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A NETWORK FLOW FORMULATION FOR AN EGALITARIAN P2P SATELLITE REFUELING STRATEGY

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Abstract

A variation of the P2P strategy, known as egalitarian P2P (E-P2P) refueling strategy, relaxes the restriction on the active satellites to return to their original orbital slots after undergoing the fuel transactions. The E-P2P refueling problem can be formulated as a three-index assignment problem on an undirected tripartite constellation graph, which can be solved using, say, a Greedy Random Adaptive Search Procedure (GRASP). In this paper we consider again the E-P2P problem, which we formulate as a minimum cost flow problem on a directed graph, along with some additional constraints. The solution of the corresponding integer program yields the optimal satellite assignment.

INTRODUCTION

The current practice when the fuel on-board a satellite is exhausted is to simply replace the satellite with a new one. Replacing old satellites involves significant costs in production as well as launching operations. An alternative to satellite replacement is to refuel a satellite when the on-board fuel is depleted. Periodic refueling enhances the lifetime of satellite constellations. The traditional approach for satellite refueling involves a single service vehicle that refuels all fuel-deficient satellites in the constellation in a sequential manner.[1] Recently, an alternative scenario for distributing fuel amongst a large number of satellites has been proposed.[2,3] In this scenario, there is no designated spacecraft with the responsibility of refueling all satellites depleted of fuel. Instead, all fuel-sufficient satellites share the responsibility of refueling the fuel-deficient ones. We call this the P2P refueling strategy. The single service vehicle refueling strategy and the P2P refueling strategy can be combined to form a mixed refueling strategy. In a mixed refueling strategy a refueling spacecraft, either launched from the Earth or transferred from a different orbit, refuels part of the satellites in the constellation, and these satellites subsequently refuel the remaining fuel-deficient ones via P2P refueling. Such a mixed refueling strategy can be a competitive alternative to the single-service vehicle refueling strategy and, in fact, outperforms the latter for a large number of satellites in the constellation.[4]

In the standard P2P formulation,[3–6] is assumed that all active satellites return to their original orbital slots after the refueling process is over. In this paper, we relax this constraint

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and allow the active satellites to return to any available orbital slots left vacant by other (active) satellites. We assume that all satellites are similar, that is, they have the same structure, same operational characteristics, and perform the same functions, so that any satellite can be used in lieu of any other satellite in the constellation. We call this the egalitarian P2P (E-P2P) refueling strategy. In this paper, we show that the E-P2P refueling strategy leads to lesser fuel expenditure during the refueling process, compared to the standard P2P refueling strategy.

The E-P2P strategy can be formulated as a three-index assignment problem on an undirected tripartite constellation graph. Several sub-optimal heuristics exist for solving the three-index assignment problem.[7–9] Reference 10 discusses the application of a Greedy Random Adaptive Search Procedure for determining the optimal scheduling for the E-P2P satellite refueling strategy.

Typically, in a constellation graph, the nodes represent the satellites, and the edges represent the maneuvers between pairs of satellites. Since either of the two satellites involved in a refueling transaction can be active, two different P2P maneuvers can be associated with a satellite pair. Therefore, a sense of direction naturally comes along with each edge (the direction being from the active satellite to the passive satellite), implying that the constellation graph needs to be a directed one. In previous studies[2,3,5] this issue was bypassed by considering only the cheaper of the two maneuvers associated with a satellite pair. Hence the P2P problem was formulated on an undirected constellation graph. In this paper, we reformulate the E-P2P problem over a *directed* constellation graph (digraph). We show that this problem can be posed as a minimum cost flow problem with additional constraints in the constellation digraph. In the following sections, we describe in detail the problem formulation. With the help of numerical examples we show that our proposed refueling strategy results in lesser fuel expenditure, compared to the baseline P2P refueling strategy. We also provide a comparison with the GRASP method of Ref. 10 and we show the numerical efficiency of the network flow formulation.

PROBLEM FORMULATION

We consider a constellation with n satellites distributed over n orbital slots in a circular orbit. Let $\mathcal{S} = \{s_i : i = 0, 1, 2, \ldots, n\}$ denote the set of satellites, where s_0 represents a fictitious satellite. Let $\Phi = \{\phi_i \in [0, 2\pi) : i = 1, 2, \ldots, n,\ \phi_i \neq \phi_j\}$ be the set of orbital slots. We introduce a mapping $\sigma_t : \Phi \mapsto \mathcal{S}$ that, at time $t \geq 0$ assigns to each orbital slot a satellite from $\mathcal{S}$. Specifically, $\sigma_t(\phi_j) = s_i$ implies that the satellite s_i occupies the orbital slot ϕ_j at time t. If the slot ϕ_j is empty at time t, we write $\sigma_t(\phi_j) = s_0$. Let the fuel content of satellite s_i at time t be denoted by $f_{i,t}$, let the minimum fuel content for satellite s_i to remain operational be denoted by $\underline{f}_i$, and let the maximum fuel capacity of satellite s_i be $\bar{f}_i$. Let the initial fuel content of satellite s_i be denoted by f_i^-, that is, $f_i^- = f_{i,0}$. Satellites having an amount of fuel more than or equal to the amount required to remain operational are termed *fuel-sufficient*, while the ones having fuel less than the required amount to remain operational are termed *fuel-deficient* satellites.

We follow a notation similar to that in Ref. 10. To this end, let $\mathcal{I} = \{1, 2, \ldots, n\}$, and let $\mathcal{I}_{s,t} = \{i : f_{i,t} \geq \underline{f}_i\}$ denote the index set of all fuel-sufficient satellites at time t, and

$\mathcal{I}_{d,t} = \{i : f_{i,t} < \underline{f}_i\}$ denote the index set of all fuel-deficient satellites at time t. In a P2P refueling transaction between a fuel-sufficient and a fuel-deficient satellite, one of them (henceforth referred to as the *active* satellite) performs an orbital transfer to rendezvous with the other satellite (henceforth referred to as the *passive* satellite). After a fuel exchange takes place between the active and the passive satellite, the active satellite returns to one of the available orbital slots. We will denote the index set of active satellites by $\mathcal{I}_a \subseteq \mathcal{I}$ and the index set of passive satellites by $\mathcal{I}_p \subset \mathcal{I}$. We also use $\mathcal{J}_{s,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{s,t}\}$ to denote the index set of orbital slots occupied by fuel-sufficient satellites at time t, and $\mathcal{J}_{d,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{d,t}\}$ to denote the index set of orbital slots occupied by fuel-deficient satellites at time t. Also, $\mathcal{J}_a = \{j : \sigma_0(\phi_j) = s_i, i \in \mathcal{I}_a\}$ will denote the index set of orbital slots occupied by the active satellites before any orbital maneuver commences, and $\mathcal{J}_p = \{j : \sigma_0(\phi_j) = s_i, i \in \mathcal{I}_a\}$ will denote the index set of orbital slots occupied by the passive satellites before any orbital maneuver commences. Finally, let $\mathcal{J}_r$ denote the index set of orbital slots available for the active satellites to return to after they have undergone fuel transactions with the passive satellites. Note that $\mathcal{J}_r = \mathcal{J}_a$.

The Constellation Digraph

We define a tripartite constellation graph $\mathcal{G}$ consisting of three partitions. The first partition consists of nodes that correspond to the elements of the index set $\mathcal{J}_a$, the second partition consists of nodes that correspond to the index set $\mathcal{J}_p$ and the third partition consists of nodes that correspond to the index set $\mathcal{J}_r$. Note that the set of active satellites and the set of passive satellites are not known a priori. We therefore consider $\mathcal{J}_a = \mathcal{J}_p = \mathcal{J}_r = \mathcal{I}$. A directed edge (i, j) where $i \in \mathcal{J}_a, j \in \mathcal{J}_p$ denotes an orbital transfer from the orbital slot ϕ_i to the orbital slot ϕ_j. Similarly, a directed edge (j, k) where $j \in \mathcal{J}_p, k \in \mathcal{J}_r$ denotes an orbital transfer from the orbital slot ϕ_j to the orbital slot ϕ_k. By an E-P2P maneuver, we mean that satellite $s_\mu = \sigma_0(\phi_i)$ performs an orbital transfer from slot ϕ_i to slot ϕ_j in order to undergo fuel exchange with the satellite $s_\nu = \sigma_0(\phi_j)$. After the fuel exchange satellite s_μ performs another orbital transfer from the slot ϕ_j to an unoccupied orbital slot ϕ_k. Here the active satellite is s_μ, while the passive one is s_ν. The transfer from ϕ_i to ϕ_j constitutes the forward trip of s_μ and the transfer from

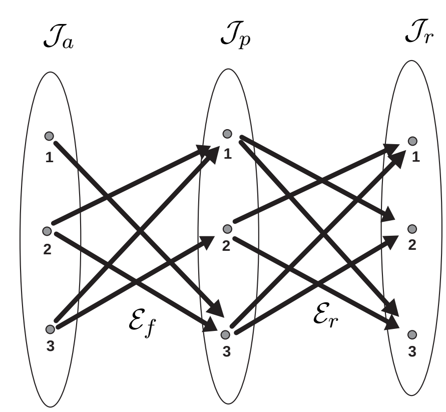


Figure 1: Directed constellation graph.

ϕ_j to ϕ_k constitutes its return trip. We represent the E-P2P maneuver by the triplet (i, j, k). On the constellation digraph $\mathcal{G}$ we represent an E-P2P maneuver (i, j, k) by the directed edges (i, j) and (j, k), where $i \in \mathcal{J}_a$, $j \in \mathcal{J}_p$, and $k \in \mathcal{J}_r$. Note that a fuel transaction can only be between a fuel-sufficient and a fuel-deficient satellite, that is, for a E-P2P maneuver (i, j, k), either $i \in \mathcal{J}_{s,0}$ and $j \in \mathcal{J}_{d,0}$, or $i \in \mathcal{J}_{d,0}$ and $j \in \mathcal{J}_{s,0}$. Therefore, the set of edges representing all possible forward trips is given by

$$\mathcal{E}_f = \{(i, j) : i \in \mathcal{J}_{s,0} \cap \mathcal{J}_a, j \in \mathcal{J}_{d,0} \cap \mathcal{J}_p\} \cup \{(i, j) : i \in \mathcal{J}_{d,0} \cap \mathcal{J}_a, j \in \mathcal{J}_{s,0} \cap \mathcal{J}_p\}. \tag{1}$$

The return maneuver from the orbital slot ϕ_j to the orbital slot ϕ_k, where $k \neq j$, can be represented by a directed edge $(j, k) \in \mathcal{J}_p \times \mathcal{J}_r, j \neq k$. We can therefore denote the set of all possible return trips by

$$\mathcal{E}_r = \{(j, k) : j \in \mathcal{J}_p, k \in \mathcal{J}_r, j \neq k\}. \tag{2}$$

Thus, the set of nodes of the constellation digraph is given by $\mathcal{V} = \mathcal{J}_a \cup \mathcal{J}_p \cup \mathcal{J}_r$, while the set of edges is given by $\mathcal{E} = \mathcal{E}_f \cup \mathcal{E}_r$. We define the constellation digraph as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Figure 1 shows the digraph for a constellation, with nodes representing orbital slots of satellites and edges representing orbital maneuvers. Note that a pair of directed edges $(i, j) \in \mathcal{E}_f$ and $(\ell, k) \in \mathcal{E}_r$ represents an E-P2P maneuver if and only if $\ell = j$.

Cost Assignment

With each orbital transfer represented by a directed edge $(i, j) \in \mathcal{E}$, we associate a cost c_{ij} as follows

$$c_{ij} = \Delta V_{ij} \text{ for all } (i, j) \in \mathcal{E}, \tag{3}$$

where ΔV_{ij} is the required velocity change for a satellite to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j. Note that the calculation of ΔV_{ij} requires, in general, the solution of a two-impulse multi-revolution Lambert problem.[11]

We should point out here that – ideally – the cost c_{ij} should be the fuel consumption during the transfer. However, the amount of fuel depends on the mass of the satellite performing the transfer, which may not be known a priori. For instance, recall that the edge $(j, k) \in \mathcal{E}_r$ represents a valid return trip for any of the E-P2P maneuvers in which an edge $(i, j) \in \mathcal{E}_f$ represents a forward trip. The set of possible active satellites that can carry out the orbital transfer from the slot ϕ_j to the slot ϕ_k is given by $\{\sigma_0(\phi_i) : (i, j) \in \mathcal{E}_f\}$. For each of these active satellites, the fuel expenditure for the return trip represented by the edge $(j, k) \in \mathcal{E}_r$ is different. Therefore, if fuel expenditure is used to define the cost of edges, no unique value can be assigned to an edge $(j, k) \in \mathcal{E}_r$. This is the reason we use (3) for c_{ij}, tacitly recognizing the fact that the results will necessarily be suboptimal in terms of actual fuel consumption.

Constellation Network Flow

Given the constellation digraph $\mathcal{G}$, we will now set up the constellation network $\mathcal{G}_n$ and show that the E-P2P problem can be formulated as a minimum cost flow problem on the constellation network $\mathcal{G}_n$. To this end, we add a source node s and a sink node t to the constellation digraph $\mathcal{G}$. For all $i \in \mathcal{J}_a$, we also add an arc (s, i) with associated cost $c_{si} = 0$. We denote the set of these arcs by $\mathcal{E}_s$. Similarly, for all $k \in \mathcal{J}_r$, we add an arc (k,t) with associated cost $c_{kt} = 0$. We denote the set of these arcs by $\mathcal{E}_t$. The set of nodes for $\mathcal{G}_n$ is $\mathcal{V}_n = \{s\} \cup \mathcal{V} \cup \{t\}$, while the set of arcs (directed edges) of $\mathcal{G}_n$ is $\mathcal{E}_n = \mathcal{E}_s \cup \mathcal{E} \cup \mathcal{E}_t$. That is, $\mathcal{G}_n = (\mathcal{V}_n, \mathcal{E}_n)$. A depiction of $\mathcal{G}_n$ is given in Figure 2. Let us now consider a $s \to t$ flow in the network $\mathcal{G}_n$. By a $s \to t$ flow,

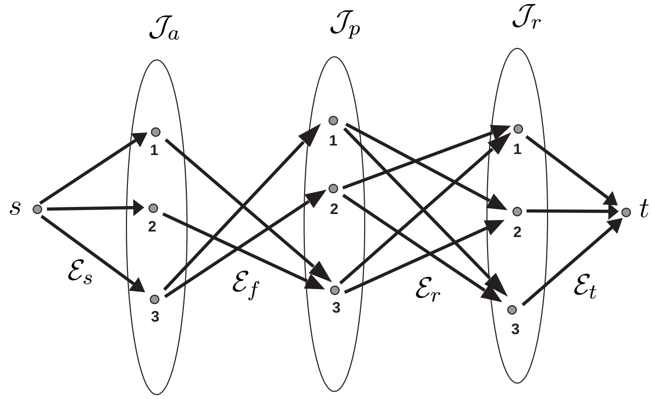


Figure 2: Constellation flow network.

we mean a flow from the source s to the sink t passing through the nodes $i \in \mathcal{J}_a$, $j \in \mathcal{J}_p$ and $k \in \mathcal{J}_r$ in that order, that is, a flow along the directed path $\{s \to i \to j \to k \to t\}$. Note that the $s \to t$ flow passes through the arcs $(s,i) \in \mathcal{E}_s$, $(i,j) \in \mathcal{E}_f$, $(j,k) \in \mathcal{E}_r$ and $(k,t) \in \mathcal{E}_t$. Of these, the arcs (i,j) and (j,k) constitute an E-P2P maneuver (i,j,k), and the sum of the costs of all these edges is the total cost of the E-P2P maneuver (i,j,k). The remaining arcs (s,i) and (k,t) have zero cost and therefore the cost of a unit flow along the path $\{s \to i \to j \to k \to t\}$ is the total cost of the corresponding E-P2P maneuver. We can therefore associate an E-P2P maneuver with a unique $s \to t$ flow.

Network Flow Minimization Problem

Recall that we are interested in a set $\mathcal{M} \subset \mathcal{J}_a \times \mathcal{J}_p \times \mathcal{J}_r$ of $|\mathcal{I}_{d,0}|$ maneuvers such that all fuel-deficient satellites are involved in fuel transactions. Corresponding to this set of maneuvers, let $A_{\mathcal{M}} = \{i : (i,j,k) \in \mathcal{M}\}$ denote the set of indices of the orbital slots of active satellites, let $P_{\mathcal{M}} = \{j : (i,j,k) \in \mathcal{M}\}$ denote the set of indices of the orbital slots of passive satellites, and let $R_{\mathcal{M}} = \{k : (i,j,k) \in \mathcal{M}\}$ denote the set of indices of available return slots. We introduce a flow variable x_{ij} for each arc $(i,j) \in \mathcal{E}_n$. The flow variable x_{ij} equals the amount of flow through the edge (i,j). The capacity u_{ij} of each edge is the maximum amount of flow that is permissible through that edge, that is, $0 \le x_{ij} \le u_{ij}$. We assume $u_{ij} = 1$ for all $(i,j) \in \mathcal{E}_n$. In addition, let b_i denote the amount of supply at node $i \in \mathcal{V}_n$, such that $b_i < 0$ denotes demand at that node. For all nodes $i \in \mathcal{N} \backslash \{s,t\}$, we have $b_i = 0$. For the source and sink nodes, we have $b_s = |\mathcal{I}_{d,0}|$ and $b_t = -|\mathcal{I}_{d,0}|$, respectively. This implies that we wish to send a flow equal to $|\mathcal{I}_{d,0}|$ through the network from the source to the sink, given that no edge allows more than one unit of flow through it.

All nodes in the constellation network $\mathcal{G}_n$ are required to satisfy the usual flow balance equations

$$\sum_{j:(i,j)\in\mathcal{E}_n} x_{ij} - \sum_{j:(j,i)\in\mathcal{E}_n} x_{ji} = b_i \text{ for all } i \in \mathcal{V}_n. \tag{4}$$

However, our initial consideration $\mathcal{J}_a = \mathcal{J}_p = \mathcal{J}_r = \mathcal{I}$ requires the introduction of additional constraints. First, note that $A_{\mathcal{M}} = R_{\mathcal{M}}$. Hence, if the flow passes through a node $i \in \mathcal{J}_a$, then the flow has to pass through the node $i \in \mathcal{J}_r$. Moreover, if the flow does not pass through the node $i \in \mathcal{J}_a$, no flow should then pass through $i \in \mathcal{J}_r$. This constraint can be written as

$$x_{si} = x_{it} \text{ for all } i \in \mathcal{J}_a = \mathcal{J}_r. \tag{5}$$

Second, note that $i \in A_{\mathcal{M}}$ implies $i \notin P_{\mathcal{M}}$. Hence, the network should not allow two $s \to t$ flows, one that passes through node $i \in \mathcal{J}_a$ and the other that passes through $i \in \mathcal{J}_p$. That is, the satellite originally occupying the orbital slot ϕ_i cannot be simultaneously the active satellite and the passive satellite with respect to two different P2P maneuvers. This implies the following constraint

$$x_{sj} + \sum_{i:(i,j)\in\mathcal{E}_f} x_{ij} \leq 1 \text{ for all } j \in \mathcal{J}_p. \tag{6}$$

Finally, given the constellation network $\mathcal{G}_n$, we seek to find the minimum cost flow in the network

$$\min \sum_{(i,j)\in\mathcal{E}_n} c_{ij} x_{ij} \tag{7}$$

subject to the constraints (4)-(6). It is to be noted here that the integrality property [12] states that if all arc capacities and supplies/demands of the nodes are integers, the minimum cost flow problem has an integral optimal solution, that is, although we allow $0 \leq x_{ij} \leq 1$ for all $(i,j) \in \mathcal{E}_n$, the minimum cost flow ensures that $x_{ij} = 0$ or 1 in the solution. In other words, we can arrive at the optimal solution by considering only integer values of the decision variables.

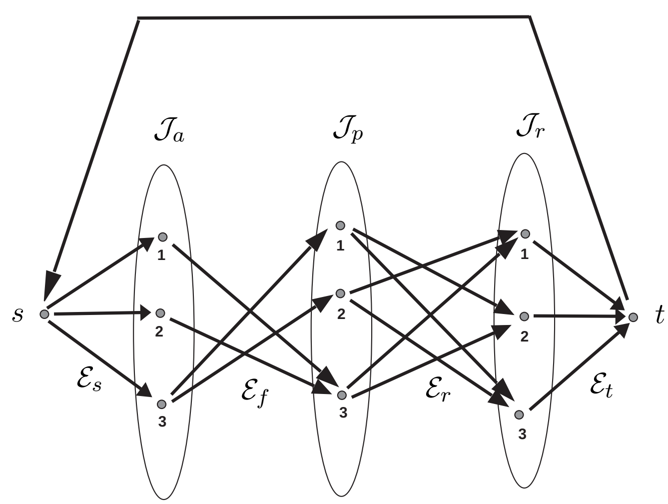


Figure 3: Constellation flow network with an additional (t, s) arc.

Note that in Fig. 2 the source sends a total flow equal to $|\mathcal{I}_{d,0}|$ to the sink via the network. Since the capacity of each edge is unity, the minimum cost flow will be comprised of $|\mathcal{I}_{d,0}|$

flows from s to$t$, and these will correspond to the optimal assignment $\mathcal{M}^*$. We now prove that the configuration of the network assures that all fuel-deficient satellites are involved in fuel transactions. The flow from the sink reaches $|\mathcal{I}_{d,0}|$ nodes in $\mathcal{J}_a$. Clearly, these nodes are given by $A_{\mathcal{M}^*}$ and $|A_{\mathcal{M}^*}| = |\mathcal{I}_{d,0}|$. The indices of the original orbital slots of the active fuel-sufficient satellites are given by $A_{\mathcal{M}^*} \cap \mathcal{J}_{s,0}$ and those for the active fuel-deficient satellites are given by $A_{\mathcal{M}^*} \cap \mathcal{J}_{d,0}$. The set of nodes in $\mathcal{J}_p$ through which the flow passes are given by $P_{\mathcal{M}^*}$. Evidently, the indices of the original orbital slots of the passive fuel-sufficient satellites are given by $P_{\mathcal{M}^*} \cap \mathcal{J}_{s,0}$, while those of the passive fuel-deficient satellites are given by $P_{\mathcal{M}^*} \cap \mathcal{J}_{d,0}$. Because a fuel transaction can only be between a fuel-sufficient and a fuel-deficient satellite, the number of passive fuel-deficient satellites will equal the number of active fuel-sufficient satellites, that is, we have $|P_{\mathcal{M}^*} \cap \mathcal{J}_{d,0}| = |A_{\mathcal{M}^*} \cap \mathcal{J}_{s,0}|$. However, the total number of active satellites is $|\mathcal{I}_{d,0}|$, so that we have $|A_{\mathcal{M}^*} \cap \mathcal{J}_{s,0}| + |A_{\mathcal{M}^*} \cap \mathcal{J}_{d,0}| = |\mathcal{I}_{d,0}|$. It follows that $|A_{\mathcal{M}^*} \cap \mathcal{J}_{d,0}| + |P_{\mathcal{M}^*} \cap \mathcal{J}_{d,0}| = |\mathcal{I}_{d,0}|$, which implies that the flow from the source reaches all nodes corresponding to the orbital slots of all fuel-deficient satellites. In other words, all fuel-deficient satellites are involved in fuel transaction during the E-P2P maneuvers represented by the optimal flow in the network.

Remark 1. Note that in the network flow formulation for the E-P2P problem, the supply or demand at each node representing an orbital slot of a satellite is zero. If we now let $b_s = b_t = 0$, but add an arc (t, s) in the network $\mathcal{G}_n$ and impose a flow $|\mathcal{I}_{d,0}|$ through this arc from the sink to the source, then the problem remains unaltered. All nodes in the augmented network (see Fig. 3) now have zero demand/supply and the flow in the network has to be a circulation. We know that a circulation can always be decomposed into cycles.[12] Hence, the optimal cost flow should be in the form of cycles.

NUMERICAL EXAMPLES

In this section, we determine the optimal assignments for P2P refueling of sample constellations when the active satellites are not restricted to return to their original orbital slots. The determination of the optimal assignments requires the solution of the integer program corresponding to the minimum cost flow problem (given by (7),(4)-(6)). Its solution was obtained using the binary integer programming solver (`bintprog`) of MATLAB. This solver uses branch-and-bound to solve integer programs. The fuel expenditure incurred during the orbital transfers given by the optimal assignment of the E-P2P strategy can be calculated as in Ref. 10. We also compare the results against the baseline P2P strategy, namely, when the active satellites are constrained to return to their original orbital slots. With the help of numerical examples, we show how the E-P2P refueling strategy leads to considerable reductions in fuel expenditure. The sample constellations investigated in this section are given in Table 1.

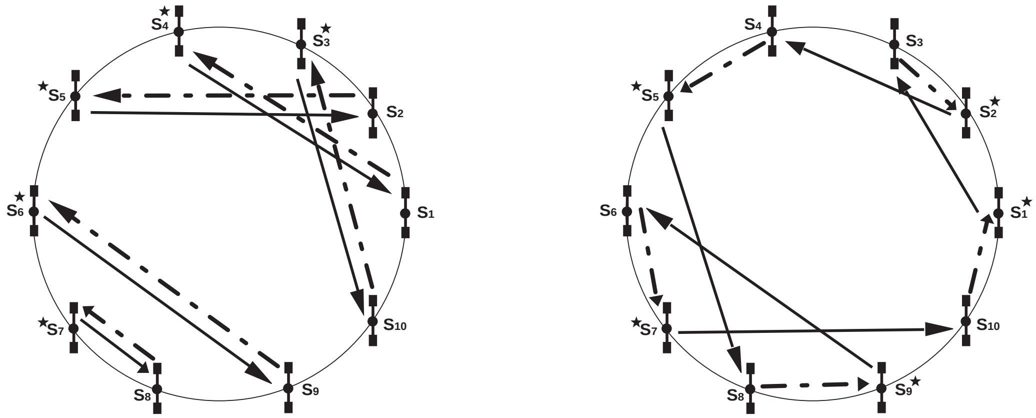
Table 1: Sample Constellations.

Label	Description
C_1	10 satellites, Altitude = $35,786$ Km, $T = 12$ f_i^-: $30, 30, 6, 6, 6, 6, 6, 30, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 12$, $m_{si} = 70$ for all satellites
C_2	16 satellites, Altitude = $1,200$ Km, $T = 30$ f_i^-: $30, 30, 30, 30, 30, 30, 10, 10, 10, 10, 10, 10, 10, 10, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_3	12 satellites, Altitude = $2,000$ Km, $T = 30$ f_i^-: $30, 30, 30, 10, 10, 10, 10, 10, 10, 30, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_4	18 satellites, Altitude = $6,000$ Km, $T = 25$ f_i^-: $25, 25, 25, 25, 25, 25, 25, 25, 25, 6, 6, 6, 6, 6, 6, 6, 6, 6$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_5	12 satellites, Altitude = $12,000$ Km, $T = 20$ f_i^-: $25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_6	14 satellites, Altitude = $1,400$ Km, $T = 35$ f_i^-: $25, 25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8, 8$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_7	16 satellites, Altitude = $30,000$ Km, $T = 15$ f_i^-: $10, 10, 10, 10, 10, 10, 10, 10, 28, 28, 28, 28, 28, 28, 28, 28$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites

Example 1.

Consider the constellation C_1 given in Table 1. This constellation consists of 10 satellites evenly distributed in a circular orbit. The maximum allowed time for refueling is $T = 12$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 12$ units, while their maximum amount of fuel each can hold is $\bar{f}_i = 30$ units. Each satellite s_i has a permanent structure of $m_{si} = 70$ units, and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 2, 8, 9, 10\}$ and those of the fuel-deficient satellites are $\mathcal{I}_{d,0} = \{3, 4, 5, 6, 7\}$. For the baseline P2P refueling strategy, the optimal assignment is $s_4 \rightarrow s_1$, $s_5 \rightarrow s_2$, $s_7 \rightarrow s_8$, $s_6 \rightarrow s_9$, $s_3 \rightarrow s_{10}$, and the total fuel consumption for all P2P maneuvers is 26.07 units. This represents 14.48% of the total initial fuel in the constellation. The indices of the active satellites in this case are $\mathcal{I}_a = \{3, 4, 5, 6, 7\}$. Note that $\mathcal{I}_a = \mathcal{I}_{d,0}$, that is, the fuel-deficient satellites are the active ones for the baseline P2P refueling strategy. The baseline P2P assignment assignments is shown in Fig. 4(a). The active satellites are marked by '$\star$'. The forward trips are marked by solid arrows, while the return trips are marked by dotted arrows.

For the case in which the active satellites are allowed to interchange their orbital slots, the optimal assignment for E-P2P refueling is $s_1 \rightarrow s_3 \rightarrow s_2$, $s_2 \rightarrow s_4 \rightarrow s_5$, $s_5 \rightarrow s_8 \rightarrow s_9$, $s_7 \rightarrow s_{10} \rightarrow s_1$, $s_9 \rightarrow s_6 \rightarrow s_7$. The fuel expenditure during the E-P2P refueling process is 19.11 units, which is less than the fuel expenditure for the baseline P2P case. This represents



(a) Baseline P2P strategy

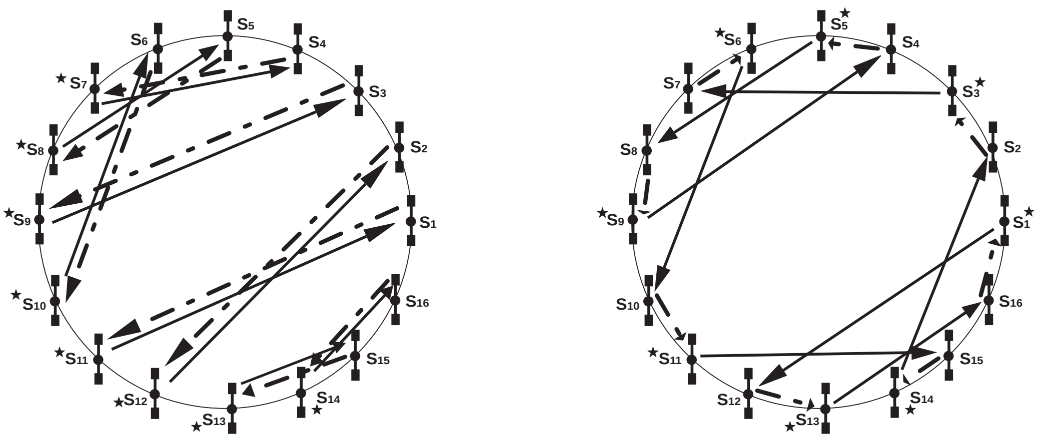
(b) E-P2P strategy

Figure 4: Optimal assignments for Constellation C_1

10.62% of the total initial fuel in the constellation. Figure 4(b) shows the optimal assignments for the E-P2P case. For the optimal assignment for the proposed E-P2P refueling strategy, it is observed that each active satellite, after undergoing a fuel transaction with the corresponding passive satellite, returns to an available orbital slot in the vicinity of the passive satellite with which it was involved in the transaction. For instance, satellite s_1 undergoes a fuel transaction with satellite s_3, and then returns to the orbital slot initially occupied by active satellite s_2. Moving to an orbital slot in the vicinity involves an orbital transfer through a smaller transfer angle and thereby likely results in a lesser fuel expenditure during the return trip. Hence, the active satellites, having the freedom to return to any available orbital slot, they opt to move to a nearby orbital slot during the return trip. In the baseline P2P strategy, such freedom is not available, and some of the active satellites have to perform orbital transfers that incur higher cost. Another observation is the fact that some of the active satellites are also fuel-sufficient. For instance, satellites s_1, s_2 and s_9 are fuel-sufficient and active. Figure 4(b) also shows that the optimal solution comprises of a Hamiltonian cycle $\{s_1 \rightarrow s_3 \rightarrow s_2 \rightarrow s_4 \rightarrow s_5 \rightarrow s_8 \rightarrow s_9 \rightarrow s_6 \rightarrow s_7 \rightarrow s_{10} \rightarrow s_1\}$ in the constellation.

Example 2.

In this example, we consider the constellation C_2 given in Table 1. This is a constellation of 16 satellites, evenly distributed in a circular orbit. The maximum allowable time for refueling is $T = 30$ orbital periods. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 15$ units, a maximum fuel capacity of $\bar{f}_i = 30$ units, permanent structure of $m_{si} = 70$ units and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 2, 3, 4, 5, 6, 15, 16\}$, while those of the fuel-deficient satellites are $\mathcal{I}_{d,0} = \{7, 8, 9, 10, 11, 12, 13, 14\}$. If the active satellites are constrained to return to their original orbital slots after refueling, the optimal assignment is $s_{11} \rightarrow s_1$, $s_{12} \rightarrow s_2$, $s_9 \rightarrow s_3$, $s_7 \rightarrow s_4$, $s_8 \rightarrow s_5$, $s_{10} \rightarrow s_6$, $s_{13} \rightarrow s_{15}$, $s_{14} \rightarrow s_{16}$ and the total fuel consumption is 37.46 units. This represents 11.71% of the total initial fuel in the constellation. In this case, only the fuel-deficient satellites are the active ones, that is, $\mathcal{I}_a = \{7, 8, 9, 10, 11, 12, 13, 14\} = \mathcal{I}_{d,0}$. This is similar to the previous example. The standard P2P assignment for C_2 is shown in Fig. 5(a).



(a) Baseline P2P strategy (b) E-P2P strategy

Figure 5: Optimal assignments for Constellation C_2

If the active satellites are allowed to interchange orbital slots, then the optimal assignment is $s_1 \to s_{12} \to s_{13}$, $s_3 \to s_7 \to s_6$, $s_5 \to s_8 \to s_9$, $s_6 \to s_{10} \to s_{11}$, $s_9 \to s_4 \to s_5$, $s_{11} \to s_{15} \to s_{14}$, $s_{13} \to s_{16} \to s_1$, $s_{14} \to s_2 \to s_3$. Here, $\mathcal{I}_a = \{1, 3, 5, 6, 9, 11, 13, 14\}$. Relaxing the return orbital position constraint reduces the fuel expenditure to 24.82 units. This represents 7.76% of the total initial fuel in the constellation. Figure 5(b) shows the constellation and the optimal assignments for the E-P2P case. The active satellites are marked by '$\star$'. Similar to Example 1, it is observed that the active satellites, after undergoing fuel transactions with the corresponding passive satellites, return to an available orbital slot in their vicinity. For instance, satellite s_1 undergoes a fuel transaction with satellite s_{12}, and then returns to the orbital slot occupied by active satellite s_{13}. Also, note that the active satellites include fuel-sufficient ones. Here, s_1, s_3, s_5 and s_6 are fuel-sufficient and active. Figure 5(b) shows that the optimal solution corresponds to three cycles in the constellation, namely, $\{s_1 \to s_{12} \to s_{13} \to s_{16} \to s_1\}$, $\{s_3 \to s_7 \to s_6 \to s_{10} \to s_{11} \to s_{15} \to s_{14} \to s_2 \to s_3\}$ and $\{s_5 \to s_8 \to s_9 \to s_4 \to s_5\}$.

We have also tested the proposed methodology on other constellations as depicted in Table 1. The optimal assignments for these constellations show considerable reduction in fuel consumption against the baseline P2P strategy. For instance, for constellation C_3, the baseline P2P refueling strategy yields an optimal assignment $s_4 \to s_1$, $s_5 \to s_2$, $s_7 \to s_{10}$, $s_6 \to s_3$, $s_{11} \to s_8$, $s_9 \to s_{12}$ with a fuel expenditure of 26.73 units, with the fuel-deficient satellites being the active ones. Our proposed methodology yields the optimal assignment $s_1 \to s_4 \to s_5$, $s_3 \to s_6 \to s_7$, $s_5 \to s_2 \to s_3$, $s_7 \to s_{10} \to s_{11}$, $s_9 \to s_{12} \to s_1$, $s_{11} \to s_8 \to s_9$, that reduces the fuel expenditure to 18.87 units. In this case, the optimal solution consists of a Hamiltonian cycle $\{s_1 \to s_4 \to s_5 \to s_2 \to s_3 \to s_6 \to s_7 \to s_{10} \to s_{11} \to s_8 \to s_9 \to s_{12} \to s_1\}$. Similarly, for the other constellations, the fuel expenditure reduces from 41.06 units to 26.26 units in case of C_4, from 28.38 to 18.87 in case of C_5, from 28.77 units to 19.26 units in case with C_6, and from 34.97 units to 22.75 units in case of C_7. Figure 6 summarizes these results.

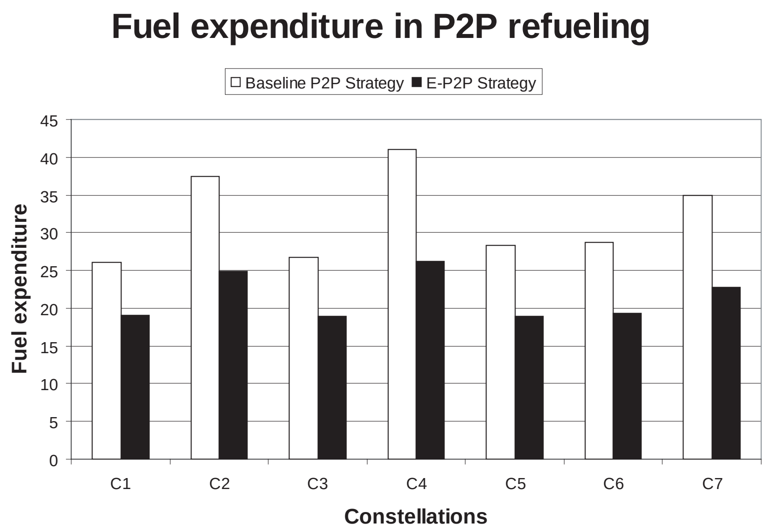


Figure 6: Comparison of E-P2P and baseline P2P refueling strategies.

Comparison with GRASP

We now compare the results obtained using the network flow formulation with those given by the GRASP method[10] for the constellations given in Table 1. Such a comparison is depicted in Fig. 7. For constellations C_2 and C_4, the optimal assignment is the same for both methods. For C_1, C_3, C_5, C_6 and C_7, the results using GRASP are better than those given by the network flow formulation. This is to be expected since GRASP in most of the cases finds the actual optimal solution.

An important aspect that needs to be mentioned here is the computational time required to determine the optimal assignment. Figure 8 compares the computational time required by the two methods. The network flow formulation runs several times faster than the GRASP method. Nevertheless, it needs to be mentioned that the GRASP method has capabilities for parallelization which can decrease its running time. Parallel implementation issues of the GRASP have not been considered in our work. As a final point we should also mention that the GRASP method requires a good initial feasible solution that is improved upon in subsequent steps of the GRASP. The solution obtained by the network flow formulation can be used as an initial feasible solution that can speed up the GRASP by several orders of magnitude.

CONCLUSIONS

In this paper, we have studied the egalitarian peer-to-peer (E-P2P) refueling strategy for satellite constellations, in which the active satellites are allowed to interchange orbital positions during their return trip. A directed constellation graph is introduced in order to take into account the direction in which the orbital maneuvers are executed. The problem is formulated

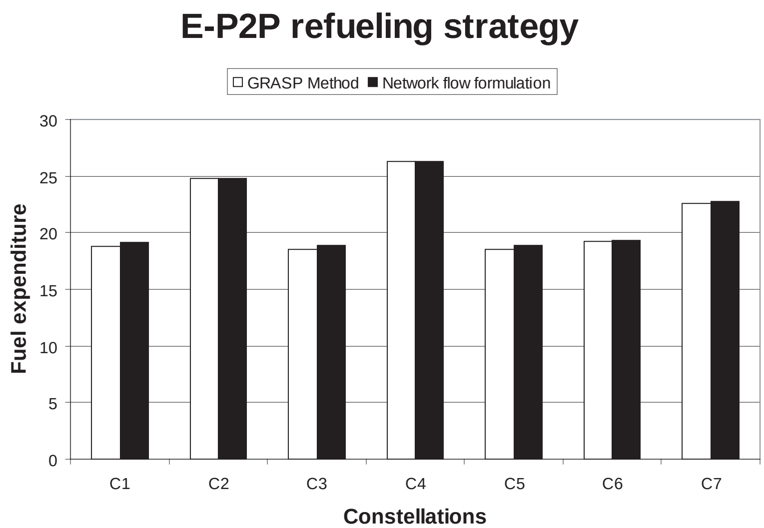


Figure 7: Comparison with results from GRASP method.

as a minimum cost network flow problem with additional constraints. The solution of the corresponding integer program yields the optimal assignment for the E-P2P strategy. With the help of numerical examples, it is shown that the proposed E-P2P strategy results in considerable reduction in the fuel expenditure incurred during the refueling process. It is also shown that each active satellite opts to return to an available orbital slot in the vicinity of the passive satellite with which it is involved in a fuel transaction. The results obtained using the network flow formulation are found to be comparable to those obtained using the GRASP method, however the optimal assignments using the network flow formulation are faster to compute than those using GRASP.

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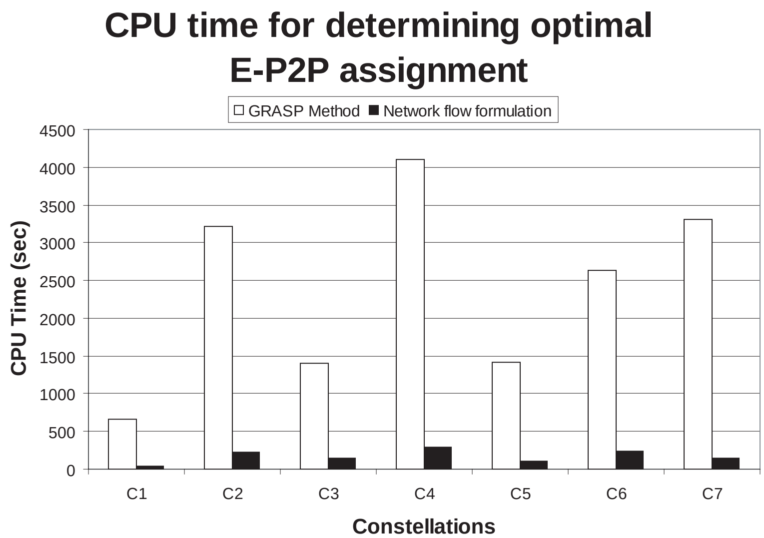

Figure 8: Comparison of CPU Time with GRASP method.

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