Stability of Linear Time-Delay Systems: A Delay-Dependent Criterion with a Tight Conservatism Bound *

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Abstract

The stability of linear time-delay systems is investigated via the robustness analysis of a related delay-free comparison system with an uncertain real parameter. By exploiting its phase properties, the delay element is removed from the system via a parameter-dependent Padé approximation. We then present a simple yet rigorous condition for delay-dependent stability of the original time-delay system. The novelty of this result is that it explicitly provides an *a priori* upper bound of how conservative this condition can be, and this bound depends only on the order of Padé approximation and can be reduced to any desired degree. Furthermore, the delay margin provided by this condition can be computed explicitly without incurring any additional conservatism for the single delay case. This condition can also be checked with some (typically small) additional conservatism by reducing it to finite-dimensional linear matrix inequalities (LMIs). Finally, several numerical examples demonstrate that this simplified LMI criterion can be significantly less conservative than those existing in the literature.

Keywords. Time-delay systems; stability; Padé approximation.

1 Introduction

The analysis of time-delay systems has attracted much interest over a half century, especially in the last decade. The recent book [5] contains an extensive collection of papers dealing with both delay-dependent and delay-independent stability conditions. Much interest in the literature has focused on searching for sufficient conditions which are numerically tractable but are not too conservative. Many such conditions involve, either explicitly or implicitly, covering the delay elements with some (convex) sets so as to obtain numerically tractable stability conditions [15]. Furthermore, the conservatism of the analysis can be reduced by choosing appropriate covering sets, based on delay elements' properties. This insight has been used in [16] to develop several less conservative LMI conditions for delay-dependent stability.

In this paper, this insight will be further exploited. The delay element is eliminated from the system by covering it using a parameter-dependent Padé approximation. The obtained comparison system is a delay-free system with a real parametric uncertainty. A simple delay-dependent sufficient stability condition, is then presented. Two approaches are then presented to avoid a parameter sweep. One approach provides an *explicit* formula to compute the delay margin provided by this condition without incurring any additional conservatism for the single delay case. The other approach is to reduce this condition with some (typically small) conservatism to finite-dimensional LMIs.

The traditional manner in using Padé approximations, such as [14], is to simply replace delay element $e^{-\tau s}$ with the approximation, by assuming *small delays* and some dynamical properties (such as bandwidth) of the system, because the Padé approximations are accurate only when $|\tau s|$ is sufficiently small. This does not guarantee, in general, the stability of the original systems. However, the approach of this paper can be used for any system with finite time-invariant delays. Moreover, the conditions derived here rigorously guarantee the stability of the time-delay system.

The contribution of this paper is that it presents a stability criterion whose degree of conservatism is guaranteed to be no more than an *a priori* upper bound. This upper bound depends only on the order of Padé approximation. The conservatism of the criterion can be reduced to any desired degree by increasing the order of the Padé approximation. To the best of the authors' knowledge, this is the first result for analysis of time-delay systems that guarantees a desired accuracy.

Notation. Let $\Re^{n\times m}$ ($\mathbf{C}^{n\times m}$) be the set of all real (complex) $n\times m$ matrices, I_n be $n\times n$ identity matrix, and W^T be the transpose of real matrix W. $X>0$ indicates that X is a symmetric and positive definite matrix. For matrices $M=(m_{ij})\in\Re^{n_1\times n_1}$ and $N\in\Re^{n_2\times n_2}$, the Kronecker product is defined by $M\otimes N := (m_{ij}N)$ and the Kronecker sum is defined by $M\oplus N := M\otimes I_{n_2} + I_{n_1}\otimes N$. $\lambda^+_{\max}(M)$ is the maximum positive real eigenvalue of M and $\lambda^+_{\max}(M)=0^+$ when M does

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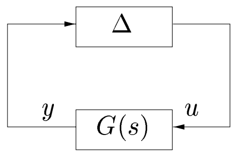

Figure 1: An interconnection system.

not have any positive real eigenvalues.

2 Preliminaries

Consider the linear time-delay system

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau) \tag{1}$$

where $\tau \in [0, \bar{\tau}]$ and $\bar{A} := A + A_d$ is Hurwitz. Let $A_d = HF$ where $H \in \Re^{n\times q}, F \in \Re^{q\times n}$ have full rank. In the sequel, $\Psi(s,\tau) := \det(sI_n - A - A_d e^{-\tau s})$ is the characteristic function associated with the system (1).

Definition 1 *The* ***actual delay margin*** $\bar{\tau}^*$ *for the system (1) is defined by*

$$\bar{\tau}^* := \sup_{\bar{\tau}>0}\{\bar{\tau}|(1) \textit{ is asymptotcially stable } \forall\tau \in [0,\bar{\tau}]\}.$$

The system (1) is said to be ***delay-dependent*** *if* $\bar{\tau}^* < \infty$, *and* ***delay-independent*** *otherwise.*

The delay-independent stability of time-delay systems has been studied extensively in the literature, see [2, 12, 9], etc. Herein, we investigate the delay-dependent stability of (1).

Definition 2 *Suppose* $\mathcal{P}$ *is a condition that ensures that (1) is asymptotically stable for all* $\tau \in [0,\bar{\tau}]$. *If (1) is delay-dependent with actual delay margin* $\bar{\tau}^*$, *then the* ***degree of conservatism (d.o.c.) of*** P *is defined by*

$$d.o.c. := \frac{\bar{\tau}^* - \bar{\tau}^*_{\mathcal{P}}}{\bar{\tau}^*}$$

where $\bar{\tau}^*_{\mathcal{P}} := \sup_{\bar{\tau}>0}\{\bar{\tau}|\mathcal{P} \text{ is true}\}$ *is said to be the* ***delay margin provided by*** P.

Definition 3 *Consider a linear, time-invariant (finite-dimensional) system* $G(s)$ *interconnected with an uncertain block* $\Delta \in \underline{\Delta}$ *(*$\underline{\Delta}$ *is a set of linear time-invariant stable systems), as shown in Figure 1, denoted as* $\sum[G(s),\Delta(s)]$. *Then the system is said to be* ***robustly stable*** *if* $G(s)$ *is internally stable, the interconnection is well-posed and it remains internally stable for all* $\Delta \in \underline{\Delta}$.

Lemma 1 *The system (1) is asymptotically stable for all* $\tau \in [0,\bar{\tau}]$ *if and only if*

$$\Psi(j\omega,\tau) \neq 0,\ \forall\omega \geq 0, \tau \in [0,\bar{\tau}].$$

Proof. This follows from the work of [3]. ∎

Corollary 1 *The system (1) is asymptotically stable for all* $\tau \in [0,\bar{\tau}]$, *if and only if*

$$\det[I_q - G(j\omega)\Phi(j\tau\omega)] \neq 0,\ \forall\omega \geq 0, \tau \in [0,\bar{\tau}], \tag{2}$$

where $G(s) = F(sI_n - \bar{A})^{-1}H$ *and* $\Phi(\tau s) = \phi(\tau s)I_q$, $\phi(\tau s) = e^{-\tau s} - 1$.

Examining the stability of (1) by checking the condition (2) directly is nontrivial, because (2) involves solving a transcendental equation. An indirect but intuitive approach of examining whether (2) holds, is to cover $\Phi(j\tau\omega)$ with another set $\underline{\Phi}(\omega)$, that is,

$$\Phi(j\tau\omega) \in \underline{\Phi}(\omega),\ \forall\omega \geq 0, \tau \in [0,\bar{\tau}].$$

Then (2) holds if

$$\det[I_q - G(j\omega)\Delta(j\omega)] \neq 0,\ \forall\omega \geq 0, \Delta(j\omega) \in \underline{\Phi}(\omega).$$

which is satisfied if the interconnection $\sum[G(s),\Delta(s)]$ (referred to as the comparison system in the sequel) is robustly stable. The conservatism of this approach mainly arises from the manner in which the covering set $\underline{\Phi}$ is chosen based on the properties of the delay element [15]. In [16], various covering sets, based on a shifted disk and/or weighting filter were introduced to reduce the conservatism of the analysis. Herein, we introduce a new less conservative covering set for the delay element $\Phi(j\tau\omega)$. We consider the mth order ($m \geq 3$) *diagonal* Padé approximation to e^{-s} as follows [11]

$$R_m(s) = \frac{N_m(s)}{N_m(-s)}$$

where

$$N_m(s) = \sum_{l=0}^{m} \frac{(2m-l)!(-s)^l}{l!(m-l)!}.$$

Let $\mathrm{Arg}(\cdot)$ denote the argument (phase) of $R_m(j\omega)$ such that it is *continuous* for all $\omega \geq 0$ and $\mathrm{Arg}(R_m(j\omega))\,|_{\omega=0} = 0$. The authors are grateful to Mr. V. Maymeskul and Prof. E.B. Saff for providing a proof (to be presented at the full version of this paper) for the following Lemma.

Lemma 2 *The function* $\frac{d}{d\omega}Arg(R_m(j\omega))$ *can be expressed in the following form:*

$$\frac{d}{d\omega}\mathrm{Arg}(R_m(j\omega)) = -\frac{T_m(\omega)}{\omega^{2m} + T_m(\omega)},\ \forall\omega \in \Re$$

where $T_m(\omega) = \sum_{k=0}^{m-1} a_k\omega^{2k}$, *and* $a_k > 0$, $k = 0, \cdots, m-1$.

Now, we define the following sets:

$$\Omega_A(\omega,\bar\tau) := \{e^{-j\tau\omega} | \tau \in [0,\bar\tau]\},$$
$$\Omega_B(\omega,\bar\tau) := \{R_m(j\theta\alpha_m\omega) | \theta \in [0,\bar\tau]\},$$
$$\Omega_C(\omega,\bar\tau) := \{R_m(j\theta\omega) | \theta \in [0,\bar\tau]\}.$$

where $\alpha_m := \frac{1}{2\pi}\min\{\omega > 0 | R_m(j\omega) = 1\}$. It can be found that for $m = 3, 4$ and 5, $\alpha_m \approx 1.2329, 1.0315$, and 1.00363 respectively.

The function $R_m(s)$ and the above sets have several important properties which are summarized as the following lemma.

Lemma 3 *For every integer $m \geq 3$, the following statements hold:*

(a) All poles of $R_m(s)$ are in the open left half complex plane.

(b) $\Omega_C(\omega,\bar\tau) \subseteq \Omega_A(\omega,\bar\tau) \subseteq \Omega_B(\omega,\bar\tau)$, $\forall\omega \geq 0$.

(c) $\lim_{\mathbf{m}\to\infty} \alpha_m = 1$.

Proof. (a) is a well-known result, see [11]. (c) follows directly from Theorem 3 of [6]. (b) can be shown by using Lemma 2. The details are omitted due to limited space. ■

3 Main Results

Now, we derive a delay-dependent stability condition for system (1). For convenience, denote the interconnection systems $\sum[G(s),\ (R_m(\theta\alpha_m s) - 1)I_q]$ and $\sum[G(s),\ (R_m(\theta s) - 1)I_q]$ as $\sum_B(\theta)$ and $\sum_C(\theta)$, respectively. Let (A_P, B_P, C_P, D_P) be the minimal realization of $P(s) := [R_m(\alpha_m s) - 1]I_q$ and denote n_P as the order of A_P. Also we let $A_s := \bar A + HD_PF, B_s := B_PF$, and $C_s =: HC_P$.

The following theorem gives a *sufficient* condition for stability of (1).

Theorem 1 *The system (1) is asymptotically stable for all $\tau \in [0,\bar\tau]$, if the comparison system $\sum_B(\theta)$ is robustly stable for $\theta \in [0,\bar\tau]$.*

Using above theorem, we obtain the following eigenvalue test for the stability of (1).

Corollary 2 *Let*

$$A_L(\theta) := \begin{bmatrix} A_s & \theta^{-\frac{1}{2}}C_s \\ \theta^{-\frac{1}{2}}B_s & \theta^{-1}A_P \end{bmatrix}. \tag{3}$$

Then the system (1) is asymptotically stable for any constant time-delay $\tau \in [0,\bar\tau]$, if $A_L(\theta)$ is Hurwitz for all $\theta \in (0,\bar\tau]$.

The following theorem presents a *necessary* condition for stability of (1). Later we will find that it plays a key role for checking the d.o.c. of our new result. The proof of this theorem is rather technical due to the singularity issue when θ is zero and it is omitted to keep our presentation straightforward.

Theorem 2 *If (1) is asymptotically stable for all $\tau \in [0,\bar\tau]$, then $\sum_C(\theta)$ is robustly stable for $\theta \in [0,\bar\tau]$.*

Now, we show that the d.o.c. of Theorem 1 (or Corollary 2) is bounded by a function of α_m.

Theorem 3 *The d.o.c. of Theorem 1 (or Corollary 2) satisfies*

$$d.o.c. \leq \frac{\alpha_m - 1}{\alpha_m}. \tag{4}$$

Moreover, d.o.c. $\to 0$ as $m \to \infty$.

Proof. Let $\bar\tau_B^*$ be the delay margin provided by Theorem 1. Let $\bar\tau_C^* := \sup_{\bar\tau>0}\{\bar\tau | \sum_C(\theta)$ is robustly stable for $\theta \in [0,\bar\tau]\}$. Then, clearly, we have $\bar\tau_C^* = \alpha_m\bar\tau_B^*$. In addition, from Theorem 2, $\sum_C(\theta)$ is asymptotically stable for all $\theta \in [0,\bar\tau]$ whenever (1) is asymptotically stable for all $\tau \in [0,\bar\tau]$. Therefore,

$$\bar\tau_C^* \geq \bar\tau^*$$

which immediately yields (4). ■

Remark 1 *For $k = 3, 4$ and 5, $\frac{\alpha_m - 1}{\alpha_m} \approx 18.9\%, 3.05\%$ and 0.361%, respectively. This bound can be reduced to any desired degree by choosing m sufficiently large at the expense of higher computational effort. This bound depends only on the order of Padé approximation used. It is independent of $\bar\tau^*$, A and A_d, and hence the d.o.c. of Theorem 1 is guaranteed for any linear system with a time-invariant state-delay.*

If one has already determined that (1) is asymptotically stable for all $\tau \in [0,\bar\tau_a]$, then the following corollary can be used instead of using Corollary 2.

Corollary 3 *Suppose that the system (1) is asymptotically stable for all $\tau \in [0,\bar\tau_a]$, where $\bar\tau_a > 0$. Then it is asymptotically stable for any constant time-delay $\tau \in [0,\bar\tau]$, if $A_L(\theta)$ is Hurwitz for all $\theta \in [\frac{\bar\tau_a}{\alpha_m},\bar\tau]$, where $A_L(\theta)$ is given by (3).*

Remark 2 *Any existing delay-dependent criteria, such as those of [10, 7, 8, 12, 16] etc., can be applied to obtain $\bar\tau_a$. Our result can be used to further reduce the conservatism of the analysis.*

The reader may be concerned that employing Theorem 1 requires performing a parameter-sweep for θ. Remarkably, the condition of Theorem 1 can be checked rigorously without this parameter-sweep. To this end, two different approaches will be presented. One approach, based on Corollary 3 and the work of [1], shows that the delay margin $\bar\tau_B^*$ provided by Theorem 1 (or Corollary 3) can be *explicitly calculated without incurring any additional conservatism in the single delay case.*

Theorem 4 *Suppose that the system (1) is asymptotically stable for all $\tau \in [0, \bar{\tau}_a]$, where $\bar{\tau}_a > 0$. Then the delay margin provided by Theorem 1 (or Corollary 3) is given by*

$$\bar{\tau}_B^* = \frac{\bar{\tau}_a}{\alpha_m} + \frac{1}{\lambda^+_{\max}(-(A_0 \oplus A_0)^{-1}(A_1 \oplus A_1))}, \quad (5)$$

where $A_0 := \begin{bmatrix} \frac{\bar{\tau}_a}{\alpha_m} A_s & C_s \\ \frac{\bar{\tau}_a}{\alpha_m} B_s & A_P \end{bmatrix}$ *and* $A_1 := \begin{bmatrix} A_s & 0 \\ B_s & 0 \end{bmatrix}$.

The other approach is to solve a (finite) set of LMIs. Some conservatism may be introduced, but in the following section we will see that this simplified criterion still can provide a very tight lower bound of the delay margin for system (1).

Theorem 5 *The system (1) is asymptotically stable for any constant time-delay $\tau \in [0, \bar{\tau}]$, if there exist matrices $X_0 > 0$, X_0, $X_1 \in \Re^{n \times n}$, $X_{22} > 0$, $X_{22} \in \Re^{n_P \times n_P}$ and $X_{12} \in \Re^{n \times n_P}$ such that*

$$\Pi(0) < 0, \ \Pi(\bar{\tau}) < 0$$

and

$$\begin{bmatrix} X_0 + \bar{\tau} X_1 & \bar{\tau} X_{12} \\ \bar{\tau} X_{12}^T & \bar{\tau} X_{22} \end{bmatrix} > 0$$

where

$$\Pi(\theta) := \begin{bmatrix} \Pi_{11}(\theta) & \Pi_{12}(\theta) \\ * & \Pi_{22}(\theta) \end{bmatrix},$$

$\Pi_{11}(\theta) := (X_0 + \theta X_1) A_s + X_{12} B_s + A_s^T (X_0 + \theta X_1) + B_s^T X_{12}^T$, $\Pi_{12}(\theta) := (X_0 + \theta X_1) C_s + X_{12} A_P + \theta A_s^T X_{12} + B_s^T X_{22}$, *and* $\Pi_{22}(\theta) := \theta X_{12}^T C_s + \theta C_s^T X_{12} + X_{22} A_P + A_P^T X_{22}$.

Remark 3 *The formula (5) only applies to the single delay case, and to use it, one usually has to use other methods (perhaps Theorem 5) to obtain $\bar{\tau}_a$. Although it introduces some conservatism, the above Theorem, does not need another test to obtain $\bar{\tau}_a$. More importantly, the LMI-based approach can be easily extended to linear systems with multiple delays and/or dynamical uncertainties.*

4 Numerical Examples

We now use the $5th$ order Padé approximation for several numerical examples. Since from Theorem 3, the d.o.c. of Theorem 1 is known to be no more than 0.361%, here we only examine the conservatism of the simplified condition, Theorem 5. We compare the results from Theorem 5 with those from similar criteria published elsewhere [10, 7, 8, 12] including some previous results by the authors [16].

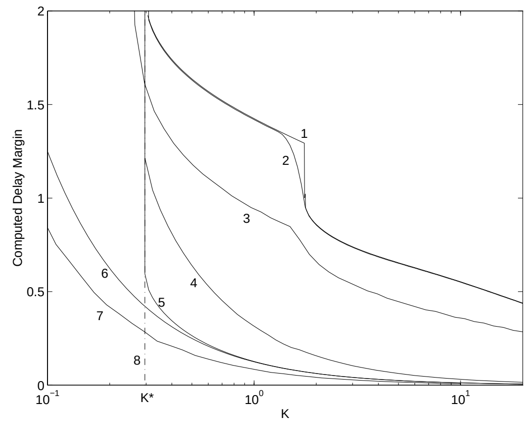

Figure 2: Calculated delay margin vs. K. (1) Actual value from Nyquist Criterion. (2) Theorem 5. (3) Result of [16] using both a filter and a shifted disk. (4) Result of [16] using a shifted disk. (5) Result of [10]. (6) Result of [7]. (7) Result of [8]. (8) Delay-independent result [12].

Example #1: Chatter Dynamics.

Consider the following system motivated by the dynamics of machining chatter with the matrices A and A_d given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(10.0 + K) & 10.0 & 0 & 0 \\ 5.0 & -15.0 & 0 & -.25 \end{bmatrix}$$

$$A_d = [\ 0 \quad 0 \quad K \quad 0\ ]^T [\ 1 \quad 0 \quad 0 \quad 0\ ].$$

The generalized Nyquist criterion [4] was used to calculate the actual delay margin. The delay margins calculated with the results of [10, 7, 8, 12] and [16] are shown in Figure 2 as a function of K. We can see that Theorem 5 provides a delay margin very close to the actual value obtained from the Nyquist Criterion.

Example #2 [10].

Consider the system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

For this example, the results of [8], [7], [10], and [16] provide the delay margins of 0.956, 0.9984, 4.3588, and 5.542, respectively. The generalized MIMO Nyquist Criterion [13, 4] provides the actual delay margin of 6.172. Using Theorem 5, the delay margin obtained is 6.150, which has a d.o.c. of only 0.36%. This delay margin coincides with the value calculated by Theorem 4, and hence in this case using the affine basis function in Theorem 5 does not introduce additional conservatism.

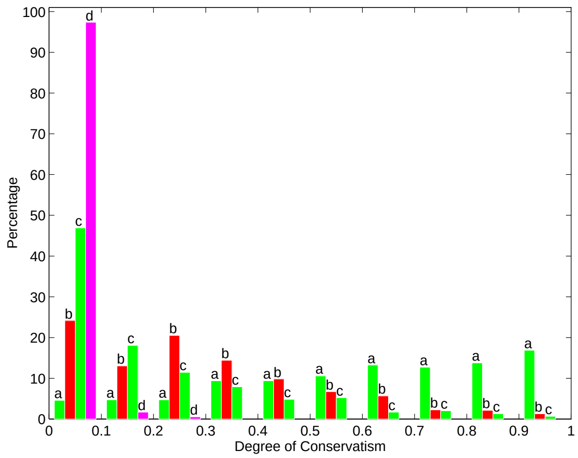

Figure 3: Performance of several criteria. (a) Result of [7]. (b) Result of [10]. (c) Result of [16]. (d) Theorem 5.

Example #3: Statistical Performance.

Finally, we compare the statistical performance of our result with that of [7], [10], and [16] by examining 1000 randomly-generated 2nd order systems[1]. The computed delay margins are compared with the actual values from the MIMO Nyquist Criterion and their distribution is shown in Figure 3. We find that for 97.3% of these systems, our new result gives the d.o.c. less than 10%. We note that with the next best performing criterion of [16], less than 50% of the cases have d.o.c. below 10%. The average d.o.c. for Theorem 5 is 1.52%.

5 Conclusions

The stability of linear time-delay systems can be analyzed rigorously and accurately via covering the delay element value set with a parameter-dependent Padé approximation. We presented a simple sufficient delay dependent stability condition for the time-delay system. Then we demonstrated that the degree of conservatism of this condition is no more than an *a priori* upper bound and this bound can be reduced by choosing a higher order approximation. Furthermore, the delay margin of this condition can be computed explicitly in the case of a single delay. This condition was also reduced to finite-dimensional LMIs with small additional conservatism. It should be noted that this result can be extended to the case of multiple delays and/or dynamical uncertainties.

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[1]For each test case, $A + A_d$ is Hurwitz and the system is delay-dependent.

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