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A COOPERATIVE EGALITARIAN PEER-TO-PEER STRATEGY FOR REFUELING SATELLITES IN CIRCULAR CONSTELLATIONS

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We address the problem of peer-to-peer (P2P) refueling of satellites in a circular constellation. In particular, we propose a Cooperative Egalitarian P2P (CE-P2P) strategy that combines the ideas of Cooperative and Egalitarian P2P refueling strategies introduced in our previous work. During a CE-P2P maneuver, a fuel-sufficient satellite and a fuel-deficient satellite engage in a cooperative rendezvous, exchange fuel, and then return to any available orbital slots left vacant by other active satellites. We propose a methodology, based on a network flow formulation, to determine the CE-P2P maneuvers that use the minimum amount of fuel during the ensuing orbital transfers. Since the methodology may yield sub-optimal solutions, we provide estimates of sub-optimality of these solutions. Finally, and with the help of numerical examples, we compare the CE-P2P, E-P2P and C-P2P alternatives, and demonstrate the benefits of CE-P2P maneuvers in terms of reducing the overall fuel expenditure.

INTRODUCTION

The traditional practice in the space industry has been the development of large and complex monolithic spacecraft. The result of this philosophy is high launching and maintenance costs. In recent times, the need for several small satellites performing the equivalent job of a larger monolithic spacecraft has been recognized. This new paradigm provides a means for reducing the overall cost of space operations, while adding flexibility to space-based missions. The areas of formation flight, satellite clustering,[1,2,3] and the more recently proposed fractionated spacecraft architecture,[4] have been receiving significant attention in this context. We are interested in the problem of on-orbit servicing of *multiple* spacecraft. As a first step in this direction, we consider first the simple case of a constellation comprised of several satellites in a circular orbit. Even for this seemingly simple problem, devising optimal servicing strategies is far from trivial, as it requires the solution of a large-scale optimization problem.

Traditionally, the practice in space industry has been the replacement of a spacecraft after its design lifetime. Still, there have been instances when on-orbit servicing (OOS) has proven to be beneficial. The first instances of OOS can be traced to the servicing missions for the SkyLab Space Station in 1970s. OOS missions were also been undertaken for the Solar Maximum Mission (SMM) and the Russian Space Station. The most visible instance of an OOS mission was the repair of the Hubble Space Telescope (HST).[5,6,7,8] All of the above involve servicing of a single spacecraft. The problem of servicing several spacecraft has only recently been looked at. The Orbital Express program of DARPA[9] considered the development of an architecture that will allow servicing operations of multiple spacecraft. It involves the launching of small propellant and other units to a low-Earth orbit, from where they can be transferred to a client spacecraft by a service vehicle.

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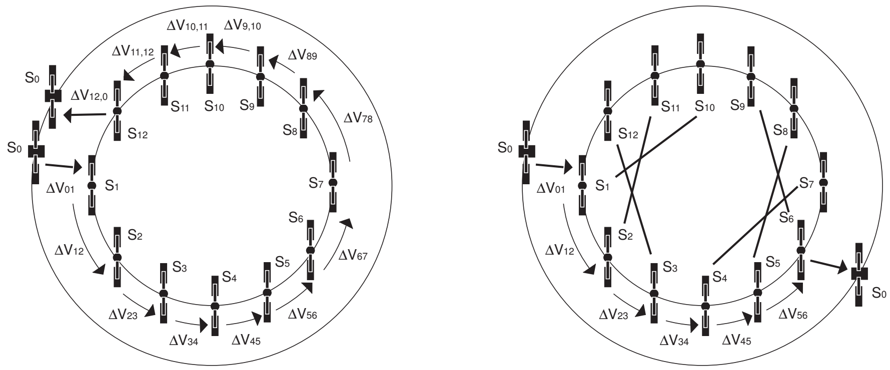


(a) Single Service Vehicle Strategy

(b) Mixed strategy

Figure 1. Refueling Strategies.

Waltz defines OOS as work done in space by man or machine or by both. He classifies the objectives of OOS into three broad categories: assembly, maintenance, and servicing.[5] Reynerson[10] introduced the notion of the cost for on-orbit servicing as one for which the benefits of OOS outweigh the associated cost. A recent customer-centric approach to studying OOS classifies the objectives of OOS into three functions, namely life extension, upgrade, and modification.[6,7]

Replenishment of consumables is one aspect of OOS. Satellites need a regular fuel-budget for stationkeeping. Providing fuel-deficient satellites with propellant extends their lifetime, enables extraordinary mission flexibility by allowing for frequent orbital maneuvering, and also reduces space debris. The potential profitability of refueling relatively lightweight geostationary communication satellites with long lifetimes has been emphasized in Reference 11. An account of technical and economic feasibility of on-orbit satellite servicing can also be found in Reference 12. Saleh et al.[7] provide numerical examples that point out the promise of refueling for OOS operations. In particular, refueling presents little risk, but offers immense gains if it is performed at the end of the spacecraft lifetime.[7]

The conventional notion of refueling fuel-deficient satellites in a constellation is to have a refueling spacecraft visit the former one by one, and impart fuel to them.[13] Figure 1(a) depicts such a scenario, in which the service vehicle s_0 sequentially visits twelve satellites to deliver fuel to them. Recently, an alternative refueling strategy has been investigated by the authors. This is the so-called peer-to-peer (P2P) refueling strategy,[14,15,16,17] in which satellites distribute fuel amongst themselves in the absence of a single refueling service vehicle. This is achieved by having satellites with excess fuel sharing their resource (propellant) with those with either insufficient amount of fuel or those that are completely depleted of fuel. P2P often comes as a natural choice in distributing fuel in the constellation in a mixed refueling strategy.[15,16] In such a scenario, an external refueling spacecraft, either launched from Earth or coming from a different orbit, replenishes part (perhaps half) of the satellites in the constellation, before returning back to its original orbit. The satellites which receive fuel from the external refueling spacecraft distribute the fuel amongst other satellites in the constellation via P2P refueling. Figure 1(b) depicts such a scenario, in which the service vehicle s_0 refuels six satellites, which subsequently engage in P2P maneuvers with the remaining satellites. Numerical studies have shown that the mixed refueling strategy is a competitive alternative to the single-service vehicle refueling strategy and, in fact, outperforms the latter, as the number of satellites in the constellation increases and/or the time to refuel decreases.[15] By incorporating additional cost-reducing strategies such as the Coasting Time Allocation (CTA) strategy and Asynchronous P2P maneuvers (A-P2P), one can increase the fuel savings even more.[16]

An extension of the baseline P2P refueling, known as the Egalitarian P2P (E-P2P) refueling, allows the active satellites to return to any available orbital slots during their return trip. This strategy has been shown to further reduce the fuel expenditure during the overall refueling process.[18, 19, 20] Another extension of the P2P strategy, known as the Cooperative P2P (C-P2P) strategy, allows the satellites participating in a refueling transaction to engage in cooperative rendezvous. This strategy is particularly beneficial in reducing the fuel expenditure when the fuel-deficient satellites do not have enough fuel to be active.[21] Typically, the optimal set of maneuvers are obtained by solving a discrete optimization problem.[18, 19, 21]

In this paper we combine the ideas of Cooperative and Egalitarian P2P refueling to develop a strategy which we call Cooperative Egalitarian P2P (CE-P2P) refueling. During a CE-P2P maneuver, a fuel-sufficient and a fuel-deficient satellite rendezvous in any available orbital slot, exchange fuel, and then return to any available orbital slots left vacant by other *active* satellites taking part in refueling transactions. For the determination of the set of CE-P2P maneuvers that consume the minimum fuel during the refueling process, we need to solve a NP-hard discrete optimization problem, in which the cost associated with each decision variable is obtained by solving one or more time-fixed, minimum-fuel, non-cooperative or cooperative rendezvous orbital transfer problem. Assuming that the satellites use chemical propulsion, the optimal orbital transfers typically employ multiple-impulses, and require the solution of a non-linear programming problem (NLP).[22, 23, 24] Solving an NLP is computationally intensive and there can also be issues with the convergence to a local minimum rather than the global minimum. For our case, numerous rendezvous problems need to be solved just to set up the discrete optimization problem. In order to minimize these computations, we bypass the solution of NLPs by considering transfers with just two impulses. This simplification leads to sub-optimal solutions, which can be computed much faster. Our approach is justified because P2P refueling is a discrete optimization problem; even if the numerical values of the costs associated with the decision variables are not exact, the optimal matching between satellites for refueling most likely will not change.

In the next section, we discuss a network flow formulation to find the satellite pairs for the optimal CE-P2P refueling strategy. We outline the optimization problem that yields the CE-P2P maneuvers corresponding to the minimum total ΔV during all ensuing orbital transfers. Recognizing the potential sub-optimality of the solutions obtained by working with ΔV instead of the actual fuel expenditure, we also derive bounds on the optimal fuel expenditure during CE-P2P refueling. Finally, we demonstrate the benefits of a CE-P2P strategy with the help of numerical examples.

FORMULATION

In this section, we discuss in detail the mathematical formulation of the CE-P2P refueling strategy. We introduce the basic notations, discuss the representation of CE-P2P maneuvers and outline the optimization problem required to determine the optimal CE-P2P strategy.

Notation

Let us consider a circular constellation consisting of n satellites, distributed over n orbital slots in a circular orbit of radius R. Let the set of n satellites be given by $\mathcal{S} = \{s_i : i = 0, 1, 2, \ldots, n\}$, where s_0 represents a fictitious satellite, the purpose of which will become clear shortly. Let us consider a set of $n' \geq n$ slots in the circular orbit, given by the set $\Phi' = \{\phi_i \in [0, 2\pi) : i = 1, 2, \ldots, n', \ \phi_i \neq \phi_j\}$. Out of these n' slots, n are occupied by the satellites. Let the set of slots occupied by the satellites be denoted by Φ. Clearly, $\Phi \subseteq \Phi'$. For convenience, let $\mathcal{I} = \mathcal{J} == \{1, 2, \ldots, n\}$, and $\mathcal{J}' = \{1, 2, \ldots, n'\}$. Note that $\mathcal{J} \subseteq \mathcal{J}'$. We introduce a mapping $\sigma_t : \Phi' \mapsto \mathcal{S}$ that, at time $t \geq 0$, assigns to each orbital slot a satellite from $\mathcal{S}$. In particular, $\sigma_t(\phi_j) = s_i$ implies that the satellite s_i occupies the orbital slot ϕ_j at time t. If the slot ϕ_j is empty at time t, we write $\sigma_t(\phi_j) = s_0$.

The initial fuel content of satellite s_i will be denoted by f_i^- and the final fuel content be denoted by f_i^+. Also, $\underline{f}_i$ will denote the minimum amount of fuel for the satellite s_i to remain operational, and $\bar{f}_i$ will denote the maximum fuel capacity of the same satellite. *Fuel-sufficient* satellites are those that have at least the required amount of fuel; the remaining satellites are *fuel-deficient*. Let

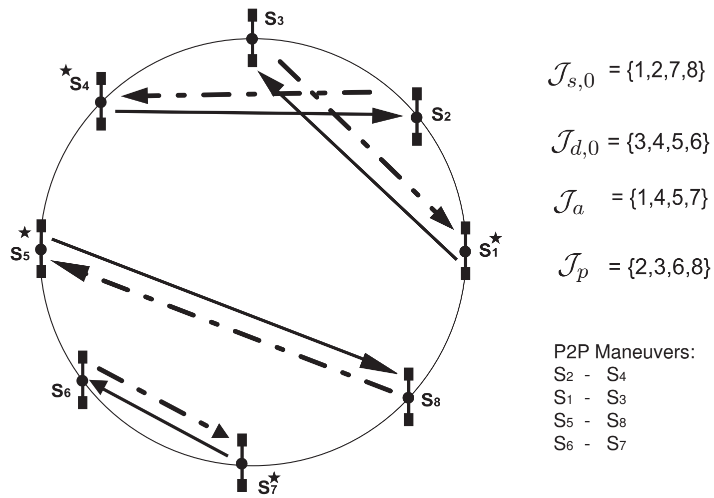


Figure 2. Notations explanation for P2P refueling.

$\mathcal{I}_{s,0}$ denote the set comprised of indices of the fuel-sufficient satellites, and let $\mathcal{I}_{d,0}$ denote the set having as elements the indices of the fuel-deficient ones. The objective of P2P refueling is therefore to achieve $f_i^+ \geq \underline{f}_i$ for all $i \in \{1, 2, \ldots, n\}$ by expending the minimum amount of fuel during the ensuing orbital transfers. For convenience, let $\mathcal{J}_{s,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{s,t}\}$ denote the index set of orbital slots occupied by fuel-sufficient satellites at time t, and let $\mathcal{J}_{d,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{d,t}\}$ denote the index set of orbital slots occupied by fuel-deficient satellites at time t. Also, let $\mathcal{J}_a$ denote the index set of orbital slots occupied by the active satellites before any orbital maneuver commences, and let $\mathcal{J}_c$ denote the set of slots where rendezvous takes place for the various refueling transactions. Then, the set of indices of the orbital slots of the passive satellites is given by $\mathcal{J} \cap \mathcal{J}_c$. Finally, we denote the set of indices of return slots by $\mathcal{J}_r$. Note that the available return slots are the same as the slots initially occupied by the active satellites. We therefore have $\mathcal{J}_r = \mathcal{J}_a$. Figure 2 illustrates these concepts. For the situation depicted in Figure 2, we assume $\mathcal{J} = \mathcal{J}'$ and $\sigma_0(\phi_i) = s_i$. Also, satellites s_1, s_2, s_7 and s_8 are the fuel-sufficient satellites and the remaining ones are the fuel-deficient satellites. The active satellites are marked with '$\star$', the forward trips are marked by a solid arrow, while the return trips are marked by a dashed arrow.

Let us consider a CE-P2P maneuver between two satellites $s_\mu = \sigma_0(\phi_{i_1})$ and $s_\nu = \sigma_0(\phi_{i_2})$, occupying the orbital slots ϕ_{i_1} and ϕ_{i_2} respectively, where $i_1, i_2 \in \mathcal{J}$. Without loss of generality, assume s_μ to be the fuel-sufficient satellite and s_ν to be the fuel-deficient satellite, that is, $i_1 \in$

$\mathcal{J}_{s,0}$ and $i_2 \in \mathcal{J}_{d,0}$. Let these satellites engage in a rendezvous at the orbital slot ϕ_j, where $j \in \mathcal{J}_c$. After the refueling transaction, the satellites s_μ and s_ν return to the orbital slots ϕ_{k_1} and ϕ_{k_2} respectively, where $k_1, k_2 \in \mathcal{J}_r$. Given $i_1, i_2 \in \mathcal{J}_a$, $j \in \mathcal{J}_c$, and $k_1, k_2 \in \mathcal{J}_r$, we can represent an assignment for a CE-P2P maneuver by (i_1, i_2, j, k_1, k_2). An assignment (i_1, i_2, j, k_1, k_2) is feasible if the satellites s_μ and s_ν engaging in the CE-P2P refueling transaction end up being fuel-sufficient after the maneuver is complete. Let $\mathcal{P}$ denote the set of all feasible CE-P2P assignments in the constellation. Let $\mathcal{M}_{ce} \subseteq \mathcal{P}$ denote the set of $|\mathcal{J}_{d,0}|$ feasible CE-P2P maneuvers such that all fuel-deficient satellites are included in the refueling transactions. The cost of a CE-P2P solution is the total fuel expenditure incurred during all the orbital transfers taking place. Let p^μ_{ij} denote the fuel used by satellite s_μ during its transfer from the orbital slot ϕ_i to the slot ϕ_j. Therefore, the cost of the CE-P2P solution is given by

$$\mathcal{C}(\mathcal{M}_{ce}) = \sum_{(i_1,i_2,j,k_1,k_2)\in\mathcal{M}_{ce}} p^\mu_{i_1 j} + p^\nu_{i_2 j} + p^\mu_{j k_1} + p^\nu_{j k_2}. \tag{1}$$

Also note that, if $i_1 = j = k_1$ or $i_1 = j = k_1$, for a CE-P2P assignment $(i_1, i_2, j, k_1, k_2) \in \mathcal{P}$ then the assignment represents an E-P2P maneuver (non-cooperative). Let $\mathcal{P}_e$ denote the set of feasible E-P2P maneuvers in the constellation. Clearly, $\mathcal{P}_e \subseteq \mathcal{P}$. Similarly, if $i_1 = k_1$ and $i_2 = k_2$, for the CE-P2P assignment $(i_1, i_2, j, k_1, k_2) \in \mathcal{P}$ then the assignment represents a C-P2P maneuver (non-Egalitarian). Let $\mathcal{P}_c$ denote the set of feasible C-P2P maneuvers in the constellation. Clearly, $\mathcal{P}_c \subseteq \mathcal{P}$. Furthermore, let $\mathcal{M}^*_{ce}$ denote the optimal set of assignments that minimizes the fuel expenditure during CE-P2P refueling. We therefore have

$$\mathcal{C}(\mathcal{M}^*_{ce}) = \min_{\mathcal{M}_{ce}\subseteq\mathcal{P}} \mathcal{C}(\mathcal{M}_{ce}). \tag{2}$$

Similarly, let $\mathcal{M}^*_c \subseteq \mathcal{P}_c$ and $\mathcal{M}^*_e \subseteq \mathcal{P}_e$ denote the optimal set of assignments for C-P2P and E-P2P refueling. We therefore have,

$$\mathcal{C}(\mathcal{M}^*_c) = \min_{\mathcal{M}_{ce}\subseteq\mathcal{P}_c} \mathcal{C}(\mathcal{M}_{ce}), \tag{3}$$

and

$$\mathcal{C}(\mathcal{M}^*_e) = \min_{\mathcal{M}_{ce}\subseteq\mathcal{P}_e} \mathcal{C}(\mathcal{M}_{ce}). \tag{4}$$

CE-P2P Maneuver Costs

Let us consider a CE-P2P maneuver (i_1, i_2, j, k_1, k_2). During the first phase of the maneuver, the two satellites $s_\mu = \sigma_0(\phi_{i_1})$ and $s_\nu = \sigma_0(\phi_{i_2})$ transfer to the orbital slot ϕ_j. The fuel consumed by the active satellite s_μ to transfer from the orbital slot ϕ_{i_1} to the orbital slot ϕ_j is given by

$$p^\mu_{i_1 j} = \left(m_{\mathrm{s}\mu} + f^-_\mu\right)\left(1 - e^{-\frac{\Delta V_{i_1 j}}{c_{0\mu}}}\right), \tag{5}$$

where $m_{\mathrm{sp}\mu}$ denotes the mass of the permanent structure of the satellite s_μ, $c_{0\mu}$ denotes the characteristic constant for the satellite s_μ, and $\Delta V_{i_1 j}$ denotes the optimal velocity change required for the transfer from the slot ϕ_{i_1} to ϕ_j. The characteristic constant is defined by $c_{0\mu} = g_0 I_{\mathrm{sp}\mu}$, where g_0 denote the gravitational acceleration on the surface of the earth, and $I_{\mathrm{sp}\mu}$ denote the specific thrust of the engine of the satellite s_μ. Similarly, the fuel expenditure for satellite s_ν to transfer from the orbital slot ϕ_{i_2} to the orbital slot ϕ_j is given by:

$$p^\nu_{i_2 j} = \left(m_{\mathrm{s}\nu} + f^-_\nu\right)\left(1 - e^{-\frac{\Delta V_{i_2 j}}{c_{0\nu}}}\right). \tag{6}$$

The fuel content of satellite s_μ after its forward trip (but before the fuel exchange takes place) is $f_\mu^- - p_{i_1j}^\mu$, while that of satellite s_ν is $f_\nu^- - p_{i_2j}^\nu$. The amount of fuel that s_μ delivers to s_ν is g_μ^ν. Hence, the fuel content of satellite s_μ just after the fuel exchange takes place is $f_\mu^- - p_{i_1j}^\mu - g_\mu^\nu$, while that of satellite s_ν is $f_\nu^- - p_{i_2j}^\nu + g_\mu^\nu$. After the fuel exchange, and in the second phase of the P2P maneuver, satellites s_μ and s_ν transfer to the orbital slots ϕ_{k_1} and ϕ_{k_2}, respectively. During the return trip, the fuel expenditure of satellite s_μ to transfer from slot ϕ_j to slot ϕ_{k_1} is given by

$$p_{jk_1}^\mu = \left(m_{s\mu} + f_\mu^- - p_{i_1j}^\mu - g_\mu^\nu\right)\left(1 - e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}\right), \tag{7}$$

while that of satellite s_ν to transfer from slot ϕ_j to slot ϕ_{k_1} is given by

$$p_{jk_2}^\nu = \left(m_{s\nu} + f_\nu^- - p_{i_2j}^\nu + g_\mu^\nu\right)\left(1 - e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}}\right). \tag{8}$$

The amount of fuel exchanged affects the return trip fuel expenditure. Following an analysis similar to the one in Reference 21, it can be shown that the fuel expenditure during the CE-P2P is minimized if the amount of fuel exchanged by the satellites is given by

$$g_\mu^\nu = \begin{cases} g_\mu^\nu|_\ell, & e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} < e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}, \\ g_\mu^\nu|_u, & e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} > e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}, \end{cases} \tag{9}$$

where,

$$g_\mu^\nu|_\ell = \left(m_{s\nu} + \underline{f}_\nu\right) e^{\frac{\Delta V_{jk_2}}{c_{0\nu}}} - \left(m_{s\nu} + f_\nu^- - p_{i_2j}^\nu\right), \tag{10}$$

and

$$g_\mu^\nu|_u = \left(m_{s\mu} + f_\mu^- - p_{i_1j}^\mu\right) - \left(m_{s\mu} + \underline{f}_\mu\right) e^{\frac{\Delta V_{jk_1}}{c_{0\mu}}}. \tag{11}$$

Also, if $e^{-\frac{\Delta V_{jk_2}}{c_{0\nu}}} = e^{-\frac{\Delta V_{jk_1}}{c_{0\mu}}}$, g_μ^ν can assume any value in the interval $g_\mu^\nu|_\ell \leq g_\mu^\nu \leq g_\mu^\nu|_u$. For the maneuver to be feasible we must have $g_\mu^\nu|_\ell \leq g_\mu^\nu|_u$, that is, there exists a fuel exchange that would result in both satellites to be fuel-sufficient at the end of the maneuver. Furthermore, for feasibility of the CE-P2P maneuver, we must also have $p_{i_1j}^\mu < f_\mu^-$ and $p_{i_2j}^\nu < f_\nu^-$, that is, both satellites must have enough fuel to complete their forward trips.

Constellation Digraph

We can represent a CE-P2P maneuver using a directed graph. To this end, let us define a constellation graph $\mathcal{G}$ consisting of three partitions $\mathcal{J}_a$, $\mathcal{J}_c$ and $\mathcal{J}_r$. The nodes of $\mathcal{G}$ are given by $\mathcal{J}_a \cup \mathcal{J}_c \cup \mathcal{J}_r$. However, we do not know a priori which satellites are active, which are passive, and which slots are used for cooperative rendezvous. That is, we do not know the sets $\mathcal{J}_a$, $\mathcal{J}_c$ and $\mathcal{J}_r$ a priori. We therefore let $\mathcal{J}_a = \mathcal{J}_r = \mathcal{J}$ and $\mathcal{J}_c = \mathcal{J}'$. We will denote an orbital transfer using a *directed* edge, with the direction of edge signifying the direction of the orbital transfer. Let an edge (i, j), where $i \in \mathcal{J}_a$ and $j \in \mathcal{J}_c$, denote a forward trip from the slot ϕ_i to the slot ϕ_j, and let the associated cost for this transfer be denoted by c_{ij}. Let an edge (j, k), where $j \in \mathcal{J}_c$ and $k \in \mathcal{J}_r$, denote a return trip from the slot ϕ_j to ϕ_k, and let the associated cost for this transfer be denoted by c_{jk}. A set of edges (i_1, j), (i_2, j), (j, k_1) and (j, k_2) represents a CE-P2P maneuver. Figure 3 depicts a constellation digraph, and four directed edges corresponding to a CE-P2P maneuver. The two edges between the partitions $\mathcal{J}_a$ and $\mathcal{J}_c$ correspond to forward trips of the active satellites, while the edges between $\mathcal{J}_c$ and $\mathcal{J}_a$ correspond to their return trips. Note that any edge (i, j) having $\phi_i = \phi_j$ does not represent a physical transfer, since it would mean that the active satellite occupies the same orbital slot during its forward/return trip. Naturally, the cost associated with such an edge is zero. Hence, if we have $\phi_{i_1} = \phi_j$ or $\phi_{i_2} = \phi_j$, then the maneuver is actually non-cooperative, because one of the satellites involved in the refueling transaction remains in its orbital slot throughout the maneuver. In other words, our representation of a CE-P2P maneuver allows an E-P2P maneuver to be treated as a special case of a CE-P2P maneuver in which one forward edge and one return edge does not actually represent a maneuver, and each of these edges has a zero cost.

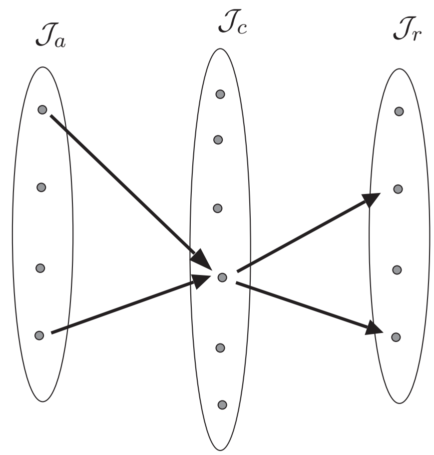


Figure 3. Directed constellation graph.

Ideally, the cost of the edges in the graph $\mathcal{G}$ has to be the fuel expenditure during the orbital transfers. However, the calculation of the fuel expenditure is dependent on the mass of the satellite performing the orbital transfer. Since we do not know a priori which satellites are going to pair up for the refueling transactions, the return trip fuel expenditure cannot be uniquely determined for the return trip edges on the graph $\mathcal{G}$. Instead of the fuel expenditure, we can use the velocity change ΔV required for the corresponding orbital transfer because the ΔV can be uniquely determined for all edges. The minimization of ΔV would yield sub-optimal results since the true objective is to minimize fuel expenditure. However, it was observed in our numerical simulations that solutions are only marginally sub-optimal when we minimize ΔV. Furthermore, in order to avoid solutions in which a fuel-deficient satellite does not have enough fuel to complete the desired rendezvous, we only allow those forward edges (i, j) in the graph $\mathcal{G}$ for which we have $p_{ij}^{\mu} < f_{\mu}^{-}$, where $s_\mu = \sigma_0(\phi_i)$ and $\phi_j \in \Phi'$.

A Network Flow Formulation

We now present a network flow formulation for the solution of CE-P2P problem. We set up a constellation network $\mathcal{G}_n$ using the constellation digraph $\mathcal{G}$. To this end, we add a source node s and a sink node t to the constellation digraph $\mathcal{G}$. For all $i \in \mathcal{J}_a$, we also add an arc (s, i) with associated cost $c_{si} = 0$. We denote the set of these arcs by $\mathcal{E}_s$. Similarly, for all $k \in \mathcal{J}_r$, we add an arc (k, t) with associated cost $c_{kt} = 0$. We denote the set of these arcs by $\mathcal{E}_t$. Let us now consider two $s \rightarrow t$ flows in the network $\mathcal{G}_n$, that pass through the same node $j \in \mathcal{J}_c$. A pair of such flows

$s \to i_1 \to j \to k_1 \to t$ and $s \to i_2 \to j \to k_2 \to t$ represent a CE-P2P maneuver (i_1, i_2, j, k_1, k_2). The total cost of the flows equal the total ΔV required for all the orbital transfers during a CE-P2P maneuver. We seek $|\mathcal{J}_{d,0}|$ pairs of flows in the constellation network with minimum total cost, such that all flows also pass through all the fuel-deficient satellites in the constellation. Note that each assignment (i_1, i_2, j, k_1, k_2) in a CE-P2P solution $\mathcal{M}_{ce}$ corresponds to a set of edges (s, i_1), (s, i_2), (i_1, j), (i_2, j), (j, k_1), (j, k_2), (k_1, t), and (k_2, t) in $\mathcal{G}_n$. The total cost of these edges is therefore the total ΔV required for all the orbital transfers corresponding to the assignment (i_1, i_2, j, k_1, k_2). Let the set of edges in the network corresponding to all assignments in the CE-P2P solution $\mathcal{M}_{ce}$ be denoted by $\mathcal{M}$. Also, let the set of slots where the cooperative rendezvous takes place corresponding to the solution $\mathcal{M}_{ce}$ be given by $\mathcal{Y}$. Let us now introduce the following decision variables for our optimization problem. Corresponding to each edge (i, j), we introduce a flow variable x_{ij} defined by

$$x_{ij} = \begin{cases} 1, & \text{if } x_{ij} \in \mathcal{M}, \\ 0, & \text{otherwise}. \end{cases} \tag{12}$$

Also, corresponding to each slot for cooperative rendezvous, let us introduce the decision variables y_j, as follows

$$y_j = \begin{cases} 1, & \text{if } j \in \mathcal{Y}, \\ 0, & \text{otherwise}. \end{cases} \tag{13}$$

We need $|\mathcal{J}_{d,0}|$ CE-P2P maneuvers in order to refuel all fuel-deficient satellites. Hence, the total flow that goes out of the source is $2|\mathcal{J}_{d,0}|$ and the flow distributes itself into $|\mathcal{J}_{d,0}|$ fuel-sufficient satellites and $|\mathcal{J}_{d,0}|$ fuel-deficient satellites. Noting that $\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} = \mathcal{J}$, we have,

$$\sum_{i \in \mathcal{J}} x_{si} = 2|\mathcal{J}_{d,0}|, \tag{14}$$

and

$$\sum_{i \in \mathcal{J}_{s,0}} x_{si} = |\mathcal{J}_{d,0}|. \tag{15}$$

An amount of flow equal to the flow originating from the source must be collected at the sink node, that is,

$$\sum_{k \in \mathcal{J}} x_{kt} = 2|\mathcal{J}_{d,0}|. \tag{16}$$

The flow balance equations at the different nodes yield the following constraints

$$x_{si} = \sum_{j \in \mathcal{J}_c} x_{ij}, \text{ for all } i \in \mathcal{J}_a, \tag{17}$$

$$x_{kt} = \sum_{j \in \mathcal{J}_c} x_{jk}, \text{ for all } i \in \mathcal{J}_r, \tag{18}$$

and

$$\sum_{i \in \mathcal{J}_a} x_{ij} = \sum_{k \in \mathcal{J}_r} x_{jk}, \text{ for all } j \in \mathcal{J}_c. \tag{19}$$

The orbital slots available for return are exactly the orbital slots for the active satellites. Hence, we have,

$$x_{si} = x_{it}, \text{ for all } i \in \mathcal{J}. \tag{20}$$

The total number of slots for rendezvous is the total number of CE-P2P maneuvers, which in turn equals the number of fuel-deficient satellites in the constellation. We therefore have,

$$\sum_{j\in\mathcal{J}_c} y_j = |\mathcal{J}_{d,0}|. \tag{21}$$

If a slot is selected for cooperative rendezvous, two satellites must transfer to that location (unless it is a non-cooperative maneuver). Hence, we have the following constraint:

$$\sum_{i\in\mathcal{J}} x_{ij} = 2y_j, \text{ for all } j \in \mathcal{J}_c. \tag{22}$$

The two satellites transferring to the slot ϕ_j must be a fuel-sufficient and a fuel-deficient satellite. In other words, we have at most one fuel-sufficient satellite ending up in the slot ϕ_j, that is,

$$\sum_{i\in\mathcal{J}_{s,0}} x_{ij} \leq 1, \text{ for all } j \in \mathcal{J}_c. \tag{23}$$

Given the decision variables defined in (12) and (13), and the set of constraints (14)-(23), we are required to minimize the total ΔV for the CE-P2P maneuvers, that is,

$$\text{(CE-P2P) } : \min \sum_{(i,j)\in\mathcal{E}_n} c_{ij}x_{ij}. \tag{24}$$

BOUNDS ON THE OPTIMAL FUEL EXPENDITURE

The set of CE-P2P maneuvers obtained by solving the optimization problem (CE-P2P) corresponds to the minimum total ΔV required for the orbital transfers taking place during refueling. Let this solution be denoted by $\mathcal{M}_{ce}^{\mathrm{H}}$. Our true objective is to minimize fuel expenditure, and hence the solution $\mathcal{M}_{ce}^{\mathrm{H}}$ is potentially sub-optimal. In this section, we provide a measure of the sub-optimality of the solution $\mathcal{M}_{ce}^{\mathrm{H}}$ by deriving bounds on the optimal fuel expenditure for CE-P2P refueling. In particular, we show that a conservative lower bound on the total fuel expenditure $\mathcal{C}\left(\mathcal{M}_{ce}^{*}\right)$ can be obtained by solving a bipartite assignment problem.

To this end, let us consider the undirected bipartite graph $\mathcal{G}_\ell = \{\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}, \mathcal{E}_\ell\}$. We will represent a P2P maneuver between two satellites by an *undirected* edge in the graph $\mathcal{G}_\ell$. In particular, we say that there exists an (undirected) edge $\langle i_1, i_2\rangle$ between two nodes $i_1 \in \mathcal{J}_{s,0}$ and $i_2 \in \mathcal{J}_{d,0}$ if and only if the satellites s_μ and s_ν, occupying initially the orbital slots ϕ_{i_1} and ϕ_{i_2}, respectively, can engage in a feasible CE-P2P maneuver. By this, we mean the satellites can engage in a rendezvous at a slot ϕ_j, where $j \in \mathcal{J}'$, and return respectively to the orbital slots ϕ_{k_1} and ϕ_{k_2}. The set of all such edges in the graph is given by $\mathcal{E}_\ell = \{\langle i_1, i_2\rangle \,:\, \text{there exists } j \in \mathcal{J}'$, and $\phi_{k_1}, \phi_{k_2} \in \mathcal{J}$ such that either $(i_1, i_2, j, k_1, k_2) \in \mathcal{P}\}$. To each edge $\langle i_1, i_2\rangle$, we associate a cost $c^{\ell}_{i_1 i_2}$ that takes into account the fuel expenditure during the forward and return trips of the satellites, among all possible slots for cooperative rendezvous and return positions. The minimum fuel consumption for all possible return slots corresponding to the cooperative rendezvous slot ϕ_j, where $j \in \mathcal{J}'$, is given by

$$\left[p^{\mu}_{i_1 j} + p^{\nu}_{i_2 j} + \min_{k_1,k_2\in\mathcal{J},k_1\neq k_2} \left(p^{\mu}_{jk_1} + p^{\mu}_{jk_2}\right)\right]. \tag{25}$$

Therefore, the cost of the edge $\langle i_1, i_2 \rangle \in \mathcal{E}_\ell$ is taken as

$$c^{\ell}_{i_1 i_2} = \min_{j \in \mathcal{J}_c} \left[p^{\mu}_{i_1 j} + p^{\nu}_{i_2 j} + \min_{k_1, k_2 \in \mathcal{J}, k_1 \neq k_2} \left(p^{\mu}_{j k_1} + p^{\mu}_{j k_2} \right) \right]. \tag{26}$$

It represents the minimum possible fuel expenditure if the satellites s_μ and s_ν engage in a CE-P2P maneuver.

We are interested in a subset $\mathcal{M}_\ell$ of $\mathcal{E}_\ell$ with $|\mathcal{J}_{d,0}|$ edges, such that no two edges share the same node. This ensures that a satellite can be assigned to only one CE-P2P maneuver. Let us associate with each edge $\langle i, j \rangle \in \mathcal{E}_\ell$ the binary variable x_{ij} given by

$$x_{ij} = \begin{cases} 1, & \text{if } x_{ij} \in \mathcal{M}_\ell, \\ 0, & \text{otherwise}. \end{cases} \tag{27}$$

We now define the following optimization problem on $\mathcal{G}_\ell$*:

$$\text{(CE-P2P-LB): } \min \sum_{\langle i,j \rangle \in \mathcal{E}_\ell} c^{\ell}_{ij} x_{ij}, \tag{28}$$

subject to

$$\sum_{j : \langle i,j \rangle \in \mathcal{E}_\ell} x_{ij} \leq 1 \text{ for all } i \in \mathcal{J}_{s,0}, \tag{29}$$

$$\sum_{i : \langle i,j \rangle \in \mathcal{E}_\ell} x_{ij} = 1 \text{ for all } j \in \mathcal{J}_{d,0}. \tag{30}$$

The constraint (29) implies that each fuel-sufficient satellite can be assigned to, at most, one fuel-deficient satellite, while the constraint (30) implies that each fuel-deficient satellite has to be assigned to a fuel-sufficient satellite. Let the optimal solution to the problem (CE-P2P-LB) be $\mathcal{M}^*_\ell$ and the optimal value of the objective given in (28) be denoted by $\mathcal{C}_{\text{LB}}$. We then have

$$\mathcal{C}_{\text{LB}} = \sum_{\langle i,j \rangle \in \mathcal{M}^*_\ell} c^{\ell}_{ij}. \tag{31}$$

We now state the following theorem.

Theorem 1. *The total fuel expenditure $\mathcal{C}(\mathcal{M}^*_{ce})$ corresponding to the optimal CE-P2P solution $\mathcal{M}^*_{ce}$ is bounded below by the optimal value $\mathcal{C}_{\text{LB}}$ of the objective function in the bipartite assignment problem (CE-P2P-LB). Moreover, $\mathcal{C}(\mathcal{M}^*_{ce})$ is bounded above by the optimal fuel expenditure $\mathcal{C}(\mathcal{M}^*_{e})$ obtained via E-P2P refueling or $\mathcal{C}(\mathcal{M}^*_{c})$ obtained via C-P2P refueling, whichever is smaller. Therefore, $\mathcal{C}_{\text{LB}} \leq \mathcal{C}(\mathcal{M}^*_{ce}) \leq \min\{\mathcal{C}(\mathcal{M}^*_{e}), \mathcal{C}(\mathcal{M}^*_{c})\}$.*

Proof. The optimal CE-P2P solution $\mathcal{M}^*_{ce}$ consists of $|\mathcal{J}_{d,0}|$ assignments. For an assignment given by $(i_1, i_2, j, k_1, k_2) \in \mathcal{M}^*_{ce}$, the satellites $s_\mu = \sigma_0(\phi_{i_1})$ and $s_\nu = \sigma_0(\phi_{i_2})$ represent the fuel-sufficient and fuel-deficient satellites respectively. Since $\mathcal{M}^*_{ce} \subseteq \mathcal{P}$, s_μ and s_ν can engage in a feasible CE-P2P maneuver, which implies that the edge $\langle i_1, i_2 \rangle$ exists in $\mathcal{G}_\ell$. We therefore define the mapping $\mathcal{Q} : \mathcal{P} \mapsto \mathcal{E}_\ell$ that gives an edge in $\mathcal{E}_\ell$ for every assignment in $\mathcal{P}$. For instance,

*CE-P2P-LB stands for CE-P2P - Lower Bound

$\mathcal{Q}(i_1, i_2, j, k_1, k_2) = \langle i_1, i_2 \rangle$. Note that the CE-P2P solution $\mathcal{M}^*_{ce}$ corresponds to $|\mathcal{J}_{d,0}|$ distinct fuel-sufficient and all $|\mathcal{J}_{d,0}|$ fuel-deficient satellites involved in refueling transactions (refer to (14) and (15)). Let us now consider the following assignment in $\mathcal{G}_\ell$: $x_{qr} = 1$ for all $\langle q, r \rangle \in \mathcal{Q}(\mathcal{M}^*)$ and 0 otherwise. For all the $|\mathcal{J}_{d,0}|$ fuel-sufficient satellites included in CE-P2P solution $\mathcal{M}^*_{ce}$, we have

$$\sum_{r:\langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} = 1,$$

whereas for the remaining $|\mathcal{J}_{s,0}| - |\mathcal{J}_{d,0}|$ fuel-sufficient satellites not included in any refueling transaction, we have

$$\sum_{r:\langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} = 0.$$

Combining the above two equations, we have

$$\sum_{r:\langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} \le 1 \text{ for all } q \in \mathcal{J}_{s,0}.$$

All the fuel-deficient satellites are included in the CE-P2P solution and each of them engages in a refueling transaction with a distinct fuel-sufficient satellite (refer to (14),(15), and (23)). We therefore have,

$$\sum_{q:\langle q,r\rangle \in \mathcal{E}_\ell} x_{qr} = 1 \text{ for all } r \in \mathcal{J}_{d,0}.$$

Hence, the optimal CE-P2P solution $\mathcal{M}^*_{ce}$ corresponds to a feasible solution $\mathcal{Q}(\mathcal{M}^*_{ce})$ for the optimization problem (CE-P2P-LB). Hence, we have

$$\sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}^*_{ce})} c^\ell_{qr} \ge \sum_{\langle q,r\rangle \in \mathcal{M}^*_\ell} c^\ell_{qr}. \tag{32}$$

Now, let us consider the fuel expenditure $\mathcal{C}(\mathcal{M}^*_{ce})$. We have

$$\begin{aligned} \mathcal{C}(\mathcal{M}^*_{ce}) &= \sum_{(i_1,i_2,j,k_1,k_2)\in\mathcal{M}^*_{ce}} p^\mu_{i_1 j} + p^\nu_{i_2 j} + \left(p^\mu_{jk_1} + p^\mu_{jk_2}\right) \\ &\ge \sum_{\{i_1,i_2,j\}:(i_1,i_2,j,k_1,k_2)\in\mathcal{M}^*_{ce}} \left[p^\mu_{i_1 j} + p^\nu_{i_2 j} + \min_{k_1,k_2\in\mathcal{J},k_1\neq k_2} \left(p^\mu_{jk_1} + p^\mu_{jk_2}\right)\right] \\ &\ge \sum_{\{i_1,i_2\}:(i_1,i_2,j,k_1,k_2)\in\mathcal{M}^*_{ce}} \left[\min_{j\in\mathcal{J}_c}\left(p^\mu_{i_1 j} + p^\nu_{i_2 j} + \min_{k_1,k_2\in\mathcal{J},k_1\neq k_2} \left(p^\mu_{jk_1} + p^\mu_{jk_2}\right)\right)\right]. \end{aligned} \tag{33}$$

Using (26), we have from (33),

$$\mathcal{C}(\mathcal{M}^*_{ce}) \ge \sum_{\langle q,r\rangle \in \mathcal{Q}(\mathcal{M}^*_{ce})} c^\ell_{qr}. \tag{34}$$

Finally, comparing Eq. (32) and Eq. (34), we have

$$\mathcal{C}(\mathcal{M}_{ce}) \ge \mathcal{C}_{\text{LB}}. \tag{35}$$

For the upper bound, recall that $\mathcal{P}_c \subseteq \mathcal{P}$ and $\mathcal{P}_e \subseteq \mathcal{P}$. Therefore, from the definition of $\mathcal{C}(\mathcal{M}^*_{ce})$, $\mathcal{C}(\mathcal{M}^*_c)$ and $\mathcal{C}(\mathcal{M}^*_e)$, given in (2)-(4), we have

$$\mathcal{C}(\mathcal{M}^*_{ce}) \leq \mathcal{C}(\mathcal{M}_c) \text{ and } \mathcal{C}(\mathcal{M}^*_{ce}) \leq \mathcal{C}(\mathcal{M}_e). \tag{36}$$

The inequalities (35) and (36) give the desired result. □

The fuel expenditure associated with the (CE-P2P) solution, obtained by solving the optimization problem (CE-P2P), is given by $\mathcal{C}(\mathcal{M}^{\rm H}_{ce})$. Since $\mathcal{M}^{\rm H}_{ce}$ might be a sub-optimal solution, we have $\mathcal{C}(\mathcal{M}^{\rm H}_{ce}) \geq \mathcal{C}(\mathcal{M}^*_{ce})$. Considering the bounds given by Theorem 1, we obtain an estimate of sub-optimality of these results. Specifically, we may define the maximum percentage of sub-optimality of $\mathcal{M}^{\rm H}_{ce}$ by the following expression

$$\eta = \frac{\mathcal{C}(\mathcal{M}^{\rm H}_{ce}) - \mathcal{C}_{\rm LB}}{\mathcal{C}_{\rm LB}} \times 100\%. \tag{37}$$

Note that because the solution of the CE-P2P-LB problem may correspond to an infeasible CE-P2P solution, η is a worst case (conservative) estimate of the suboptimality of $\mathcal{M}^{\rm H}_{ce}$. However, we can guarantee that the solution is no worse than η, but it could also be better. In fact, there are indeed cases in which the solution of the (CE-P2P-LB) does lead to a feasible solution. In such cases, the solution is globally optimal.

EXAMPLES

In this section we discuss a few numerical examples that show the benefit of a cooperative refueling strategy for different satellite constellations. These constellations vary in the number of satellites, the mass and fuel content of the satellites, and the constellation orbit. The details of these constellations are given in Table 1.

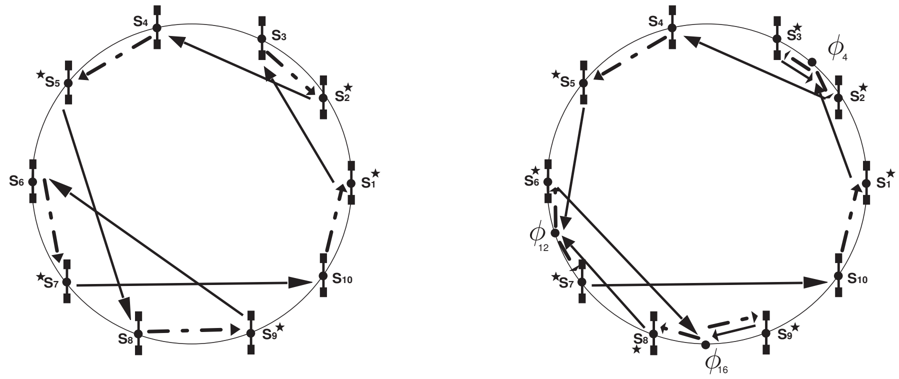
Example 1. *CE-P2P strategy for a constellation of* 10 *satellites.*

Let us consider the constellation C_1 given in Table 1. It consists of 10 satellites evenly distributed in a circular orbit. The initial fuel content of the satellites $s_1, s_2, \ldots, s_{10}$ are 30, 30, 6, 6, 6, 6, 6, 30, 30, 30 units respectively. The maximum allowed time for refueling is $T = 12$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 12$ units, while the maximum amount of fuel for each satellite is $\bar{f}_i = 30$ units. Each satellite has a permanent structure of $m_{{\rm s}i} = 70$ units, and a characteristic constant of $c_0 = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 2, 8, 9, 10\}$ and those of the fuel-deficient satellites are $\mathcal{I}_{d,0} = \{3, 4, 5, 6, 7\}$. Let Φ' be a set of 20 evenly distributed slots, out of which 10 are occupied by the satellites. We have, $\mathcal{J}' = \{1, 2, \ldots, 20\}$, and the satellites occupy the slots $\mathcal{J} = \{1, 3, \ldots 19\}$ respectively, that is, we have $s_i = \sigma_0(\phi_{2i-1})$ for all $i \in \{1, 2, \ldots, 10\}$. An E-P2P strategy for this constellation yields the following optimal assignments: $s_1 \rightarrow s_3 \rightarrow s_2$, $s_2 \rightarrow s_4 \rightarrow s_5$, $s_5 \rightarrow s_8 \rightarrow s_9$, $s_7 \rightarrow s_{10} \rightarrow s_1$, $s_9 \rightarrow s_6 \rightarrow s_7$, where the assignment $s_1 \rightarrow s_3 \rightarrow s_2$ implies that the satellite s_1 undergoes an orbital transfer to rendezvous with s_3, exchanges fuel, and then returns to the orbital slot originally occupied by the satellite s_2. Figure 4(a) depicts these E-P2P maneuvers. The fuel expenditure during the E-P2P refueling process is 19.11 units. This represents 10.62% of the total initial fuel in the constellation. Figure 4(a) shows the optimal assignments for the E-P2P case. A C-P2P strategy for this constellation yields a higher fuel expenditure than the E-P2P case. Let us now consider

Table 1. Sample Constellations.

Label	Description
C_1	10 satellites, Altitude $= 35,786$ Km, $T = 12$ f_i^-: $30, 30, 6, 6, 6, 6, 6, 30, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{\text{s}i} = 70$ for all satellites
C_2	16 satellites, Altitude $= 1,200$ Km, $T = 30$ f_i^-: $30, 30, 30, 30, 30, 30, 10, 10, 10, 10, 10, 10, 10, 10, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{\text{s}i} = 70$ for all satellites
C_3	16 satellites, Altitude $= 1,200$ Km, $T = 30$ f_i^-: $30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{\text{s}i} = 70$ for all satellites
C_4	16 satellites, Altitude $= 1,200$ Km, $T = 30$ f_i^-: $30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4, 30, 0.4$ $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{\text{s}i} = 70$ for all satellites
C_5	12 satellites, Altitude $= 12,000$ Km, $T = 20$ f_i^-: $25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{\text{s}i} = 75$ for all satellites
C_6	14 satellites, Altitude $= 1,400$ Km, $T = 35$ f_i^-: $25, 25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8, 8$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{\text{s}i} = 75$ for all satellites
C_7	14 satellites, Altitude $= 30,000$ Km, $T = 15$ f_i^-: $1.2, 1.2, 1.2, 1.2, 1.2, 1.2, 1.2, 25, 25, 25, 25, 25, 25, 25$ $\bar{f}_i = 25$, $\underline{f}_i = 10$, $m_{\text{s}i} = 75$ for all satellites

a CE-P2P strategy for refueling satellites in this constellation. First, let us look at the solution provided by the problem (CE-P2P-LB). The lower bound on CE-P2P expenditure is found to be $\mathcal{C}_{\text{LB}} = 17.05$ units. The corresponding optimal matching is the following satellites pairs: $s_1 \leftrightarrow s_4$, $s_2 \leftrightarrow s_3$, $s_8 \leftrightarrow s_5$, $s_9 \leftrightarrow s_6$, and $s_{10} \leftrightarrow s_7$ with their preferred slots for rendezvous being ϕ_1, ϕ_3, ϕ_{15}, ϕ_{17}, and ϕ_{19} respectively. Note that in all of these matchings between the fuel-sufficient and fuel-deficient satellites, the fuel-deficient satellite performs a non-cooperative rendezvous with the corresponding fuel-sufficient satellite. The preferred return locations for these active satellites are ϕ_3, ϕ_7, ϕ_{17}, ϕ_{19}, and ϕ_1 respectively. All these are slots adjacent to the corresponding rendezvous slot. Note that these slots are occupied by the passive satellites and it is not possible for all of the active satellites to return to their most preferred choice of orbital slots. Hence, the solution of (CE-P2P-LB) is not a feasible CE-P2P solution. We therefore solve the optimization problem (CE-P2P) yielding the following assignments: $(s_1, s_3) \rightarrow \phi_4 \rightarrow (s_2, s_3)$, $s_2 \rightarrow s_4 \rightarrow s_5$, $(s_5, s_8) \rightarrow \phi_{12} \rightarrow (s_6, s_7)$, $(s_6, s_9) \rightarrow \phi_{16} \rightarrow (s_8, s_9)$ and $s_7 \rightarrow s_{10} \rightarrow s_1$. Figure 4(b) depicts this solution. Note that, like the E-P2P case, all active satellites transfer to available slots in the vicinity during their return trips. The fuel expenditure during the cooperative E-P2P refueling process is 18.65 units, which represents 2.5% fuel savings over the E-P2P refueling strategy. This example demonstrates the utility of the CE-P2P refueling strategy in reducing the fuel expenditure incurred during a (non-cooperative) E-P2P strategy or a (non-Egalitarian) C-P2P strategy. The solution determined is potentially sub-optimal. Comparing with the lower bound on fuel expenditure, we have $\eta = 9.38\%$. This means that our solution is at most 9.38% sub-optimal. Furthermore, looking at



(a) E-P2P strategy (b) CE-P2P strategy

Figure 4. Optimal assignments.

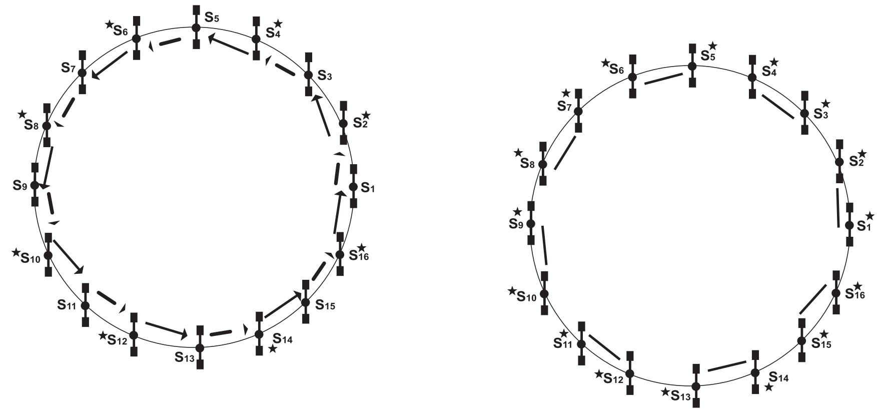
the optimal CE-P2P solution, we find that two of the maneuvers are actually non-cooperative E-P2P maneuvers. Satellites s_2, s_4 and s_7, s_{10} engage in (non-cooperative) E-P2P maneuvers, while the remaining transactions are all cooperative. Hence, s_4 and s_{10} are the passive satellites for the CE-P2P refueling strategy, that is, they remain in their orbital slots throughout the refueling process.

Example 2. *Global minimum in the case of a constellation of* 16 *satellites.*

Let us consider the constellation C_3 in Table 1 consisting of 16 satellites, evenly distributed in a circular orbit. The fuel content of satellites $s_1, s_2, \ldots, s_{16}$ are 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10, 30, 10 respectively. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 5, 7, 9, 11, 13, 15\}$ and those of the fuel-deficient satellites are $\mathcal{I}_{d,0} = \{2, 4, 6, 8, 10, 12, 14, 16\}$. Let us consider Φ' to be a set of 32 orbital slots evenly distributed on the orbit, out of which 16 are initially occupied by the satellites. We therefore have, $\mathcal{J}' = \{1, 2, \ldots, 32\}$. The satellites occupy the slots $\phi_1, \phi_3, \ldots \phi_{31}$ respectively, so that $s_i = \sigma_0(\phi_{2i-1})$ for all $i \in \{1, 2, \ldots, 16\}$. If we solve (CE-P2P-LB), we have the lower bound on the CE-P2P fuel expenditure to be $\mathcal{C}_{\rm LB} = 9.08$ units of fuel. The optimal matching yielded by (CE-P2P-LB) is the following satellites pairs: $s_1 \leftrightarrow s_{16}$, $s_2 \leftrightarrow s_3$, $s_4 \leftrightarrow s_5$, $s_6 \leftrightarrow s_7$, $s_{10} \leftrightarrow s_{11}$, $s_{12} \leftrightarrow s_{13}$, and $s_{14} \leftrightarrow s_{15}$. For all of these matchings, the fuel-deficient satellite performs a non-cooperative rendezvous with the corresponding fuel-sufficient satellite and returns to an orbital slot previously occupied by a different active satellite. Furthermore, the active satellites rendezvous with their preferred choice of fuel-sufficient satellite in its vicinity, and return to their preferred choice of orbital slots without any conflict. Thus, the solution of (CE-P2P-LB) yields a feasible, and hence the global optimum, CE-P2P solution. Figure 5(a) depicts this global minimum. In particular, we find that the global minimum is also the optimal (non-cooperative) E-P2P solution. The (non-Egalitarian) C-P2P solution has a higher fuel expenditure (10.34 units) in this case.

Example 3. *Fuel-deficient satellites have insufficient fuel to engage in non-cooperative rendezvous.*

Let us consider the constellation C_4 given in Table 1. This is similar to the constellation C_3, except that now the fuel-deficient satellites have much less amount of fuel so that they cannot engage in a non-cooperative rendezvous. If we solve (CE-P2P-LB), the optimal matching obtained is the following set of satellites pairs: $s_1 \leftrightarrow s_2$, $s_3 \leftrightarrow s_4$, $s_5 \leftrightarrow s_6$, $s_7 \leftrightarrow s_8$, $s_9 \leftrightarrow s_{10}$, $s_{11} \leftrightarrow s_{12}$,



(a) Fuel-deficient satellites can initiate non-cooperative rendezvous

(b) Fuel-deficient satellites cannot initiate non-cooperative rendezvous

Figure 5. Global Minimum for a Constellation of 16 satellites.

$s_{13} \leftrightarrow s_{14}$, and $s_{15} \leftrightarrow s_{16}$. The lower bound obtained is $C_{\text{LB}} = 9.48$ units of fuel. In each of these assignments, the fuel-deficient satellite engages in a cooperative rendezvous with a neighboring fuel-sufficient satellite and after undergoing a fuel-exchange, returns to its original orbital slot. For each pair of active satellites engaging in a fuel exchange, the slot for cooperative rendezvous is midway between the original slots of the satellites. In fact, all fuel-deficient satellites rendezvous with their preferred choice of fuel-sufficient satellites and return to their preferred orbital slots, without any conflict. The solution of (CE-P2P-LB) is therefore a feasible CE-P2P solution and, hence, also the global optimal solution. Figure 5(b) depicts the matching between the satellites

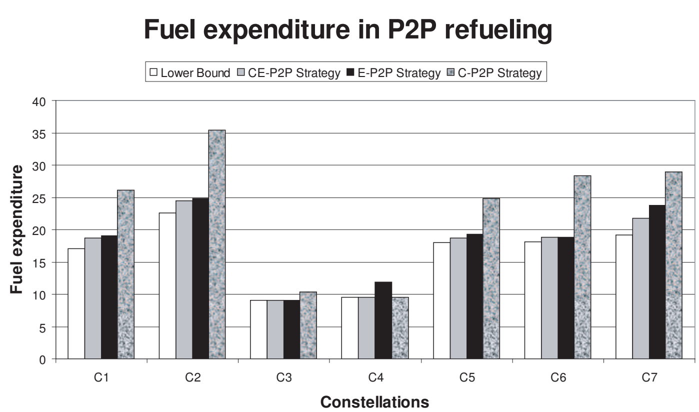


Figure 6. Refueling Expenditures.

required for refueling. The global minimum in this case is the optimal C-P2P solution. For this constellation, the (non-cooperative) E-P2P solution has a higher fuel expenditure of 11.85 units.

Figure 6 provides a comparison of the CE-P2P, E-P2P and C-P2P refueling strategies for the constellations depicted in Table 1. It also shows the lower bound given by the (CE-P2P-LB) solution for all constellations. In general, it is observed that the CE-P2P strategy provides an improvement over either the E-P2P or the C-P2P strategies.

CONCLUSIONS

In this paper, we have studied a Cooperative Egalitarian P2P (CE-P2P) strategy for refueling satellites in a circular constellation. We have presented a network flow formulation for determining the optimal set of CE-P2P maneuvers in the constellation and we have computed a lower bound on the fuel expenditure for the optimal set of CE-P2P maneuvers. The bound is determined by solving a bipartite assignment problem, the solution of which may or may not correspond to a feasible CE-P2P solution. In case it does, we have a globally optimal CE-P2P solution. Otherwise, the bound helps in providing an estimate of the sub-optimality of the CE-P2P solution obtained by our proposed methodology. The CE-P2P strategy is found to be a better refueling strategy compared to either a (non-cooperative) Egalitarian P2P (E-P2P) strategy or a (non-Egalitarian) Cooperative P2P strategy (C-P2P). In fact, the CE-P2P strategy allows for the benefits of both Egalitarian P2P refueling and Cooperative P2P refueling. On one hand, active satellites can perform smaller-ΔV (and hence lower fuel expenditure) orbital transfers since they are allowed to return to any available orbital slot. On the other hand, the CE-P2P strategy reduces the fuel expenditure by allowing satellites to engage in cooperative rendezvous. This is particularly advantageous when the fuel-deficient satellite does not have enough fuel to initiate a non-cooperative rendezvous.

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