An Auction Algorithm for Allocating Fuel in Satellite Constellations Using Peer-to-Peer Refueling

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***Abstract*— The problem of assuring the operability of a satellite constellation by internal (peer-to-peer/P2P) refueling is addressed. Specifically, a formulation is presented that seeks to match the fuel-sufficient satellites in the constellation with the fuel-deficient ones, so that the former can refuel the latter, while at the same time minimize the total fuel consumption incurred during the ensuing orbital rendezvous. Asymmetric auctions are introduced as a means for solving this problem, owing to their advantages for decentralized implementation. Examples are provided to demonstrate the proposed methodology.**

I. Introduction

As commercial space applications mature, increased attention is being placed on constellations of relatively inexpensive satellites as a robust–and possibly cheaper–alternative to large, multi-million dollar satellites. Applications of this paradigm have been considered in Internet routing [1], altimetry [2], and missile warning and defense [3] to name a few. For constellations with a large number of satellites the option of replacing them every few years or so–after their onboard fuel is depleted–may not be a viable option. Extending the lifespan of satellites over longer periods of time than it is currently possible, necessitates the use of suitable refueling strategies.

Coordinated refueling of satellite constellations has been recently addressed in [4], [5], [6], [7]. Therein, the authors investigated optimal scheduling for external refueling [4], and later peer-to-peer refueling within a constellation [7]. The latter paper focused on equalizing the fuel distribution within the constellation by exchanging fuel between the satellites in the constellation ("peer-to-peer" refueling). The goal was to minimize the 1-norm of the deviation of the satellites' fuel content from the constellation average fuel. In [5], [6] the authors showed that a mixed P2P-tug refueling strategy can be more fuel-efficient for refueling a large number of satellites compared with the use of a single-vehicle refueling strategy.

In this paper we approach the coordinated satellite refueling problem from a different standpoint. Instead of seeking to equalize fuel amongst all the satellites in the constellation, we assume instead that each satellite needs a minimum amount of fuel in order to remain operational. We also assume that several of the satellites have an excess amount of fuel. This may be, for instance, due to an earlier stage of a mixed refueling strategy [5], [6]. We may therefore define two disjoint sets of satellites: one set consisting of those satellites that have enough fuel (the "fuel sufficient" satellites), and the second set consisting of those satellites which do not have enough fuel (the "fuel deficient" satellites). The question we seek to answer is the following: can we use the fuel sufficient satellites to refuel the fuel deficient ones so that *all* the satellites will have the required minimal amount of fuel by the end of the refueling process?

A P2P refueling strategy seeks to match fuel-deficient with fuel sufficient satellites, while minimizing the orbital transfer cost. The dual role of fuel/propellant on board the satellites as commodity and exchange constraint is what makes the P2P problem both interesting and challenging. An additional complication arises from the fact that the fuel consumption for each transfer depends strongly on the allowable time to refuel as well as the number of impulses [8]. For simplicity, here we only assume two-impulse transfers.

The current paper extends the work of [7], [5] by considering optimal refueling strategies that result in all satellites in the constellation being fuel sufficient after a given time T.

To solve the P2P refueling problem, we first formulate it as an assignment problem over the so-called *constellation graph* [7]. Implicit or explicit operational constraints can be directly incorporated in the formation of the constellation graph. Under some mild assumptions, the constellation graph is a bi-partite graph, a fact that allows us to introduce elements from the theory of (asymmetric) auctions in order to solve this problem [9], [10]. The ability to calculate the optimal satellite assignments in a decentralized/distributed manner is the main attractive feature of auction algorithms. This property is ideal for constellations with a large number of satellites, since each satellite has to keep track only of its own best interest and its own information, while still optimizing the resulting overall cost. A key additional benefit is the possibility of asynchronous implementation, which greatly increases robustness [10].

II. P2P Refueling Problem Formulation

A. Notation and Basic Assumptions

Consider a constellation $\mathcal{C} = \{s_k : k = 1, 2, \ldots, N\}$ of N satellites. We call satellite s_k *fuel sufficient* if

$$f_k^- > \underline{f}_k, \tag{1}$$

where f_k^- denotes the fuel content of the satellite prior to refueling, and $\underline{f}_k$ denotes the minimum fuel required for the

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satellite to be operational until the next external refueling. Otherwise, the satellite will be called *fuel deficient*. Let $\mathcal{C}_s = \{s_k : f_k^- > \underline{f}_k\}$ be the set of fuel-sufficient satellites and $\mathcal{C}_d = \mathcal{C}\backslash\mathcal{C}_s$ the set of fuel-deficient satellites. In the sequel we will relabel the elements in the sets $\mathcal{C}_s$ and $\mathcal{C}_d$, and we will be using the index i to denote a fuel-deficient satellite, and the index j to denote a fuel-sufficient satellite.

Let $\mathcal{D} = \{i : i = 1, 2, \ldots, m\}$ denote the index set of all fuel-deficient satellites, and $\mathcal{S} = \{j : j = 1, 2, \ldots, n\}$ denote the index set of all fuel-sufficient satellites. Note that, by definition, $N = m + n$. We seek to find a refueling strategy that will ensure that, for all $s_k \in \mathcal{C}$,

$$f_k^+ \geq \underline{f}_k, \tag{2}$$

while minimizing fuel consumption during the ensuing rendezvous. Here f_k^+ denotes the fuel content of satellite s_k after the refueling transaction is completed[1].

The sets $\mathcal{D}$ and $\mathcal{S}$ induce a natural bi-partition on the set $\mathcal{C}$. Given the index sets $\mathcal{D}$ and $\mathcal{S}$ we may define a bi-partite graph $\mathcal{G} = \{\mathcal{D} \cup \mathcal{S}, \mathcal{E}\}$ over $\mathcal{C}$, where the set of edges $\mathcal{E} = \{(i, j) : i \in \mathcal{D}, j \in \mathcal{S}\}$ in the graph has as vertices pairs of fuel sufficient and fuel deficient satellites. If there are no other operational constraints, $\mathcal{G}$ is a complete graph.

For each rendezvous between a fuel sufficient satellite and a fuel deficient satellite, we will assume that only one is *active*, namely only one initiates the rendezvous, and returns to its original slot after refueling. The other satellite remains *passive* during the fuel transaction. Note that this does not preclude cases when either one of the satellites can be the active one. In particular, we can partition $\mathcal{E}$ into three subsets as follows

$$\begin{aligned}
\mathcal{A} &= \{(i, j) \in \mathcal{E} : i \text{ is active}\}, && (3a)\\
\mathcal{P} &= \{(i, j) \in \mathcal{E} : i \text{ is passive}\}, && (3b)\\
\mathcal{U} &= \{(i, j) \in \mathcal{E} : (i, j) \text{ is infeasible}\}, && (3c)
\end{aligned}$$

where an infeasible pair is defined as one for which neither of the two satellites can be active. Note that $\mathcal{A} \cap \mathcal{P} \neq \varnothing$ to allow for the case when both satellites i and j may be active. Let the sets $\mathcal{U}_1 = \{(i, j) : p_{ij}^i > f_i^-\}$, $\mathcal{U}_2 = \{(i, j) : p_{ij}^j > f_j^-\}$ and $\mathcal{U}_3 = \{(i, j) : f_i^- + f_j^- - c_{ij} < \underline{f}_i + \underline{f}_j\}$ where p_{ij}^i is the fuel for the (active) satellite i to meet with (passive) satellite j, and similarly for p_{ij}^j, and c_{ij} is the total amount of fuel required to perform the rendezvous between satellites i and j. The expressions for p_{ij}^i, p_{ij}^j, and c_{ij} are given in Section III-B below. It can be readily shown that the set of infeasible pairs is given by $\mathcal{U} = \bigcup_{k=1}^3 \mathcal{U}_k$.

By keeping only the feasible pairs $\mathcal{E}_f = \mathcal{A} \cup \mathcal{P} \subseteq \mathcal{E}$ one obtains the *feasible constellation graph* $\mathcal{G}_f = \{\mathcal{D} \cup \mathcal{S}, \mathcal{E}_f\}$. The graph $\mathcal{G}_f$ thus contains exactly the pairs for which at least one of the two satellites can initiate a fuel transaction, exchange fuel with its partner, and return to its original slot, and so that both satellites satisfy condition (2). Clearly, the graph $\mathcal{G}_f$ is not, in general, a complete graph since several fuel-sufficient/fuel-deficient satellite pairs may not be feasible.

Let $\mathcal{N}(i) = \{j \in \mathcal{S} : (i, j) \in \mathcal{E}_f\}$ denote the set of fuel sufficient satellites that can perform a fuel transaction with $i \in \mathcal{D}$, and associate to each pair $(i, j) \in \mathcal{E}_f$ a cost $c_{ij} > 0$, which is the least amount of fuel required to perform a rendezvous between i and j. Since we only allow one fuel exchange per satellite, the final result of a P2P refueling strategy is a matching $\mathcal{M} \subset \mathcal{E}_f$ such that the total fuel incurred during the associated rendezvous of the satellite pairs in $\mathcal{M}$ is minimized. By matching here we mean a subset of edges in $\mathcal{E}_f$ such that no two edges share the same vertex.

B. Satellite Assignment Problem

The problem of finding the optimal matching $\mathcal{M}^*$ can be formulated as a linear program over the bi-partite graph $\mathcal{G}_f$ as follows

$$\begin{aligned}
\text{Maximize} \quad & -\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} && (4)\\
\text{Subject to:} \quad & \sum_{j=1}^{n} x_{ij} = 1, \quad \forall\, i \in \mathcal{D}, && (5)\\
& \sum_{i=1}^{m} x_{ij} \leq 1, \quad \forall\, j \in \mathcal{S}, && (6)\\
& x_{ij} = 1 \implies j \in \mathcal{N}(i), \quad i \in \mathcal{D}. && (7)\\
& x_{ij} \geq 0, \quad \forall\, i, j. && (8)
\end{aligned}$$

Equation (5) enforces the condition that every fuel deficient satellite must be paired with exactly one fuel sufficient satellite. Inequality (6) enforces the condition that every fuel sufficient satellite can be paired with at most one fuel deficient satellite. Equation (7) simply states that all pairs considered are feasible. Note that for a problem to be feasible, necessarily $n \geq m$.

Note that the original integrality constraint $x_{ij} \in \{0, 1\}$ has been replaced by (7) without loss of generality, as it is well known that assignment problems always have integral solutions [11].

III. Optimal Rendezvous Cost

A. Calculation of ΔV

Consider two satellites $i \in \mathcal{D}$ and $j \in \mathcal{S}$ in circular orbits of (possibly different) radii r_i and r_j, and separated initially by an angle θ_{ij}, as shown in Figure 1.

If satellite i is active, it will apply an impulse to rendezvous with satellite j within a given amount of time t_{ij}^i. Let $\Delta V_{ij}^i = \Delta V(t_{ij}^i, \theta_{ij})$ denote the two-impulse velocity increment required for satellite i to leave its orbital slot and rendezvous with satellite j under the time constraint t_{ij}^i. The calculation of ΔV_{ij}^i can be performed using, for example, the results of [8], [12]. Although easy to compute numerically, no analytic solution for ΔV_{ij}^i is available as it requires the solution of the multi-revolution Lambert problem.

The situation is significantly simpler for transfers to and from the same circular orbit, as is the case of the constellations under consideration. In such as case it can be

[1] A refueling transaction includes the forward and return rendezvous maneuvers, as well as the actual exchange of fuel between the two satellites.

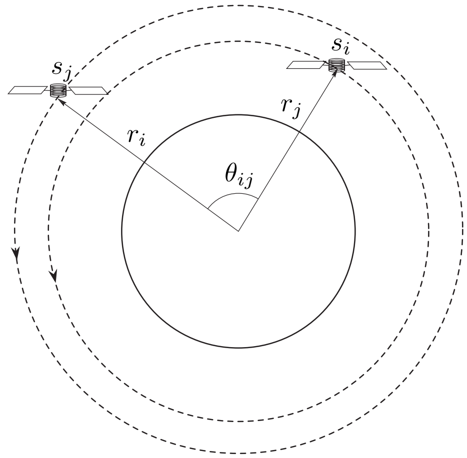

Fig. 1. A basic rendezvous configuration.

shown that for the majority of cases, the optimal two-impulse transfers correspond to "phasing maneuvers." A phasing maneuver uses tangential initial and final impulses.

Computing the cost of a phasing transfer is much simpler than computing the cost of a multiple-revolution Lambert transfer and it can be done analytically. Specifically, the ΔV_{ij}^i for satellite i to rendezvous with satellite j via phasing can be obtained analytically as follows [13]

$$\Delta V_{ij}^i = \sqrt{\frac{\mu}{r}} \left| \sqrt{2 - \left(\frac{z+\kappa}{z - \tilde{\theta}_{ij}/2\pi} \right)} - 1 \right|, \tag{9}$$

$$z = \left\lfloor \frac{t_{ij}^i}{2\pi} \sqrt{\frac{\mu}{r^3}} + \frac{\tilde{\theta}_{ij}}{2\pi} \right\rfloor, \tag{10}$$

where r is the radius of the orbit and

$$\tilde{\theta}_{ij} = \begin{cases} \theta_{ij} \bmod 2\pi, & \text{if } -\pi \le \theta_{ij} \bmod 2\pi \le \pi, \\ \theta_{ij} \bmod 2\pi - 2\pi, & \text{if } \theta_{ij} \bmod 2\pi > \pi, \\ \theta_{ij} \bmod 2\pi + 2\pi, & \text{if } \theta_{ij} \bmod 2\pi < -\pi, \end{cases} \tag{11}$$

and where,

$$\kappa = \begin{cases} -1 & \text{if } a > r \text{ and } \tilde{\theta}_{ij} > 0, \\ 1 & \text{if } a < r \text{ and } \tilde{\theta}_{ij} < 0, \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

where a is the semi-major axis of the transfer orbit.

Figure 2 depicts (in dashed line) the optimal ΔV from the solution of the multi-revolution Lambert problem (with coasting) and the ΔV obtained via phasing. Although there are a few cases when the phasing transfer is suboptimal, the degree of suboptimality is small and decreases as the allowed transfer time increases. If needed, one may calculate both the Lambert rendezvous and the phasing rendezvous costs, and choose the smaller one of the two.

Notice also from Figure 2 that no phasing rendezvous is possible before the first minimum of the Lambert ΔV curve. This is because there must be enough time for the passive satellite to arrive at the location where the first impulse was applied. This time is simply given by $t_{\min} = \omega\theta_{ij}$, where ω is the angular velocity of the orbit. Therefore, if $t_{ij}^i \le t_{\min}$ a Lambert transfer is necessary.

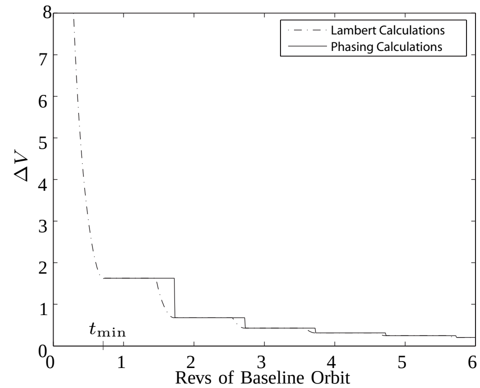

Fig. 2. ΔV comparison between the general (Lambert-transfer) and phasing rendezvous, for a separation angle $\theta_{ij} = 100°$. For the majority of cases a phasing transfer is optimal. Even in cases when it is not optimal, the degree of suboptimality is small.

B. *Fuel Cost for Rendezvous*

Given the required ΔV_{ij}^i for satellite i to visit satellite j, the corresponding fuel cost p_{ij}^i can be computed from [14]

$$p_{ij}^i = (m_i + f_i^-) \left(1 - e^{-\Delta V_{ij}^i/\sigma_i}\right), \tag{13}$$

where $\sigma_i = g_0 I_{\text{sp}i}$, m_i is the dry mass of satellite i, g_0 is the acceleration of gravity at the surface of the Earth and $I_{\text{sp}i}$ is the specific impulse of the satellite engine.

The return fuel cost p_{ji}^i can be similarly computed from

$$p_{ji}^i = (m_i + f_i^- - p_{ij}^i - g_{ij}) \left(1 - e^{-\Delta V_{ji}^i/\sigma_i}\right), \tag{14}$$

where g_{ij} denotes the amount of fuel transferred from i to j. Note that, in general, $\Delta V_{ij}^i \ne \Delta V_{ji}^i = \Delta V(t_{ji}^i, 2\pi - \theta_{ij})$, even if $t_{ij}^i = t_{ji}^i$; see, for example, [8].

From (13) and (14), it is clear that in order to minimize $p_{ij}^i + p_{ji}^i$ the quantity g_{ij} must be maximized, subject to the constraint $g_{ij} \le f_i^- - p_{ij}^i - p_{ji}^i - \underline{f}_i$, so that $f_i^+ = f_i^- - p_{ij}^i - p_{ji}^i - g_{ij} \ge \underline{f}_i$. Clearly, the optimal value is $g_{ij}^* = f_i^- - p_{ij}^i - p_{ji}^i - \underline{f}_i$, and the corresponding value for p_{ji}^i is

$$p_{ji}^i = (m_i + \underline{f}_i + p_{ji}^i) \left(1 - e^{-\Delta V_{ji}^i/\sigma_i}\right),$$

or, upon rearrangement

$$p_{ji}^i = (m_i + \underline{f}_i) \left(1 - e^{-\Delta V_{ji}^i/\sigma_i}\right) e^{\Delta V_{ji}^i/\sigma_i}. \tag{15}$$

Equations (13) and (15) give the optimal fuel cost in the case where satellite i is active, and under the assumptions that i has enough fuel to complete the first leg of the rendezvous and that there is enough fuel in j to transfer over g_{ji} units of fuel to i.

Similarly, when satellite j is active, the cost for the forward trip is given by

$$p_{ji}^j = (m_j + f_j^-) \left(1 - e^{-\Delta V_{ji}^j/\sigma_j}\right), \tag{16}$$

whereas the return transfer cost is given by

$$p_{ij}^{j} = (m_j + f_j^- - p_{ji}^j - g_{ji})\left(1 - e^{-\Delta V_{ij}^j/\sigma_j}\right). \tag{17}$$

Note that in this case $g_{ji} \geq 0$, and its value is limited by the fuel requirement for j to be able to make its return trip, and by the amount of fuel that satellite i is capable of accepting. If $\bar{f}_i$ denotes the maximum fuel capacity of satellite i then equation (17) implies that g_{ji} must be maximized in order to minimize $p_{ji}^j + p_{ij}^j$. The optimal transfer fuel from j to i in this case is given by

$$g_{ji}^* = \min\{f_j^- - \underline{f}_j - p_{ij}^j - p_{ji}^j\ ,\ \bar{f}_i - f_i^-\}, \tag{18}$$

which leads to the following expression for p_{ij}^j

$$p_{ij}^j = \begin{cases} (m_j + f_j^- - p_{ji}^j - \bar{f}_i + f_i^-)\left(1 - e^{-\Delta V_{ij}^j/\sigma_j}\right), \\ \qquad \text{if} \quad g_{ji}^* = \bar{f}_i - f_i^-, \\ (m_j + \underline{f}_j)\left(1 - e^{-\Delta V_{ij}^j/\sigma_j}\right) e^{\Delta V_{ij}^j/\sigma_j}, \\ \qquad \text{if} \quad g_{ji}^* = f_j^- - \underline{f}_j - p_{ij}^j. \end{cases} \tag{19}$$

Finally, the cost c_{ij} assigned to satellite pair (i, j) is given by

$$c_{ij} = \min\{p_{ij}^i + p_{ji}^i, p_{ji}^j + p_{ij}^j\}. \tag{20}$$

IV. A Solution to the P2P Refueling Problem via Asymmetric Auctions

A. The Asymmetric Auction Algorithm

The problem in (4)-(7) is an *asymmetric assignment* problem, that is, a problem where a set of m "persons" (in this case fuel-deficient satellites) is matched to a set of n "objects" (in this case fuel-sufficient satellites) so that every person is matched to one and only one object with $m \leq n$. Of the many existing methods for solving assignment problems, the auction algorithm naturally fits the P2P refueling problem because of its inherent distributed nature. The method is also relatively immune to time delays and asynchronous or bad communication links between the satellites that may result in out-of-date bids [15], [16]. Moreover, auction algorithms tend to be far superior than other methods when the underlying graph structure is sparse [10], as is typically the case with satellite constellations. Below we summarize the main ideas behind auction theory for solving assignment problems. A more detailed exposition can be found in [9] and [10].

To this end, consider a set of m persons $\mathcal{D} = \{i : i = 1, \ldots, m\}$ that have to be assigned to a set of n objects $\mathcal{S} = \{j : j = 1, \ldots, n\}$, where $m \leq n$, such that $(i, j) \in \mathcal{E}_f$, where $\mathcal{E}_f$ denotes the set of all allowable pairs. For every person i the set $\mathcal{N}(i)$ consists of all objects that person i can be assigned to. For each object $j \in \mathcal{N}(i)$ there is a benefit a_{ij} for matching person i with object j. The objective is to find the person/object pairs (i, j_i) so that all persons are assigned only one object, and such that the total benefit $\sum_{i\in\mathcal{D}} a_{ij_i}$ is maximized among all possible person/object pairs.

One way to do this is by using an auction mechanism. To this end, we assign to each object a *price* π_j. Each object then results in a *profit* $a_{ij} - \pi_j$ to person i. Each person i then seeks to be assigned to the object j_i which yields the greatest profit, that is,

$$a_{ij_i} - \pi_{j_i} = \max_{j\in\mathcal{N}(i)} \{a_{ij} - \pi_j\}, \tag{21}$$

a condition known as *complementary slackness*. Since the same object may be desired by more than one person, a bidding mechanism is introduced, whereby every object that is bid on by more than one person chooses the highest bidder and then raises its price to that offered by the highest bidder. Specifically, each person which is unassigned at the beginning of a given iteration bids a price increment equal to

$$\gamma_i = v_i - w_i + \varepsilon, \tag{22}$$

where,

$$v_i = \max_{j\in\mathcal{N}(i)} \{a_{ij} - \pi_j\}, \quad w_i = \max_{\substack{j\in\mathcal{N}(i)\\ j\neq j_i}} \{a_{ij} - \pi_j\}. \tag{23}$$

If j_i is the only object in $\mathcal{N}(i)$, then we let $w_i = -\infty$ The process is repeated until all persons are assigned, in which case the algorithm terminates. Troublesome cases, when an object is equally desirable by two persons, and neither one is willing to raise its bid to get the object (leading to the object being assigned to each of the two persons alternatively with each bidding iteration) are possible. This phenomenon is called cycling. Cycling can be eliminated if instead of (21) one enforces the condition

$$a_{ij_i} - \pi_{j_i} \geq \max_{j\in\mathcal{N}(i)} \{a_{ij} - \pi_j\} - \varepsilon, \tag{24}$$

where $\varepsilon > 0$, as is required implicitly by (22). This form is known as ε-complementary slackness and avoids cycling. It is guaranteed that any assignment that fulfills (24) will be within $m\varepsilon$ of optimal [10]. Since the running time of the algorithm is inversely proportional to ε, a tradeoff exists between speed of termination and optimality of solutions.

Remark 1 The auction algorithm is only one of the available methods for solving the assignment problem (4)-(8); see, for example [17]. These algorithms are very efficient and perform extremely well even for a very large number of satellites. Since the time scales for performing the orbital transfers are several orders of magnitude larger than the time it takes to solve an assignment problem (for the typical case of tens or even hundreds of satellites) the main benefit from the used of auctions stems from their distributed nature of implementation and their robustness with respect to communication delays or losses [15], rather than their convergence rates.

Remark 2 The auction algorithm used in this section to solve the assignment problem between the fuel deficient and fuel sufficient satellites is motivated by the theory of real-life *economic* auctions. In particular, the bidding process resembles what is known in the economic auctions literature [18],

[19] as the English auction[2]. Despite the similarities in terms of the motivation between auction algorithms for assignment problems as used in this work and economic auctions, the two concepts are also quite different. For instance, in economic auctions often the benefit of each object to the buyers is not known in advance. In certain cases, a buyer does not even know the actual benefit of acquiring the object. Subsequently, one is only given probabilistic information on each object's valuation to the potential buyers. The term "asymmetric auction" is often used in the field of economic auctions to describe the asymmetry of each buyer's knowledge of the other's valuation of individual objects [19], while in our case this term denotes simply the existence of more objects than persons, in accordance to the terminology of [9].

V. Numerical Examples

Two examples are considered in this section to demonstrate the proposed approach for solving the P2P refueling problem. In the first example, we consider a 20-satellite constellation in a circular orbit. We assume that the satellites are equidistantly spaced around the orbit. Each satellite has dry mass of 50 units, a maximum fuel capacity of 100 units, and an equivalent $I_{\mathrm{sp}i}$ of 197s. The total allowable time to complete the refueling is $T = 20$ time units[3].

Table I shows the fuel content and minimum fuel requirement of each satellite. Satellites 9, 11, and 13 are fuel deficient, and the rest are fuel sufficient. With a value of $\varepsilon = 1/4$, the auction algorithm yields the pairs $\mathcal{M}_f^* = \{(9,8),(11,10),(13,14)\}$, with a total cost of 5.43 units of fuel. The solution was obtained after only 4 bids. This answer is guaranteed to be within 0.75 units of fuel from the optimum. Note that for this example the cost of refueling the satellites is only 0.61% of the initial fuel content of the constellation.

TABLE I

Satellite fuel specifics for Example 1.

Satellite	f_k^-	$\underline{f}_k$	Satellite	f_k^-	$\underline{f}_k$
1	83.1	8.5	11	0.3	6.4
2	37.2	16.7	12	78.8	24.6
3	33.8	27.1	13	14.8	19.1
4	23.4	12.1	14	58.3	6.2
5	32.6	11.9	15	70.6	29.9
6	28.0	19.3	16	82.4	23.1
7	29.1	29.0	17	38.9	0.1
8	40.6	6.3	18	42.9	9.8
9	2.4	22.9	19	78.5	13.1
10	44.1	2.8	20	71.0	28.5

Table II shows the final fuel contents of the satellites, as well as the amount of fuel transferred. The active satellite for each pair is also indicated. Note that for each pair, there is one satellite that is exactly fuel sufficient at the end of the fuel transaction, while the other has extra fuel. Specifically, for all pairs the active satellite is exactly self-sufficient at the end of the fuel transaction. This is expected since for maximum efficiency the active satellite should have as little mass as possible. The only case where this might not happen is when $(i,j) \in \mathcal{P}$, and the satellite i does not have enough fuel capacity to accept all the fuel required in order to achieve $f_j^+ = \underline{f}_j$. Note also that in the pairs $(9,8)$ and $(13,14)$ the active satellite is the fuel deficient one.

TABLE II

Details of the fuel-optimal matching for Example 1.

$i \in \mathcal{D}$	$j \in \mathcal{S}$	f_i^+	f_j^+	g_{ji}^*	Active
9	8	22.9	18.38	22.21	9
11	10	39.64	2.8	39.34	10
13	14	19.1	52.23	6.06	13

Figure 3 shows the constellation graph with the feasible pairs represented by a thin line between the two satellites. The final optimal solution is also shown with bold lines.

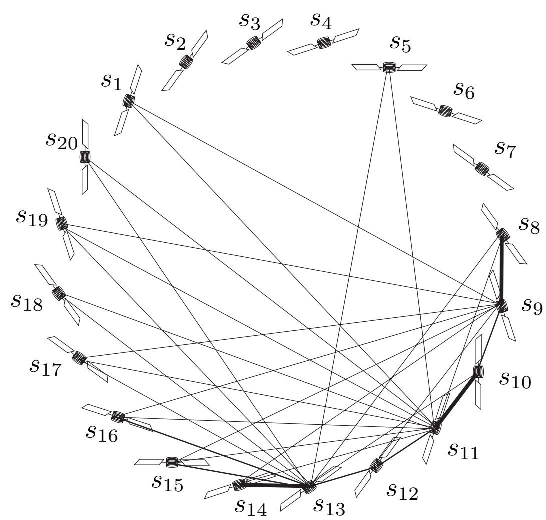

Fig. 3. The constellation graph for the first example. The bold lines indicate the final optimal pairings. The thin lines indicate all feasible pairs.

For the second example, we consider again a constellation of 20 satellites. For this example, half of the satellites are fuel sufficient and the other half are fuel deficient. The minimum amount of fuel required is 30 units for all satellites. Table III summarizes the initial fuel content for each satellite.

Table IV shows the optimal matching for this example. The total cost is 107.56 units of fuel, which corresponds to 12.37% of the total initial fuel content in the constellation (869 units). The solution in this example was obtained after 108 bids.

Figure 4 shows the constellation graph with the possible pairs represented by a thin line between the two satellites. The final matching is denoted with bold lines. Note that in both solutions, satellites tend to try to pair up with satellites that are close to themselves; see Figure 4. This fact could potentially be exploited to develop heuristics for more complicated cases.

[2]The other three commonly used auction methods are the first-price sealed-bid auction, the second price sealed-bid auction, and the Dutch auction [20].

[3]The normalized units used in this paper are the same as those introduced in [8].

TABLE III

SATELLITE FUEL SPECIFICS FOR EXAMPLE 2.

Satellite	f_k^-	$\underline{f}_k$	Satellite	f_k^-	$\underline{f}_k$
1	12	30	11	44	30
2	9	30	12	75	30
3	26	30	13	52	30
4	0	30	14	97	30
5	23	30	15	80	30
6	29	30	16	58	30
7	29	30	17	82	30
8	23	30	18	48	30
9	13	30	19	60	30
10	14	30	20	95	30

TABLE IV

DETAILS OF THE FUEL-OPTIMAL MATCHING FOR EXAMPLE 2.

$i \in \mathcal{D}$	$j \in \mathcal{S}$	f_i^+	f_j^+	g_{ji}^*	Active
1	19	30.0	37.90	22.09	1
2	17	30.0	51.09	30.90	2
3	18	30.0	32.93	15.06	3
4	20	52.76	30.0	52.76	20
5	16	30.0	31.80	26.19	5
6	11	30.0	32.30	11.69	6
7	12	30.0	63.30	11.69	7
8	13	30.0	34.69	17.30	8
9	15	30.0	51.34	28.65	9
10	14	30.0	73.27	23.72	10

VI. CONCLUSIONS

This paper introduces a new formulation for the peer-to-peer refueling problem that seeks to ensure that all satellites have a certain minimum amount of fuel after all refueling transactions have been completed. This amount of fuel may be dictated by the projected fuel usage of each satellite until the next external refueling. The solution of the P2P problem is then solved using the theory of auctions. The method ensures decentralized and robust solutions. A blanket assumption in the problem formulation is that all satellites remain operational during the whole refueling process. In practice, one may want to enforce additional constraints, such as that certain satellites remain passive to ensure a minimum level of functionality during refueling. This operational constraint is easily captured within the current framework. More challenging is the case when scheduling/sequencing constraints are imposed during refueling in order to minimize downtime. Peer-to-peer refueling subject to such scheduling constraints is currently under investigation.

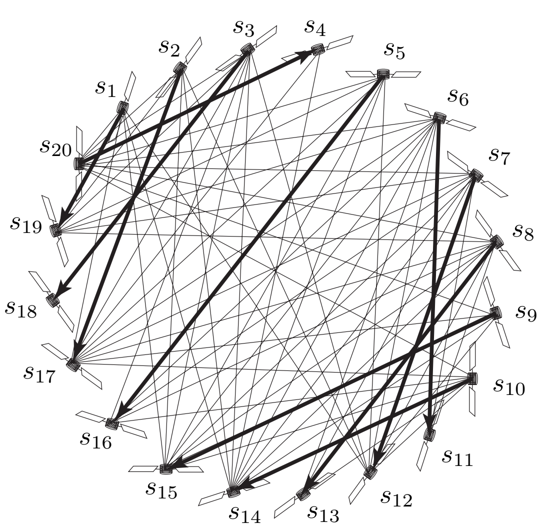

Fig. 4. The constellation graph for the second example. The bold lines indicate the final optimal pairings. The thin lines indicate all feasible pairs.

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