, that is, $(i,j) \in E$, then i, j are called *adjacent*. A *path* of length r from a vertex i to a vertex j is a sequence of $r+1$ distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. If there is a path between any two vertices of $\mathcal{G}$, then $\mathcal{G}$ is called *connected*. Otherwise it is called *disconnected*. The *degree* d_i of vertex i is the number of its neighboring vertices, that is, $d_i = \{\#j : (i,j) \in E\} = |\mathcal{N}_i|$. Let Δ be the $n \times n$ diagonal matrix with elements d_i on the diagonal. The (combinatorial) *Laplacian* of $\mathcal{G}$ is the symmetric positive semidefinite matrix $L = \Delta - A$. For a connected graph, the Laplacian has a simple zero eigenvalue and the corresponding eigenvector is the vector of ones, denoted by $\overrightarrow{\mathbf{1}}$.

B. Multiple Stationary Leaders

In this paper, the leaders are responsible for a global objective and their evolution is independent of the followers' motion. In this section we assume first that the leaders have converged to some desired final orientations with zero angular velocity, i.e., we have

$$\sigma_i = \sigma_i^d, \quad \omega_i = 0, \qquad i \in \mathcal{N}^l. \tag{5}$$

Consider the case when the leaders must "drag" the followers to a configuration where the orientations of the latter are "contained" within the convex hull of the leader orientations in the final formation configuration. In the multiple satellites scheme, this case implies, for instance, coverage of a specific area. In this case the leaders' orientations dictate the "boundary" of the area to be covered.

The control law of the followers is given by:

$$u_i = -G^{\intercal}(\sigma_i)\sum_{j\in N_i}(\sigma_i - \sigma_j) - \sum_{j\in N_i}(\omega_i - \omega_j), \quad i \in \mathcal{N}^f \tag{6}$$

Let $u, \omega, \sigma \in \mathbb{R}^{3N}$ be the stack vectors of all the control inputs, the angular velocities and the orientations of the multi-agent team, respectively. Consider

$$V(\sigma,\omega) = \sum_{i=1}^{N}\left(\frac{1}{2}\omega_i^\intercal J_i \omega_i\right) + \frac{1}{2}\sigma^\intercal \left(L \otimes I_3\right)\sigma$$

as a candidate Lyapunov function, where L is the Laplacian of the leader-follower communication graph $\mathcal{G}$. Differentiating with respect to time we obtain

$$\begin{aligned}\dot{V}\left(\sigma,\omega\right) &= \sum_{i=1}^{N}\left(\omega_i^\intercal J_i \dot{\omega}_i\right) + \sigma^\intercal\left(L\otimes I_3\right)\dot{\sigma} = \\ &= \sum_{i=1}^{N}\left(u_i^\intercal \omega_i + \sum_{j\in N_i}\left(\sigma_i - \sigma_j\right)^\intercal G\left(\sigma_i\right)\omega_i\right),\end{aligned}$$

and since $\omega_i = 0$, for all $i \in \mathcal{N}^l$, from (6) we get

$$\begin{aligned}\dot{V}\left(\sigma,\omega\right) &= \sum_{i\in\mathcal{N}^f}\omega_i^\intercal\left\{u_i + G^\intercal\left(\sigma_i\right)\sum_{j\in N_i}\left(\sigma_i-\sigma_j\right)\right\} \\ &= -\sum_{i\in\mathcal{N}^f}\omega_i^\intercal\sum_{j\in N_i}\left(\omega_i - \omega_j\right)\end{aligned} \tag{7}$$

Quoting again the fact that $\omega_i = 0$, for all $i \in \mathcal{N}^l$, we get

$$\dot{V}\left(\sigma,\omega\right) = -\omega^\intercal\left(L\otimes I_3\right)\omega \leq 0.$$

The last inequality implies that V remains bounded. The level sets of V define compact sets in the product space of the angular velocities and *relative* orientations of the agents. Specifically, the set $\Omega_c = \{(\omega,\sigma) : V(\sigma,\omega) \leq c\}$ for $c > 0$ is closed by continuity of V. For all $(\omega,\sigma) \in \Omega_c$ we have $\omega_i^\intercal J_i \omega_i \leq 2c \Rightarrow \|\omega_i\| \leq \sqrt{\frac{2c}{\lambda_{\min}(J_i)}}$. Furthermore,

$$\sigma^\intercal\left(L\otimes I_3\right)\sigma \leq 2c \Rightarrow \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i}\|\sigma_i - \sigma_j\|^2 \leq 2c.$$

Hence, $\|\sigma_i - \sigma_j\|^2 \leq 4c$, for all $(i,j) \in E$. Connectivity of $\mathcal{G}$ ensures that the maximum length of a path connecting two vertices of the graph is at most $N-1$. Hence $\|\sigma_i - \sigma_j\| \leq 2\sqrt{c\,(N-1)}$, for all $i,j \in \mathcal{N}$.

By LaSalle's invariance principle, the system converges to the largest invariant set inside the set

$$M = \left\{(\omega,\sigma) : \left(\omega^\intercal\left(L\otimes I_3\right)\omega = 0\right)\right\}.$$

Since $L \otimes I_3$ is positive semidefinite, if follows that $(L \otimes I_3)\omega = 0$ which implies that

$$L\omega^1 = L\omega^2 = L\omega^3 = 0, \tag{8}$$

where $\omega^1, \omega^2, \omega^3 \in \mathbb{R}^N$ are the stack vectors of the three coefficients of the agents' angular velocities, respectively. Connectivity of the leader-follower communication graph implies that L has a simple zero eigenvalue with corresponding eigenvector $\overrightarrow{\mathbf{1}}$. Equation (8) now implies that $\omega^1, \omega^2, \omega^3$ are eigenvectors of L that correspond to the zero eigenvalue, thus they belong to span$\{\overrightarrow{\mathbf{1}}\}$. Hence $\omega_i = \omega_j$ for all $i,j \in \mathcal{N}$, implying that all ω_i's converge to a common value ω^* at steady state. Since $\omega_i = 0$, for all $i \in \mathcal{N}^l$, we have that $\omega^* = 0$, and hence all agents assume zero angular velocities.

By virtue of (1), the control inputs of all followers tend to zero, and

$$u_i = -G^\intercal\left(\sigma_i\right)\sum_{j\in N_i}\left(\sigma_i - \sigma_j\right) = 0$$

which implies that

$$G\left(\sigma_i\right)G^\intercal\left(\sigma_i\right)\sum_{j\in N_i}\left(\sigma_i - \sigma_j\right) = 0$$

or

$$\left(\frac{1+\sigma_i^\intercal\sigma_i}{4}\right)^2\sum_{j\in N_i}\left(\sigma_i - \sigma_j\right) = 0$$

and finally,

$$\sum_{j\in N_i}\left(\sigma_i - \sigma_j\right) = 0, \quad \forall i \in \mathcal{N}^f.$$

Hence, the agents' orientations at steady state satisfy:

$$\left(L\sigma^1\right)_i = \left(L\sigma^2\right)_i = \left(L\sigma^3\right)_i, \qquad i \in \mathcal{N}^f, \tag{9a}$$

$$\sigma_i = \sigma_i^d, \qquad i \in \mathcal{N}^l, \tag{9b}$$

where for a vector a, $(a)_i$ denotes its ith element. The solutions of (9) have been studied in [3]. In particular, Theorem 2 in [3] states that for a connected leader-follower communication graph and a nonempty set of leaders, the orientation of each follower, as given by the solution of (9), lies in the convex hull of the leaders' orientations.

The previous derivations are summarized as follows:

Theorem 1: Assume that the leader-follower communication graph $\mathcal{G}$ is connected and that the subset of leaders is nonempty. Moreover assume that (5) holds. Then the control law (6) drives the followers to the convex hull of the leaders' orientations with zero angular velocities.

IV. Leader Relative Orientation Control Design

In this section we present a control algorithm that drives the team of leaders to the desired relative orientations. This is a problem that resembles the formation control problem in multi-vehicle systems. The relative orientation for each pair of leaders may be different, and can be dictated by the mission requirements. In this section, we thus impose the specification that for each pair $(i,j) \in E^l$, there exists a desired relative orientation $\sigma_{ij}^d \in \mathbb{R}^3$, to which the pair leaders should converge. Next, we provide a control law that respects the limited communication requirements dictated by the leader communication graph $\mathcal{G}^l$ in order to achieve this objective.

We first assume that a leader $L \in \mathcal{N}_l$ plays the role of a reference point around which the desired relative orientations should be satisfied. This can represent a satellite that is initially aware of the desired target area. We assume that this satellite has already been stabilized to a desired equilibrium point

$$\sigma_L = \sigma_L^d, \quad \omega_L = 0. \tag{10}$$

The control design for the case of single rigid body stabilization can be found in [19]. The result of this section is summarized in the following theorem.

Theorem 2: Assume that the leader communication graph $\mathcal{G}^l$ is connected and that (10) holds. Then the control strategy

$$u_i = -G^{\intercal}(\sigma_i) \sum_{j\in N_i^l} \left(\sigma_i - \sigma_j - \sigma_{ij}^d\right) - \sum_{j\in N_i^l} (\omega_i - \omega_j), \tag{11}$$

where $i \in \mathcal{N}^l \setminus \{L\}$ drives the leaders to the desired relative orientations.

Proof: For each leader $i \in \mathcal{N}^l$, we define the "cost function"

$$\gamma_i = \frac{1}{2} \sum_{j\in N_i^l} \left\|\sigma_i - \sigma_j - \sigma_{ij}^d\right\|^2$$

and we introduce

$$V(\sigma,\omega) = \sum_{i\in\mathcal{N}^l} \left(\frac{1}{2}\omega_i^{\intercal} J_i \omega_i\right) + \sum_{i\in\mathcal{N}^l} \left(\frac{1}{2}\gamma_i\right)$$

as a candidate Lyapunov function. We then have

$$\dot V(\sigma,\omega) = \sum_{i\in\mathcal{N}^l} (\omega_i^{\intercal} J_i \dot\omega_i) + \frac{1}{2}\left\{\sum_{i\in\mathcal{N}^l} (\nabla^{\intercal}\gamma_i)\right\}\dot\sigma.$$

Without loss of generality, we denote the leaders' indices by $1,\ldots,|\mathcal{N}^l|$ and we also note that in this section, the notation σ,ω refers to the stack vectors of the *leaders*' orientations and angular velocities respectively. With a slight abuse of notation we can now write the last term in the previous equation as

$$\nabla\gamma_i = \begin{bmatrix} \dfrac{\partial^{\intercal}\gamma_i}{\partial\sigma_1} & \cdots & \dfrac{\partial^{\intercal}\gamma_i}{\partial\sigma_{|\mathcal{N}^l|}} \end{bmatrix},$$

where,

$$\frac{\partial\gamma_i}{\partial\sigma_j} = \begin{cases} \sum_{j\in N_i^l} (\sigma_i - \sigma_j) + \sigma_{ii}^d, & i = j \\ -\left(\sigma_i - \sigma_j - \sigma_{ij}^d\right), & j \in N_i^l,\ j \neq i, \\ 0, & j \notin N_i^l. \end{cases}$$

and where we have defined $\sigma_{ii}^d = -\sum_{j\in N_i^l} \sigma_{ij}^d$. Hence,

$$\begin{aligned}
\sum_{i\in\mathcal{N}^l} \frac{\partial\gamma_i}{\partial\sigma_j} &= \frac{\partial\gamma_j}{\partial\sigma_j} + \sum_{i\in N_j^l} \frac{\partial\gamma_i}{\partial\sigma_j} \\
&= \sum_{i\in N_j^l} (\sigma_j - \sigma_i) + \sigma_{jj}^d + \sum_{i\in N_j^l} (-\sigma_i + \sigma_j + \sigma_{ij}^d) \\
&= 2\sum_{i\in N_j^l} \sigma_j - 2\sum_{i\in N_j^l} \sigma_i + 2\sigma_{jj}^d \\
&= 2d_j\sigma_j - 2\sum_{i\in N_j^l} \sigma_i + 2\sigma_{jj}^d.
\end{aligned}$$

It follows that

$$\begin{aligned}
\sum_{i\in\mathcal{N}^l} \nabla\gamma_i &= \sum_{i\in\mathcal{N}^l} \begin{bmatrix} \dfrac{\partial\gamma_i}{\partial\sigma_1} & \cdots & \dfrac{\partial\gamma_i}{\partial\sigma_{|\mathcal{N}^l|}} \end{bmatrix} \\
&= 2\begin{bmatrix} d_1\sigma_1 & \cdots & d_{|\mathcal{N}^l|}\sigma_{|\mathcal{N}^l|} \end{bmatrix} \\
&\quad - 2\begin{bmatrix} \sum_{j\in N_1^l} \sigma_j & \cdots & \sum_{j\in N_{|\mathcal{N}^l|}^l} \sigma_j \end{bmatrix} \\
&\quad + 2\begin{bmatrix} \sigma_{11}^d & \cdots & \sigma_{|\mathcal{N}^l||\mathcal{N}^l|}^d \end{bmatrix},
\end{aligned}$$

and finally,

$$\sum_{i\in\mathcal{N}^l} \nabla\gamma_i = 2\left(\left(L^l \otimes I_3\right)\sigma + c_\ell\right)^{\intercal}, \tag{12}$$

where $c_\ell = \left[\sigma_{11}^d \cdots \sigma_{|\mathcal{N}^l||\mathcal{N}^l|}^d\right]^{\intercal}$ and L^l is the Laplacian matrix of the leader communication graph. Using (12), $\dot V$ can be written as

$$\dot V(\sigma,\omega) = u^{\intercal}\omega + \left(\left(L^l \otimes I_3\right)\sigma + c_\ell\right)^{\intercal} \mathsf{G}(\sigma)\,\omega.$$

where

$$\mathsf{G}(\sigma) = \mathrm{blockdiag}\left(G(\sigma_1),\ldots,G\left(\sigma_{|\mathcal{N}^l|}\right)\right),$$

Substituting (11), we have

$$\dot V = \sum_{i\in\mathcal{N}^l} \omega_i^{\intercal}\left\{u_i + G^{\intercal}(\sigma_i)\sum_{j\in N_i^l} \left(\sigma_i - \sigma_j - \sigma_{ij}^d\right)\right\}$$

from which it follows that

$$\dot V = -\sum_{i\in\mathcal{N}^l\setminus\{L\}} \omega_i^{\intercal}\left\{\sum_{j\in N_i^l} (\omega_i - \omega_j)\right\}$$

which, using the fact that $\omega_L = 0$, yields

$$\dot V = -\omega^{\intercal}\left(L^l \otimes I_3\right)\omega \leq 0.$$

Using similar arguments with the proof of Theorem 1, we conclude that since the leader communication graph is connected, all leaders attain the same angular velocities at steady state. Since $\omega_L = 0$, this common angular velocity is zero. We thus have shown that $\omega_i = 0$ for all $i \in \mathcal{N}^l$ at steady state. This, in turn, implies that $u_i = 0$ for all $i \in \mathcal{N}^l$ and following again the arguments of the proof of Theorem 1, we get

$$\sum_{j\in\mathcal{N}_i^l} \left(\sigma_i - \sigma_j - \sigma_{ij}^d\right) = 0, \quad \forall i \in \mathcal{N}^l\setminus\{L\}$$

at steady state. This implies that the leaders orientations satisfy the following equations at steady state

$$\left(\left(L^l \otimes I_3\right)\sigma + c_l\right)_i = 0, \quad \forall i \in \mathcal{N}^l\setminus\{L\} \tag{13a}$$

$$\sigma_L = \sigma_L^d. \tag{13b}$$

For all $i \in \mathcal{N}^l\setminus\{L\}$, let σ_i^d denote the desired orientation of leader i with respect to the global coordinate frame. It is then obvious that $\sigma_{ij}^d = \sigma_i^d - \sigma_j^d$ for all $(i,j) \in E^l$. Define $\sigma_i - \sigma_j - \sigma_{ij}^d = \sigma_i - \sigma_j - (\sigma_i^d - \sigma_j^d) = \tilde\sigma_i - \tilde\sigma_j$. The condition $\left(\left(L^l \otimes I_3\right)\sigma + c_l\right)_i = 0$ for all $i \in \mathcal{N}^l\setminus\{L\}$ along with the fact that $\sigma_L = \sigma_L^d$ implies that $\left(L^l \otimes I_3\right)\tilde{\sigma} = 0$, equivalently, $L^l\tilde{\sigma}_1 = L^l\tilde{\sigma}_2 = L^l\tilde{\sigma}_3 = 0$,, where $\tilde{\sigma}^1, \tilde{\sigma}^2, \tilde{\sigma}^3$ are the stack vectors of each of the three coefficients of $\tilde{\sigma}$ of the leaders' orientations, respectively. The fact that $\mathcal{G}^l$ is connected implies that L^l has a simple zero eigenvalue with corresponding eigenvector the vector of ones. This guarantees that each one of the vectors $\tilde{\sigma}^1, \tilde{\sigma}^2, \tilde{\sigma}^3$ are eigenvectors of L^l belonging to span$\{\overrightarrow{\mathbf{1}}\}$. Therefore, all $\tilde{\sigma}_i$ are equal to a common vector value, say c. Hence $\tilde{\sigma}_i = c$ for all $i \in \mathcal{N}^l$, which implies that $\sigma_i - \sigma_j = \sigma_{ij}^d$ where $j \in N_i^l$ and for all $i \in \mathcal{N}^l$. We conclude that the leaders converge to the desired, specified configuration of relative orientations. $\diamondsuit$

V. The case of lack of a global objective

In this section we assume that no global objective is imposed by a team of leaders. In particular, we assume that $\mathcal{N}_l = \emptyset$. The objective is to build distributed algorithms that drive the team of multiple rigid bodies to a common constant orientation with zero angular velocities.

In order to ensure that all agents converge to the same constant orientation, in this section we show that it is sufficient that one agent has a damping element on the angular velocity. In contrast, the control design of [12] assumes that all agents have a damping element in their angular velocity. Without loss of generality, we assume that this is agent 1. The following theorem is the main result of this section:

Theorem 3: Assume that the leader-follower communication graph is connected. Then the control strategy

$$u_i = -G^{\intercal}(\sigma_i) \sum_{j\in\mathcal{N}_i} (\sigma_i - \sigma_j) - \sum_{j\in\mathcal{N}_i} (\omega_i - \omega_j) - a_i\omega_i, \quad (14)$$

where $i = 1, \ldots, N$ and $a_i = 1$ if $i = 1$, and $a_i = 0$ otherwise, drives the rigid bodies to the same constant orientation with zero angular velocities.

Proof: We choose again

$$V(\sigma, \omega) = \sum_{i=1}^{N} \left(\frac{1}{2}\omega_i^{\intercal} J_i \omega_i\right) + \frac{1}{2}\sigma^{\intercal}(L \otimes I_3)\sigma$$

as a candidate Lyapunov function. Differentiating with respect to time, and after some algebraic manipulation, we obtain

$$\dot{V}(\sigma, \omega) = -\omega^{\intercal}(L \otimes I_3)\omega - \|\omega_1\|^2 \leq 0.$$

It follows that ω remains bounded. By LaSalle's invariance principle, the system converges to the largest invariant set inside the set

$$M = \{(\sigma, \omega) : (\omega^{\intercal}(L \otimes I_3)\omega = 0) \wedge (\omega_1 = 0)\}.$$

Similarly to the proof of Theorem 1, the condition $\omega^{\intercal}(L \otimes I_3)\omega = 0$ guarantees that all ω_i's converge to a common value. Since $\omega_1 = 0$, this common value is zero. Following now the same steps as in the proof of Theorem 1, we conclude that the system reaches a configuration in which $\sum_{j\in N_i}(\sigma_i - \sigma_j) = 0$ for all $i \in \mathcal{N}$ and thus

$$(L \otimes I_3)\sigma = 0.$$

Connectivity of the leader follower communication graph implies now that the agents attain a common constant orientation at steady state. $\diamondsuit$

VI. Numerical Example

In this section we present a numerical example that supports the theoretical developments.

The simulation involves four rigid bodies indexed from 1 to 4. We assume that there are two leaders $\mathcal{N}^l = \{1, 2\}$ and two followers $\mathcal{N}^f = \{3, 4\}$. We further assume that leader 1 is the reference point, and according to (10) we have $\sigma_1 = \sigma_1^d = [1.02, -1.12, 0.4]$, and $\omega_1 = 0$. The reference point σ_1^d was randomly produced for this example. We also have $\mathcal{N}_2^l = \{1\}$ and $\sigma_{12}^d = [1, -1, 1]$. The control law of leader 2 is given by (11). The communication sets of the followers are given by $\mathcal{N}_3 = \{1, 4\}$ and $\mathcal{N}_4 = \{2, 3\}$ and their control laws by (6). The inertia matrices of the four rigid bodies have been chosen here as $J_1 = \mathrm{diag}(18, 12, 10)$, $J_2 = \mathrm{diag}(22, 16, 12)$, $J_3 = \mathrm{diag}(17, 14, 12)$ and $J_4 = \mathrm{diag}(15, 13, 8)$.

Figures 1, 2 show the plots of the orientations and angular velocities of the four rigid bodies with respect to time in all three coordinates. We observe that the system behaves as expected. As witnessed in Figure 1, the orientations of the leaders converge to the desired relative value while the orientations of the followers converge to the convex hull of the leaders' final orientations. The angular velocities converge to zero, as shown in Figure 2. For this example, this implies that the final orientations of the followers 3,4 converge to values that are between the final values of the two leaders 1,2 in all three orientation coordinates.

VII. Conclusions

We propose distributed control strategies that exploit graph theoretic tools for cooperative rotational control of multiple rigid bodies. We assume that the agents are divided into leaders and followers. The leaders should maintain certain relative orientations with each other, while the followers' orientations are to remain within a certain region that is dictated by the orientations of the leaders. Similarly to the case with linear dynamics, the convergence of the multi-agent system was shown to rely on the connectivity of the communication graph. Further research efforts will involve the cases of switching interconnection topology, as well as the case of unidirectional communication.

VIII. Acknowledgments

The first author was supported in part by the Swedish Research Council, the Swedish Foundation for Strategic Research, and the EU NoE HYCON.

The third author was supported by the European Commission through contract iFLY (Safety, Complexity and Responsibility based design and validation of highly automated Air Traffic Management), No TREN/07/FP6AE/S07.71574/037180.

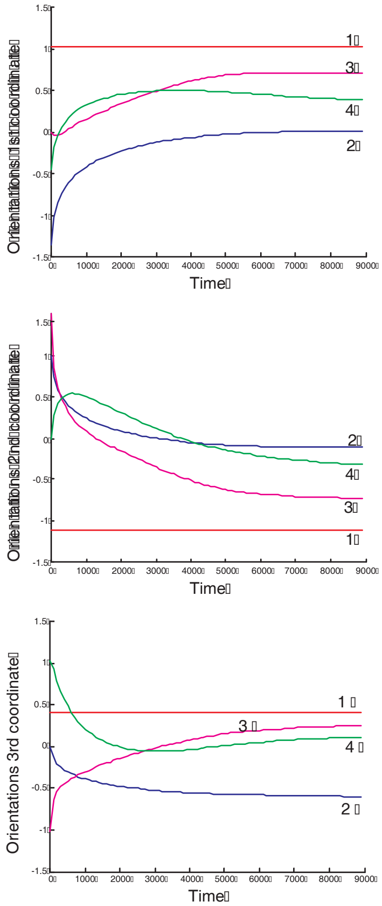

Fig. 1. Time histories of the orientations for the four rigid bodies using the leader-follower structure.

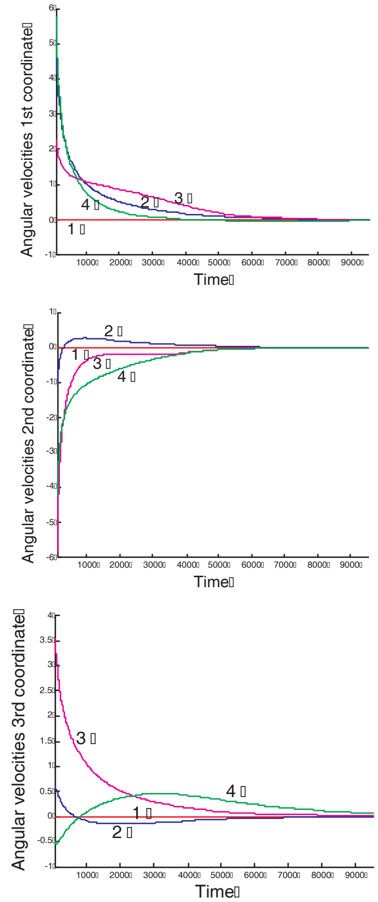

Fig. 2. Time histories of the angular velocities for the four rigid bodies using the leader-follower structure.

REFERENCES

[1] J. Cortes, S. Martinez, and F. Bullo. Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions. *IEEE Transactions on Automatic Control*, 51(8):1289– 1298, Aug. 2006.

[2] D. Dimarogonas, P. Tsiotras, and K. Kyriakopoulos. Laplacian cooperative attitude control of multiple rigid bodies. In *2006 CCA/CACSD/ISIC*, Münich, Germany, October 4-6 2006.

[3] G. Ferrari-Trecate, M. Egerstedt, A. Buffa, and M. Ji. Laplacian sheep: A hybrid, stop-go policy for leader-based containment control. In João P. Hespanha and Ashish Tiwari, editors, *HSCC*, volume 3927 of *Lecture Notes in Computer Science*, pages 212–226. Springer, 2006.

[4] C. Godsil and G. Royle. *Algebraic Graph Theory*. Graduate Texts in Mathematics # 207. Springer-Verlag, 2001. New York.

[5] A. Jadbabaie, J. Lin, and A.S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 48(6):988–1001, 2003.

[6] G. Lafferriere, A. Williams, J. Caughman, and J.J.P. Veerman. Decentralized control of vehicle formations. *Systems and Control Letters*, 54(9):899–910, 2005.

[7] J. R. Lawton and R. Beard. Synchronized multiple spacecraft rotations. *Automatica*, 38(8):1359–1364, 2002.

[8] M. Mesbahi and F.Y. Hadaegh. Formation flying control of multiple spacecraft via graphs, matrix inequalities, and switching. *Journal of Guidance,Control and Dynamics*, 24(2):369–377, 2001.

[9] S. Nair and N.E. Leonard. Stabilization of a coordinated network of rotating rigid bodies. *43rd IEEE Conf. Decision and Control*, pages 4690–4695, 2004. Paradise Island, Bahamas.

[10] R. Olfati-Saber. Flocking for multi-agent dynamic systems: Algorithms and theory. *IEEE Transactions on Automatic Control*, 51(3):401–420, 2006.

[11] R. Olfati-Saber and R.M. Murray. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control*, 49(9):1520–1533, 2004.

[12] W. Ren. Distributed attitude alignment among multiple networked spacecraft. In *American Control Conference*, pages 1760–1765, 2006. Minneapolis, MN.

[13] W. Ren. Synchronized multiple spacecraft rotations: A revisit in the context of consensus building. In *American Control Conference*, pages 3174–3179, 2007.

[14] W. Ren and R. Beard. Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Transactions on Automatic Control*, 50(5):655–661, 2005.

[15] M. D. Shuster. A survey of attitude representations. *Journal of Astronautical Sciences*, 41(4):493–517, 1993.

[16] H.G. Tanner, A. Jadbabaie, and G.J. Pappas. Stable flocking of mobile agents. *42st IEEE Conf. Decision and Control*, pages 2010–2015, 2003. Maui, HW.

[17] P. Tsiotras. New control laws for the attitude stabilization of rigid bodies. In *13th IFAC Symposium on Automatic Control in Aerospace*, pages 316–321, Sept. 14-17, 1994. Palo Alto, CA.

[18] P. Tsiotras. Stabilization and optimality results for the attitude control problem. *Journal of Guidance,Control and Dynamics*, 19(4):772–779, 1996.

[19] P. Tsiotras. Further passivity results for the attitude control problem. *IEEE Transactions on Automatic Control*, 43(11):1597–1600, 1998.

[20] P.K.C. Wang, F.Y. Hadaegh, and K. Lau. Synchronized formation rotation and attitude control of multiple free-flying spacecraft. *Journal of Guidance,Control and Dynamics*, 22(1):28–35, 1999.