AAS 07-150

A GREEDY RANDOM ADAPTIVE SEARCH PROCEDURE FOR OPTIMAL SCHEDULING OF P2P SATELLITE REFUELING

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Abstract

All studies of peer-to-peer (henceforth abbreviated as P2P) satellite refueling so far assume that all active satellites return back to their original orbital positions after undergoing fuel exchanges with the passive satellites. In this paper, we remove this restriction on the active satellites, namely, we allow the active satellites to interchange orbital slots during the return trips. We formulate the problem as a three-index assignment problem in a tripartite constellation graph. We use a greedy random adaptive search procedure (GRASP) to determine the optimal P2P refueling schedule between the active and the passive satellites. It is shown that the proposed methodology leads to considerable fuel reduction over the baseline P2P refueling strategy.

INTRODUCTION

The lifespan of satellite constellations can be extended by periodic refueling of the satellites in the constellation. The traditional approach for satellite refueling involves a single service vehicle refueling all fuel-deficient satellites in the constellation in a sequential manner. Several studies have been performed on the problem of servicing multiple satellites in a constellation.[1] Recently, an alternative scenario for distributing fuel amongst a large number of satellites has been proposed.[2,3] In this scenario, no single spacecraft is in charge of the whole refueling process. Instead, all satellites share the responsibility of refueling each other on an equal footing. We call this the peer-to-peer (P2P) refueling strategy.[3] Studies have also been performed on mixed refueling strategies, in which the P2P refueling is an integral component.[4,5] It has been shown that a mixed refueling strategy is a competitive alternative to the single service vehicle refueling strategy, outperforming the latter for a large number of satellites in the constellation.[4] Additional extensions, namely, the coasting time allocation strategy and the asynchronous P2P refueling strategy,[5] have also been developed. These extensions result in considerable reduction of the fuel expenditure in mixed refueling scenarios.

In all of the above-mentioned studies, P2P refueling was perceived as a means to equalize fuel in the constellation. In order to achieve fuel equalization in the constellation, an optimization problem was formulated, such that the deviation of each satellite's fuel from the initial

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average fuel in the constellation is penalized. Under such a formulation, the problem of finding optimal pairings of satellites reduces to a problem of finding the maximum weighted matching in the so-called constellation graph. This maximum matching problem can be solved using standard methods.[6] A decentralized approach that uses auctions has also been reported in Ref 7.

An alternative formulation for the P2P refueling problem is to impose a minimum fuel requirement for each satellite in the constellation in order to remain operational. Satellites having the required amount of fuel are fuel-sufficient, while those which do not have the required amount of fuel are fuel-deficient. We therefore seek to determine the optimal pairings of satellites so that all satellites end up being fuel-sufficient at the end of the refueling process. This is to be achieved by using as little fuel as possible in the process. In all studies of P2P refueling so far, the active satellites have been constrained to return to their original positions. In this paper, we relax this constraint. Namely, we allow the active satellites to interchange their orbital positions in the constellation after the refueling of the passive satellites has been completed.

When the active satellites are constrained to return to their original orbital positions, the problem of P2P refueling can be solved as a two-index assignment problem on a bipartite graph, between the fuel-sufficient and fuel-deficient satellites. The assignment between the fuel-sufficient and fuel-deficient satellites completely determines the P2P maneuvers. In the current formulation of the P2P refueling problem, this is not the case however. Since the active satellites are allowed to interchange orbital slots during their return trip, the assignment between fuel-sufficient satellites and fuel-deficient satellites only determines the forward trip. We also need to assign each refueling transaction to an available orbital slot for the active satellite to return. We therefore need to consider three entities in order to completely define a P2P maneuver. These entities are the fuel-sufficient satellite, the fuel-deficient satellite, and the orbital slot the active satellite will return to. The P2P refueling problem can therefore be posed as a three-index assignment problem.

It is known that the three-index assignment problem is NP-complete.[8] The general multi-index assignment problem was first stated by Pierskalla[9] as an extension of the two-index assignment problem. The three dimensional assignment problem, which is a special case of the multi-index assignment problem, can be viewed as a matching problem on a complete tripartite graph. Several sub-optimal algorithms have been proposed for this problem. A branch-and-bound algorithm was proposed to solve the three-index assignment problem by Balas and Saltzman.[10] Approximation algorithms for three-index assignment problems with triangle inequalities were addressed by Crama and Spieksma.[11] For multi-index assignment problems in k-partite graphs with decomposable costs[§], Bandelt, Crama and Spieksma[12] introduced two approximate algorithms, each of which solves a sequence of two-index assignment problems. Another class of algorithms that has been developed for solving the three-index assignment problem includes the Greedy Random Adaptive Search Procedure (GRASP).[13–15] Feo and Resende[13] discussed GRASP as a means for solving general combinatorial optimization problems. Robertson[14] introduced four GRASP implementations for the multi-index assignment problem, which are combinations of two constructive methods (i.e., randomized reduced cost greedy, and randomized maximum maximum regret) and two local search methods (i.e.,

[§]By decomposable costs, we mean that the cost of a clique in the k-partite graph is a function of the cost of the edges induced by the clique. Note that a clique is a subgraph in which all vertices are pairwise adjacent. For a k-partite graph, a clique comprises of exactly one node from each partition of the k-partite graph.

two-assignment exchange, and variable depth exchange). Aiex et al.[15] proposed the use of GRASP with path relinking. This method was able to improve the quality of the heuristic solutions proposed in Refs. 10 and 12. Moreover, the GRASP method is shown to benefit from parallelization.

In this paper, we apply the GRASP method for solving the P2P refueling problem, under the assumption that the active satellites are allowed to interchange their orbital positions after they have refueled the passive satellites. However, since our problem deviates from the standard three-index assignment problem, we need to incorporate additional validation steps when we construct a basic feasible solution for an instance of our problem. These steps are explained in great detail later on in the section that discusses the three-index assignment problem.

PROBLEM FORMULATION

We consider a constellation with n satellites distributed over n orbital slots in a circular orbit. Let $\mathcal{S} = \{s_i : i = 0, 1, 2, \dots, n\}$ denote the set of satellites, where s_0 represents a fictitious satellite, and let $\Phi = \{\phi_i \in [0, 2\pi) : i = 1, 2, \dots, n,\ \phi_i \neq \phi_j\}$ be the set of orbital slots. We introduce a mapping $\sigma_t : \Phi \mapsto \mathcal{S}$ that, at time t, assigns to each orbital slot a satellite from $\mathcal{S}$. Specifically, $\sigma_t(\phi_j) = s_i$ implies that the satellite s_i occupies the orbital slot ϕ_j at time t. If the slot ϕ_j is empty at time t, we write $\sigma_t(\phi_j) = s_0$. We designate the constellation by the triplet $\mathcal{C} = \{\mathcal{S}, \Phi, \sigma_{t\geq 0}\}$. Let the fuel content of satellite s_i at time t be denoted by $f_{i,t}$ and let the minimum fuel content for satellite s_i in order to remain operational be denoted by $\underline{f}_i$. Finally, let the initial fuel content of satellite s_i be denoted by f_i^-, that is, $f_i^- = f_{i,0}$. Satellites having amount of fuel more than or equal to the amount required to remain operational are termed *fuel-sufficient*, while the ones having fuel less than the required amount are termed *fuel-deficient* satellites. In a P2P refueling strategy, the fuel-sufficient and the fuel-deficient satellites undergo fuel transactions amongst themselves, such that, at the end of refueling, each satellite has at least the required amount of fuel in order to remain operational.

It will be convenient to keep track of the indices of the satellites participating in the refueling process under different roles. To this end, let $\mathcal{I} = \{1, 2, \dots, n\}$, and let $\mathcal{I}_{s,t} = \{i : f_{i,t} \geq \underline{f}_i\}$ denote the index set of all fuel-sufficient satellites at time t, and let $\mathcal{I}_{d,t} = \{i : f_{i,t} < \underline{f}_i\}$ denote the index set of all fuel-deficient satellites at time t. In a P2P refueling transaction between a fuel-sufficient and a fuel-deficient satellite, one of them (henceforth referred to as the *active* satellite) performs an orbital transfer to rendezvous with the other satellite (henceforth referred to as the *passive* satellite). After a fuel exchange takes place between the active and the passive satellite, the active satellite returns to one of the available orbital slots. We will denote the index set of active satellites by $\mathcal{I}_a \subseteq \mathcal{I}$ and the index set of the passive satellites by $\mathcal{I}_p \subset \mathcal{I}$.

Our current problem formulation assumes that after refueling a passive satellite, each active satellite is allowed to return to any orbital slot that has been left vacant by another active satellite. For convenience, let $\mathcal{J}_{s,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{s,t}\}$ denote the index set of orbital slots occupied by fuel-sufficient satellites at time t, let $\mathcal{J}_{d,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{d,t}\}$ denote the index set of orbital slots occupied by fuel-deficient satellites at time t, and let $\mathcal{J}_a = \{j : \sigma_0(\phi_j) = s_i, i \in \mathcal{I}_a\}$ denote the index set of orbital slots occupied by the active satellites before any orbital maneuver commences. We therefore define a complete tripartite constellation graph $\mathcal{G}$ consisting of three partitions. The first partition consists of nodes that correspond to the elements of the index set $\mathcal{J}_{s,0}$. The second partition consists of nodes that correspond to the elements of the index set $\mathcal{J}_{d,0}$, and the third partition consists of nodes that correspond to the elements of the index set $\mathcal{J}_a$. Therefore, nodes of $\mathcal{G}$ are given by $\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} \cup \mathcal{J}_a$ and the edges of $\mathcal{G}$ are all edges induced by triplets in $\mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$, that is, $\mathcal{G} = \{\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} \cup \mathcal{J}_a, \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a\}$. Let us consider a triplet $(i, j, k) \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$. We say that the triplet (i, j, k) is *feasible* if the satellites $\sigma_0(\phi_i)$ and $\sigma_0(\phi_j)$ can engage in a *feasible* P2P refueling maneuver, that is, the active satellite (which can be either $\sigma_0(\phi_i)$ or $\sigma_0(\phi_j)$) rendezvous with the passive satellite, exchanges fuel, and then returns to the orbital slot initially occupied by the active satellite $\sigma_0(\phi_k)$, such that both $\sigma_0(\phi_i)$ and $\sigma_0(\phi_j)$ end up being fuel-sufficient at the end of the process. Let $\mathcal{T} \subseteq \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$ denote the set of all feasible triplets in the constellation graph $\mathcal{G}$.

Maneuver Costs and Feasible Triplets

Let us consider a triplet $(i, j, k) \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$ in the constellation graph $\mathcal{G}$. Also, let the satellite s_μ occupy the orbital slot ϕ_i at time $t = 0$ and the satellite s_ν occupy the orbital slot ϕ_j at time $t = 0$. Hence, $s_\mu = \sigma_0(\phi_i)$ and $s_\nu = \sigma_0(\phi_j)$. Without loss of generality, assume s_μ to be a fuel-sufficient satellite and s_ν to be a fuel-deficient satellite, that is, $\mu \in \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_{d,0}$. Either of the two satellites may be active during a refueling transaction between the two satellites. Hence, two different P2P refueling transactions are possible for the triplet (i, j, k).

In the first case, the fuel-sufficient satellite is active, that is, the satellite s_μ performs the orbital maneuver to rendezvous with the passive satellite s_ν. Therefore, $\mu \in \mathcal{I}_a \cap \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_p \cap \mathcal{I}_{d,0}$. After a fuel exchange takes place between the two satellites, s_μ performs another orbital maneuver and moves to the orbital slot ϕ_k initially occupied by the active satellite $\sigma_0(\phi_k)$. Note that $k \in \mathcal{J}_a$ and $k \neq j$. The fuel consumed by the active satellite s_μ to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j is given by:

$$p_{ij}^{\mu} = \left(m_{s\mu} + f_\mu^-\right)\left(1 - e^{-\frac{\Delta V_{ij}}{c_{0\mu}}}\right), \tag{1}$$

where $m_{s\mu}$ is the mass of the permanent structure of satellite s_μ, and ΔV_{ij} is the optimal velocity change required for a two-impulse transfer from the orbital slot ϕ_i to the orbital slot ϕ_j. The parameter $c_{0\mu}$ is defined by $c_{0\mu} = g_0 I_{sp\mu}$, where g_0 is the acceleration due to gravity at the Earth's surface and $I_{sp\mu}$ is the specific thrust of satellite s_μ. The fuel content of satellite s_μ after its forward trip (but before fuel exchange takes place) is $f_\mu^- - p_{ij}^\mu$. Since the fuel consumption during the maneuver is minimized if the active satellite returns with exactly the required minimum amount of fuel to remain operational, the amount of fuel consumed during the return trip, during which satellite s_μ travels from ϕ_j to ϕ_k, is given by

$$p_{jk}^{\mu} = \left(m_{s\mu} + \underline{f}_\mu\right) e^{\frac{\Delta V_{jk}}{c_{0\mu}}}\left(1 - e^{-\frac{\Delta V_{jk}}{c_{0\mu}}}\right), \tag{2}$$

where ΔV_{jk} is the optimal velocity change required for the transfer from the orbital slot ϕ_j to the orbital slot ϕ_k. Before the return trip (but after the fuel exchange takes place), the fuel on board satellite s_μ is $\underline{f}_\mu + p_{jk}^\mu$. The fuel transferred to satellite s_ν during the fuel exchange is $(f_\mu^- - p_{ij}^\mu) - (\underline{f}_\mu + p_{jk}^\mu)$, assuming that the satellite s_ν has enough fuel capacity to accommodate this amount of fuel. The fuel on board satellite s_ν after it is refueled is $f_\nu^- + (f_\mu^- - p_{ij}^\mu) - (\underline{f}_\mu + p_{jk}^\mu)$. In order for satellite s_ν to become fuel-sufficient after the fuel transaction, we must therefore have,

$$\left(f_\nu^- + f_\mu^-\right) - \left(\underline{f}_\mu + \underline{f}_\nu\right) \geq p_{ij}^\mu + p_{jk}^\mu. \tag{3}$$

If the above condition does not hold, then the P2P refueling transaction between s_μ and s_ν is not feasible. Also, if satellite s_μ does not have enough fuel to carry out the orbital transfer during the forward trip, that is, if $p_{ij}^\mu \geq f_\mu^-$, then the P2P refueling transaction is also not feasible. Let $c_1(i,j,k)$ denote the cost of a P2P maneuver for the case when the fuel-sufficient satellite is active. Then $c_1(i,j,k)$ is given by the sum of (1) and (2). We therefore have,

$$c_1(i,j,k) = \begin{cases} p_{ij}^\mu + p_{jk}^\mu, & \text{if } p_{ij}^\mu < f_\mu^- \text{ and } p_{ij}^\mu + p_{jk}^\mu \leq \left(f_\mu^- + f_\nu^-\right) - \left(\underline{f}_\mu + \underline{f}_\nu\right), \\ \infty, & \text{otherwise.} \end{cases} \tag{4}$$

In the second case, the fuel-deficient satellite is active, that is, satellite s_ν performs the orbital maneuver to rendezvous with the passive satellite s_μ. Therefore, $\mu \in \mathcal{I}_p \cap \mathcal{I}_{s,0}$ and $\nu \in \mathcal{I}_a \cap \mathcal{I}_{d,0}$. After a fuel exchange takes place between the two satellites, s_ν performs another orbital maneuver and travels to the orbital slot ϕ_k initially occupied by the active satellite $\sigma_0(\phi_k)$. Note that $k \in \mathcal{J}_a$ and $k \neq i$. The fuel consumed for the active satellite s_ν to transfer from the orbital slot ϕ_i to the orbital slot ϕ_j is given by

$$p_{ji}^\nu = \left(m_{\mathrm{s}\nu} + f_\nu^-\right)\left(1 - e^{-\frac{\Delta V_{ji}}{c_{0\nu}}}\right), \tag{5}$$

where $m_{\mathrm{s}\nu}$ is the mass of the permanent structure of satellite s_ν, and ΔV_{ji} is the optimal velocity required for the transfer from the orbital slot ϕ_j to the orbital slot ϕ_i. The fuel content of satellite s_ν after its forward trip (but before fuel exchange takes place), is $f_\nu^- - p_{ji}^\nu$. The amount of fuel consumed during the return trip, during which the satellite s_ν travels from the orbital slot ϕ_i to the orbital slot ϕ_k, is given by

$$p_{ik}^\nu = \left(m_{\mathrm{s}\nu} + \underline{f}_\nu\right) e^{\frac{\Delta V_{ik}}{c_{0\nu}}}\left(1 - e^{-\frac{\Delta V_{ik}}{c_{0\nu}}}\right), \tag{6}$$

where ΔV_{ik} is the optimal velocity change required for the transfer from the orbital slot ϕ_i to the orbital slot ϕ_k. Before the return trip (but after the fuel exchange takes place), the fuel on board satellite s_ν is $\underline{f}_\nu + p_{ik}^\nu$. The fuel transferred to satellite s_ν during the fuel exchange is $(\underline{f}_\nu + p_{ik}^\nu) - (f_\nu^- - p_{ji}^\nu)$. The fuel on board satellite s_μ after the fuel transaction is $f_\mu^- - (\underline{f}_\nu + p_{ik}^\nu) + (f_\nu^- - p_{ji}^\nu)$. In order for the satellite s_μ to be fuel-sufficient after the fuel transaction, we must have

$$\left(f_\mu^- + f_\nu^-\right) - \left(\underline{f}_\nu + \underline{f}_\mu\right) \geq p_{ji}^\nu + p_{ik}^\nu. \tag{7}$$

If the above condition does not hold, then a P2P refueling transaction between s_μ and s_ν is not feasible. Also, if the satellite s_ν does not have enough fuel to carry out the orbital transfer during the forward trip, that is, if $p_{ji}^\nu \geq f_\nu^-$, then the P2P refueling transaction is also not feasible. Let $c_2(i,j,k)$ denote the cost of a P2P maneuver for the case when the fuel-deficient satellite is active. Then, $c_2(i,j,k)$ is given by the sum of (5) and (6). We therefore have,

$$c_2(i,j,k) = \begin{cases} p_{ji}^\nu + p_{ik}^\nu, & \text{if } p_{ji}^\nu < f_\nu^- \text{ and } p_{ji}^\nu + p_{ik}^\nu \leq \left(f_\mu^- + f_\nu^-\right) - \left(\underline{f}_\nu + \underline{f}_\mu\right) \\ \infty, & \text{otherwise.} \end{cases} \tag{8}$$

Of the two possible P2P maneuvers associated with the triplet (i, j, k), the cheaper one is of interest to us. To this end, let the total fuel expenditure incurred in the P2P maneuver associated with the triplet (i, j, k) be given by

$$c(i,j,k) = \begin{cases} c_1(i,j,k), & \text{if } c_1(i,j,k) \leq c_2(i,j,k) \\ c_2(i,j,k), & \text{otherwise.} \end{cases} \tag{9}$$

We therefore associate with each triplet $(i, j, k) \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a$ a single P2P maneuver. The set of all feasible triplets is then defined as $\mathcal{T} = \{(i, j, k) \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0} \times \mathcal{J}_a : c(i, j, k) < \infty\}$. Let now $\mathsf{Act} : \mathcal{T} \mapsto \mathcal{I}$ be a function that returns the index of the orbital slot of the active satellite, that is,

$$\mathsf{Act}(i,j,k) = \begin{cases} i, & \text{if } c_1(i,j,k) \leq c_2(i,j,k) \\ j, & \text{otherwise.} \end{cases} \tag{10}$$

Similarly, let $\mathsf{Pas} : \mathcal{T} \mapsto \mathcal{I}$ be a function that returns the index of the orbital slot of the passive satellite, that is,

$$\mathsf{Pas}(i,j,k) = \begin{cases} j, & \text{if } c_1(i,j,k) \leq c_2(i,j,k) \\ i, & \text{otherwise.} \end{cases} \tag{11}$$

Moreover, the edges induced by the triplets that are not feasible can be removed from the graph $\mathcal{G}$ in order to yield a *reduced constellation graph* $\mathcal{G}_r$. Therefore, $\mathcal{G}_r = \{\mathcal{J}_{s,0} \cup \mathcal{J}_{d,0} \cup \mathcal{J}_a, \mathcal{T}\}$. Henceforth, we restrict our discussion to the reduced constellation graph $\mathcal{G}_r$.

Using equations (1), (2), (5) and (6), we can ascertain the cost of a triplet $(i, j, k) \in \mathcal{T}$ using (9). Notice that the calculation of the optimal costs $\Delta V_{ij}, \Delta V_{ji}$, ΔV_{jk} and ΔV_{ki} in Equations (1), (2), (5) and (6) requires, in general, the solution of the two-impulse multi-revolution Lambert problem.[16]

The Three-Index Assignment Problem

Since our goal is to refuel all fuel-deficient satellites, each of them should be part of a feasible fuel transaction. We therefore seek a set of exactly $|\mathcal{I}_{d,0}|$ feasible triplets $\mathcal{M}^* \subseteq \mathcal{T}$ in the reduced constellation graph $\mathcal{G}_r$ such that none of the triplets in $\mathcal{M}^*$ share a common vertex or a common edge, and such that the sum of the costs of all these triplets is minimum.

To this end, let $\mathcal{M} \subseteq \mathcal{T}$ be a set that consists of $|\mathcal{I}_{d,0}|$ triplets. To each triplet $(i, j, k) \in \mathcal{T}$ we associate the binary variable x_{ijk} as follows

$$x_{ijk} = \begin{cases} 1, & \text{if } (i,j,k) \in \mathcal{M}, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

We can therefore formulate the problem of finding the set of feasible triplets $\mathcal{M} \subseteq \mathcal{T}$ that yield the minimum cost as follows

$$\min_{\mathcal{M} \subseteq \mathcal{T}} \sum_{(i,j,k) \in \mathcal{M}} c(i,j,k)\, x_{ijk}, \tag{13}$$

such that

$$\sum_{j \in \mathcal{J}_{d,0}} \sum_{k \in \mathcal{J}_a} x_{ijk} \leq 1, \text{ for all } i \in \mathcal{J}_{s,0}, \tag{14}$$

$$\sum_{i\in\mathcal{J}_{s,0}}\sum_{k\in\mathcal{J}_a} x_{ijk} = 1, \text{ for all } j\in\mathcal{J}_{d,0}, \tag{15}$$

$$\sum_{i\in\mathcal{J}_{s,0}}\sum_{j\in\mathcal{J}_{d,0}} x_{ijk} \leq 1, \text{ for all } k\in\mathcal{J}_a, \tag{16}$$

$$r \neq \mathsf{Pas}(i,j,k) \text{ for all } (i,j,k)\,,(p,q,r)\in\mathcal{M}. \tag{17}$$

Constraint (14) signifies that not all fuel-sufficient satellites have to be part of P2P refueling transactions, because we may have $|\mathcal{I}_{s,0}| > |\mathcal{I}_{d,0}|$. Constraint (15) implies that each fuel-deficient satellite must be part of exactly one P2P fuel transaction. Constraint (16) signifies that each of the slots left vacant by the active satellites needs to be assigned to a P2P refueling transaction. Note that the set of active satellites is not known a priori. We only know that $\mathcal{J}_a \subset \mathcal{I}$. For solving our problem, we use $\mathcal{J}_a = \mathcal{I}$ for the third partition of the constellation. Therefore, not all nodes of the third partition correspond to orbital slots of active satellites. Hence, the inequality sign in the constraint (16). Constraint (17) implies that the return orbital slot for the active satellite in a P2P maneuver cannot be the orbital slot of a passive satellite of a different P2P maneuver. To illustrate this, let us consider two triplets $(i,j,k)\in\mathcal{M}$ and $(p,q,r)\in\mathcal{M}$ such that $i\neq p$, $j\neq q$, $k\neq r$. Without loss of generality, assume $\mathsf{Act}(i,j,k)=i$ and $\mathsf{Act}(p,q,r)=p$. If $r=j$, then the fuel-sufficient satellite $\sigma_0(\phi_p)$ initially occupying orbital slot ϕ_p returns to the orbital slot ϕ_j. However, the orbital slot ϕ_j is not vacant because the satellite $\sigma_0(\phi_j)$ is passive and never leaves its slot. Constraint (17) avoids such infeasible cases. A set of triplets $\mathcal{M}\subseteq\mathcal{T}$ satisfying (12), (14)-(17) will be referred to as a *basic feasible solution* for our problem.

It should be mentioned at this point that a few differences emerge between our problem and the standard three-index assignment problem (AP3) discussed in Refs. 9–12, 14, 15. First, the AP3 is a matching problem in a complete tripartite graph, whose partitions have the same number of nodes. In the case of the constellation graph $\mathcal{G}$ or the reduced constellation graph $\mathcal{G}_r$, the three partitions do not have the same number of nodes. Secondly, in our problem, we have additional constraints given in (17), which need to be accounted for whenever a basic feasible solution is considered. Nonetheless, in our problem we can readily construct one basic feasible solution without solving a three-index assignment problem. This solution is obtained by solving the P2P refueling problem, while constraining the active satellites to return to their orbital slots after refueling. This problem can be solved as a two-index assignment problem.[3,5,7]

GREEDY RANDOM ADAPTIVE SEARCH PROCEDURE

In this section, we use a Greedy Random Adaptive Search Procedure to solve the three-index assignment problem, while taking into account the additional constraints in (17). The GRASP has been used to solve the standard AP3, and primarily consists of two phases: a construction phase that builds a basic feasible solution, and a local search phase that locates a solution in the neighborhood of the basic feasible solution with a lower cost. Reference 14 discusses two variants of implementing the construction phase (randomized greedy, maximum regret) as well as two variants of implementing the local search phase (two-exchange neighborhood search, variable depth exchange). We will use the randomized greedy method in the construction phase in order to generate a basic feasible solution, and we will perform a local search using a two-exchange neighborhood.

Construction of a Basic Feasible Solution

The construction phase iteratively builds a feasible solution $\mathcal{M}$ by selecting $|\mathcal{I}_{d,0}|$ triplets, one at a time, from a list L of eligible triplets from $\mathcal{T}$. The list L initially consists of all triplets in the reduced constellation graph $\mathcal{G}_r$, that is, $L = \mathcal{T}$, because initially all triplets are eligible for selection during the construction of $\mathcal{M}$.

Let $\mathcal{M}_\ell$ denote the constructed solution after the ℓth iteration, where $\ell \leq |\mathcal{I}_{d,0}|$. Initially $M_0 = \varnothing$. Assume $p-1 < |\mathcal{I}_{d,0}|$ triplets have been added after $p-1$ iterations, so the current constructed solution is denoted by $\mathcal{M}_{p-1} = \{(i_\ell, j_\ell, k_\ell) : \ell = 1, 2, \dots, p-1\}$. The pth triplet needs to be added to $\mathcal{M}_{p-1}$.

A parameter η, known as restricted candidate list parameter, is selected at random from the interval $[0, 1]$ and is used to form a list L_r called the *restricted candidate list* that comprises of the best (in terms of lower cost) candidate triplets available for selection during the current iteration step. The restricted candidate list $L_r \subseteq L$ is defined as

$$L_r = \{(i,j,k) \in L : c\,(i,j,k) \leq \underline{c} + \eta(\bar{c} - \underline{c})\}, \tag{18}$$

where $\underline{c}$ and $\bar{c}$ are given by

$$\underline{c} = \min_{(i,j,k)\in L} c\,(i,j,k) \;\text{ and }\; \bar{c} = \max_{(i,j,k)\in L} c\,(i,j,k)\,. \tag{19}$$

The definition of the restricted candidate list given in (18) shows the greedy nature of the algorithm. Only triplets in L having cost less than $\underline{c} + \eta\,(\bar{c} - \underline{c})$ are made eligible for selection. At the pth step the triplet (i_p, j_p, k_p) is chosen at random from L_r, provided it does not violate (17), that is,

$$k_p \neq \mathsf{Pas}(i_\ell, j_\ell, k_\ell) \text{ for all } \ell = 1, 2, \dots, p-1, \tag{20}$$

and

$$k_\ell \neq \mathsf{Pas}(i_p, j_p, k_p) \text{ for all } \ell = 1, 2, \dots, p-1. \tag{21}$$

Equation (20) implies that the return orbital slot corresponding to the triplet (i_p, j_p, k_p) cannot be the orbital slot of a passive satellite corresponding to any of the triplets in $\mathcal{M}_{p-1}$, and equation (21) implies that the orbital slot of the passive satellite corresponding to the triplet (i_p, j_p, k_p) cannot be the returning orbital slot corresponding to any of the triplets in $\mathcal{M}_{p-1}$. Once the pth triplet is selected, the set of candidate triplets L must be adjusted to take into account that (i_p, j_p, k_p) is now part of the solution. Therefore, any triplet $(i,j,k) \in L$ with $i = i_p$ or $j = j_p$ or $k = k_p$ is removed from L because any such triplet cannot be selected in the future; otherwise at least one of the constraints (14), (15), or (16) will be violated. Subsequently, the list L is updated accordingly. Finally, $\mathcal{M}_p = \mathcal{M}_{p-1} \cup (i_p, j_p, k_p)$.

The adaptive nature of the GRASP method is due to the fact that once a triplet from L_r is selected for addition to $\mathcal{M}_{p-1}$, all triplets that are made ineligible for addition to $\mathcal{M}_{p+1}$ are removed from L. The probabilistic nature of the algorithm arises from the use of the random parameter η and the random selection of a triplet from the restricted candidate list. In the most simple implementation of the algorithm the value of η is not changed during the construction phase.

Local Search

In the local search phase, the feasible solution from the construction phase is improved upon by searching its neighborhood for a better solution. If an improvement is detected, the solution is updated and a new neighborhood search is initialized. The definition of the neighborhood $\mathcal{N}(\mathcal{M})$ of $\mathcal{M}$ is crucial for the performance of the local search. Here we use the 2-exchange neighborhood suggested in Ref 15. Recall that the basic feasible solution generated by the construction phase consists of $|\mathcal{I}_{d,0}|$ triplets. For convenience, let us denote the triplet (i_ℓ, j_ℓ, k_ℓ) by t_ℓ. Let also $\mathcal{D} = \{1, 2, \ldots, |\mathcal{I}_{d,0}|\}$ denote the index set of triplets in $\mathcal{M}$. We can therefore write $\mathcal{M} = \{t_\ell : \ell \in \mathcal{D}\}$. We will denote the difference between $t_p, t_q \in \mathcal{M}$ by

$$\delta(t_p, t_q) = \{r : t_{p,r} \neq t_{q,r},\ r = 1, 2, 3\}. \tag{22}$$

The distance between the triplets t_p and t_q is then defined as

$$d(t_p, t_q) = |\delta(t_p, t_q)|. \tag{23}$$

Using (23), we can define the 2-exchange neighborhood of the triplet pair $(t_p, t_q) \in \mathcal{M} \times \mathcal{M}$ as

$$N_2(t_p, t_q) = \{(\tau, \sigma) \in \mathcal{M} \times \mathcal{M} : d(t_p, \tau) + d(t_q, \sigma) = 2\}. \tag{24}$$

The neighborhood of the solution $\mathcal{M}$ consists of the union of 2-exchange neighborhoods of all possible triplet pairs $(t_p, t_q) \in \mathcal{M}$, that is,

$$\mathcal{N}(\mathcal{M}) = \bigcup_{(t_p, t_q) \in \mathcal{M}} N_2(t_p, t_q). \tag{25}$$

During the local search phase the cost of each $\mathcal{M}' \in \mathcal{N}(\mathcal{M})$ (validated with respect to the constraints as in (17)) is compared with the cost of $\mathcal{M}$. If the cost is lower, then the current search is halted, and a search around the neighborhood of $\mathcal{M}'$ is initialized. The local search ends when no neighbor of the current solution has a lower cost.

The successive application of the construction phase and the local search phase may generate several local minima. The procedure halts either after the maximum number of iterations is reached, or if a local minimum with a value less than or equal to some pre-specified value is found.

Path Relinking

Path relinking is used to improve the quality of the solution generated by the construction and local search phases. Path relinking consists of exploring a trajectory that connects two local minima. Let us consider one of the two local minima as the initial solution and the other one as the guiding solution. A path is generated by selecting steps that introduce in the initial solution attributes of the guiding solution. Each step in the generation of the trajectory is said to be a *move*. Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be two local minima generated by the previous phases of the GRASP procedure. Let $\mathcal{M}_1 = \{t_\ell : \ell \in \mathcal{D}\}$ be the initial solution and $\mathcal{M}_2 = \{g_\ell : \ell \in \mathcal{D}\}$ be the guiding solution. Without loss of generality, we may assume $t_{\ell,2} = g_{\ell,2}$ for all $\ell \in \mathcal{D}$ because orbital slots of all fuel-deficient satellites need to be in any basic feasible solution. The symmetric difference between $\mathcal{M}_1$ and $\mathcal{M}_2$ is given by the following expressions

$$\Delta_1 = \{\ell \in \mathcal{D} : t_{\ell,1} \neq g_{\ell,1},\ t_\ell \in \mathcal{M}_1, g_\ell \in \mathcal{M}_2\}, \tag{26}$$

$$\Delta_3 = \{\ell \in \mathcal{D} : t_{\ell,3} \neq g_{\ell,3},\ t_\ell \in \mathcal{M}_1, g_\ell \in \mathcal{M}_2\}. \tag{27}$$

An intermediate solution on the path from $\mathcal{M}_1$ to $\mathcal{M}_2$ can be generated by making two kinds of moves. The sets Δ_1 and Δ_3 are used to guide such moves. Let $\mathcal{M} = \{h_\ell : \ell \in \mathcal{D}\}$ be an intermediate solution such that $t_{\ell,2} = h_{\ell,2}$ for all $\ell \in \mathcal{D}$. Initially, $\mathcal{M} = \mathcal{M}_1$. Let $r \in \Delta_1$. Also, let $h_{q,1} = g_{r,1}$ for some $q \in \mathcal{D}$. Then, in one kind of move, triplets h_r and h_q are replaced by $(h_{q,1}, h_{r,2}, h_{r,3})$ and $(h_{r,1}, h_{q,2}, h_{q,3})$ respectively in $\mathcal{M}$. The new solution generated is

$$\mathcal{M}' = \mathcal{M} \cup \{(h_{q,1}, h_{r,2}, h_{r,3}), (h_{r,1}, h_{q,2}, h_{q,3})\} \backslash \{h_r, h_q\}. \tag{28}$$

Once such a move is made, r is removed from Δ_1. Similarly, let $r \in \Delta_3$ and $h_{q,3} = g_{r,3}$ for some $q \in \mathcal{D}$. In the other kind of move, triplets h_r and h_q are replaced by $(h_{r,1}, h_{r,2}, h_{q,3})$ and $(h_{q,1}, h_{q,2}, h_{r,3})$ respectively in $\mathcal{M}$. The new solution generated is

$$\mathcal{M}'' = \mathcal{M} \cup \{(h_{r,1}, h_{r,2}, h_{q,3}), (h_{q,1}, h_{q,2}, h_{r,3})\} \backslash \{h_r, h_q\}. \tag{29}$$

Once such a move is made, r is removed from Δ_3. Provided a move generates a basic feasible solution (validated with respect to constraints as in (17)), the cost of the solution $\mathcal{M}'$ or $\mathcal{M}''$ is compared to $\mathcal{M}_1$ and $\mathcal{M}_2$. If the cost of either $\mathcal{M}'$ or $\mathcal{M}''$ is found to be lower than $\mathcal{M}$, then a local search is performed in its neighborhood to yield a better solution.

NUMERICAL RESULTS

In this section, we apply the GRASP method with path relinking in order to determine the optimal assignments required for P2P refueling of sample constellations when the active satellites are not restricted to return to their original orbital slots. We also compare the results against the baseline P2P cases, namely when the active satellites are constrained to return to their original orbital slots. With the help of numerical examples, we show how the removal of such a restriction leads to considerable reduction in the fuel expenditure required for the refueling process to be completed. In all examples, we run the GRASP procedure $10,000$ times in order to determine the optimal assignments.

Example 1.

We consider a circular constellation of 10 satellites evenly distributed in a circular orbit. The maximum allowed time for refueling is $T = 12$ orbital periods. The constellation details are given in Table 1. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 12$ units, permanent structure of $m_{si} = 70$, and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 2, 8, 9, 10\}$ and those of the fuel-deficient satelites are $\mathcal{I}_{d,0} = \{3, 4, 5, 6, 7\}$. For the baseline P2P refueling strategy, the optimal pairings are $s_4 \to s_1$, $s_5 \to s_2$, $s_7 \to s_8$, $s_6 \to s_9$, $s_3 \to s_{10}$, and the total fuel consumption for all P2P maneuvers is 26.07 units. This represents 14.48% of the total initial fuel in the constellation. The indices of the active satellites in this case are $\mathcal{I}_a = \{3, 4, 5, 6, 7\}$. Note that $\mathcal{I}_a = \mathcal{I}_{d,0}$, that is, the fuel-deficient satellites are the active ones for the baseline P2P refueling strategy. A fuel-deficient satellite has smaller mass compared to a fuel-sufficient satellite and thus uses a smaller amount of fuel during an orbital transfer. Hence, during a refueling maneuver between a fuel-sufficient and a fuel-deficient satellite, the fuel-deficient satellite is more likely to be the active satellite.

Table 1: Constellation C_1.

i	Satellites	Orbital Position (deg)	Initial fuel content
1	s_1	$\phi_1 = 0$	$f_1^- = 30$
2	s_2	$\phi_2 = 36$	$f_2^- = 30$
3	s_3	$\phi_3 = 72$	$f_3^- = 6$
4	s_4	$\phi_4 = 108$	$f_4^- = 6$
5	s_5	$\phi_5 = 144$	$f_5^- = 6$
6	s_6	$\phi_6 = 180$	$f_6^- = 6$
7	s_7	$\phi_7 = 216$	$f_7^- = 6$
8	s_8	$\phi_8 = 252$	$f_8^- = 30$
9	s_9	$\phi_9 = 288$	$f_9^- = 30$
10	s_{10}	$\phi_{10} = 324$	$f_{10}^- = 30$

For the case in which the active satellites are allowed to interchange their orbital slots, the optimal assignment for P2P refueling, determined from GRASP, is $s_8 \rightarrow s_7 \rightarrow s_6$, $s_6 \rightarrow s_9 \rightarrow s_8$, $s_3 \rightarrow s_{10} \rightarrow s_1$, $s_1 \rightarrow s_4 \rightarrow s_5$, $s_5 \rightarrow s_2 \rightarrow s_3$. The fuel expenditure during the refueling process is 18.73 units, which is less than the fuel expenditure for the baseline P2P case. This represents 10.41% of the total initial fuel in the constellation, or an improvement of 28% over the standard P2P scenario. Figure 1 shows the constellation and the optimal assignments. The active satellites are marked by '$\star$'.

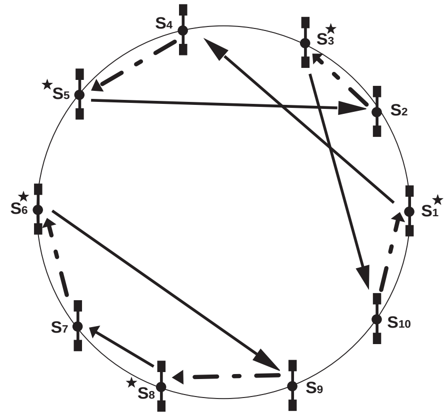

Figure 1: Constellation for Example 1.

The forward trips are marked by solid arrows, while the return trips are marked by dotted arrows. In the optimal assignment produced by the GRASP method, it is observed that each active satellite, after undergoing a fuel transaction with the corresponding passive satellite, returns to an available orbital slot in the vicinity of the passive satellite with which it was involved in the transaction. For instance, satellite s_1 undergoes a fuel transaction with the satellite s_4 and then returns to the orbital slot initially occupied by active satellite s_5. Moving to an orbital slot in the vicinity involves an orbital transfer through a smaller transfer angle and thereby results in a likely lesser fuel expenditure during the return trip. Hence, the active satellites, having the freedom to return to any available orbital slot, opt to move to a nearby

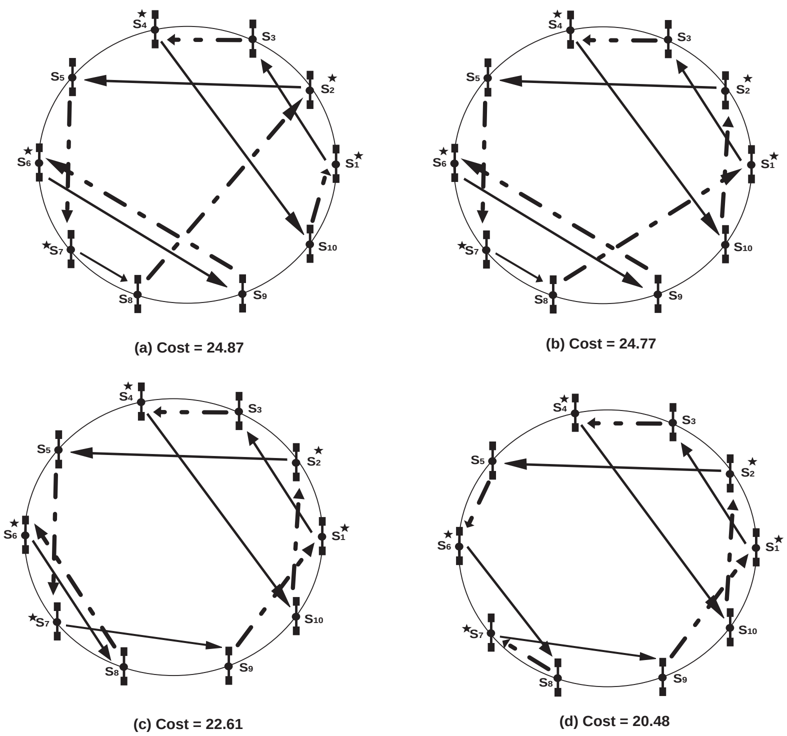

Figure 2: Assignments obtained during local searches of the GRASP method.

one during the return trip. In the baseline P2P strategy, such freedom is not available, and some of the active satellites have to perform orbital transfers that incur higher cost. Another observation is the fact that some of the active satellites are also fuel-sufficient. Note that satellites s_1 and s_8 are fuel-sufficient and active. For this problem, by having some fuel-sufficient satellites as the active satellites, it is ensured that all active satellites are able to return to the nearest orbital slot, thereby saving fuel during the return trip.

Figure 2 depicts a basic feasible solution generated by the GRASP method along with a local search performed about this solution. Figure 2(a) is the basic feasible solution and corresponds to the assignment $s_4 \to s_{10} \to s_1$, $s_1 \to s_3 \to s_4$, $s_7 \to s_8 \to s_2$, $s_2 \to s_5 \to s_7$, $s_6 \to s_9 \to s_6$. The cost of this assignment is 24.87 units of fuel. By performing a search in the neighborhood of this solution, another assignment of lower cost, shown in Figure 2(b) is obtained. In this assignment, satellite s_4 returns to the orbital slot initially occupied by s_2 instead of the orbital slot initially occupied by s_1, while satellite s_7 returns to the orbital slot initially occupied by s_1 instead of returning to the orbital slot initially occupied by s_2. The cost of this assignment is 24.77 units of fuel. A local search performed in the neighborhood of this solution yields the assignment shown in Figure 2 (c). In this assignment, satellite s_7 rendezvous with s_9 instead of s_8, while satellite s_6 rendezvous with s_8 instead of s_9. The cost of this assignment is 22.61 units of fuel. A search in the neighborhood of this solution now yields yet another assignment shown in Figure 2(d). In this assignment, satellite s_2 returns to the orbital slot initially occupied by s_6 instead of the orbital slot initially occupied by s_7, while satellite s_6 returns to the orbital slot initially occupied by s_7 instead of returning to its original orbital slot. The cost of this solution is 20.48. Finally, a local search in the neighborhood of this

Table 2: CONSTELLATION C_2.

i	Satellites	Orbital Position	Initial fuel content
1	s_1	$\phi_1 = 0$	$f_1^- = 30$
2	s_2	$\phi_2 = 22.5$	$f_2^- = 30$
3	s_3	$\phi_3 = 45$	$f_3^- = 30$
4	s_4	$\phi_4 = 67.5$	$f_4^- = 30$
5	s_5	$\phi_5 = 90$	$f_5^- = 30$
6	s_6	$\phi_6 = 112.5$	$f_6^- = 30$
7	s_7	$\phi_7 = 135$	$f_7^- = 10$
8	s_8	$\phi_8 = 157.5$	$f_8^- = 10$
9	s_9	$\phi_9 = 180$	$f_9^- = 10$
10	s_{10}	$\phi_{10} = 202.5$	$f_{10}^- = 10$
11	s_{11}	$\phi_{11} = 225$	$f_{11}^- = 10$
12	s_{12}	$\phi_{12} = 247.5$	$f_{12}^- = 10$
13	s_{13}	$\phi_{13} = 270$	$f_{13}^- = 10$
14	s_{14}	$\phi_{14} = 292.5$	$f_{14}^- = 10$
15	s_{15}	$\phi_{15} = 315$	$f_{15}^- = 30$
16	s_{16}	$\phi_{16} = 337.5$	$f_{16}^- = 30$

solution yields no other cheaper solution, thereby implying that the assignment in Figure 2(d) is a local minimum.

Example 2.

In this example we consider a circular constellation of 16 satellites, evenly distributed in a circular orbit. The maximum allowable time for refueling is $T = 30$ orbital periods. The constellation details are given in Table 2. Each satellite s_i has a minimum fuel requirement of $\underline{f}_i = 15$ units, permanent structure of $m_{si} = 70$ units and a characteristic constant of $c_{0i} = 2943$ m/s. The indices of the fuel-sufficient satellites are $\mathcal{I}_{s,0} = \{1, 2, 3, 4, 5, 6, 15, 16\}$, while those of the fuel-deficient satellites are $\mathcal{I}_{d,0} = \{7, 8, 9, 10, 11, 12, 13, 14\}$. If the active satellites are constrained to return to their original orbital slots after refueling, then the optimal pairings are $s_{11} \to s_1$, $s_{12} \to s_2$, $s_9 \to s_3$, $s_7 \to s_4$, $s_8 \to s_5$, $s_{10} \to s_6$, $s_{13} \to s_{15}$, $s_{14} \to s_{16}$, and the total fuel consumption during the P2P maneuvers is 37.46 units. This represents 11.71% of the total initial fuel in the constellation. For all the pairings in this case, only the fuel-deficient satellites are the active ones, that is, $\mathcal{I}_a = \{7, 8, 9, 10, 11, 12, 13, 14\} = \mathcal{I}_{d,0}$. This is similar to the previous example.

If the active satellites are allowed to interchange orbital slots, then the optimal assignment for the P2P refueling problem, as determined by the GRASP method, are $s_1 \to s_{12} \to s_{13}$, $s_3 \to s_7 \to s_6$, $s_5 \to s_8 \to s_9$, $s_6 \to s_{10} \to s_{11}$, $s_9 \to s_4 \to s_5$, $s_{11} \to s_{15} \to s_{14}$, $s_{13} \to s_{16} \to s_1$, $s_{14} \to s_2 \to s_3$. Here, $\mathcal{I}_a = \{1, 3, 5, 6, 9, 11, 13, 14\}$. Relaxing the return orbital position constraint reduces the fuel expenditure to 24.82 units. This represents 7.76% of the total initial fuel in the constellation or an improvement of 33% over the standard P2P scenario. Figure 3 shows the constellation and the optimal assignments. The active satellites are marked by '$\star$'. Similar to Example 1, it is observed that the active satellites, after undergoing fuel transactions with the corresponding passive satellites, return to an available orbital slot in their vicinity. For instance, satellite s_1 undergoes a fuel transaction with satellite s_{12} and returns to the orbital slot occupied by active satellite s_{13}. Also, the active satellites include fuel-sufficient

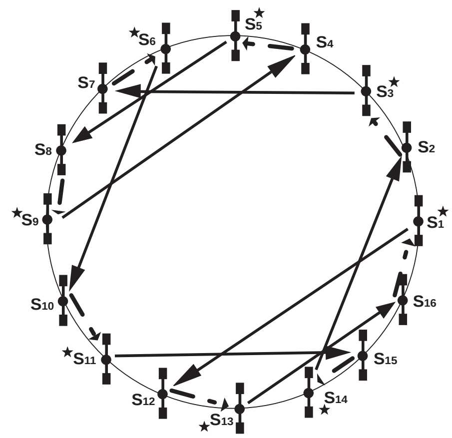

Figure 3: Constellation for Example 2.

ones. Here s_1, s_3, s_5 and s_6 are fuel-sufficient and active.

We have also tested the proposed methodology on several sample constellations. Table 3 gives the description of a few of these sample constellations (C_1 and C_2 are the constellations already described in Examples 1 and 2, respectively). The optimal assignments for these constellations show considerable reduction in fuel consumption against the baseline P2P strategy. For instance, for constellation C_3 the baseline P2P refueling strategy yields the optimal assignment $s_4 \to s_1$, $s_5 \to s_2$, $s_7 \to s_{10}$, $s_6 \to s_3$, $s_{11} \to s_8$, $s_9 \to s_{12}$ with a fuel expenditure of 26.73 units, with the fuel-deficient satellites being the active ones. Our proposed methodology yields the optimal assignment $s_9 \to s_{12} \to s_{11}$, $s_{11} \to s_8 \to s_7$, $s_4 \to s_1 \to s_2$, $s_2 \to s_5 \to s_6$, $s_6 \to s_3 \to s_4$, $s_7 \to s_{10} \to s_9$, that reduces the fuel expenditure to 18.51 units. Similarly, for

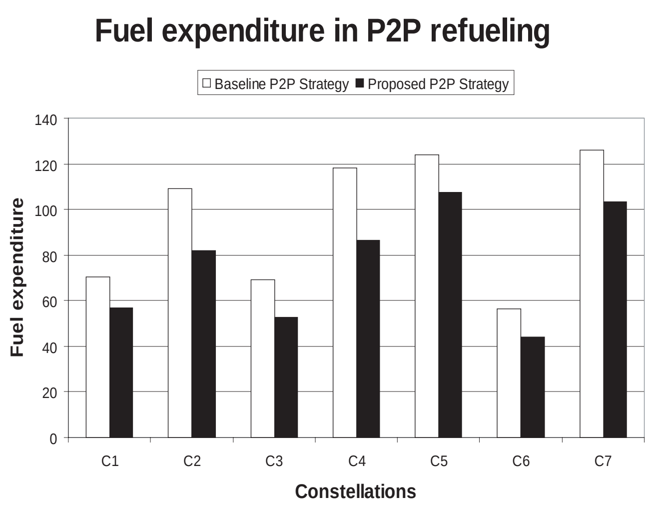

Figure 4: Comparison of results for several sample constellations.

the other constellations the fuel expenses reduce from 41.06 units to 26.26 units in case of C_4, from 28.38 to 18.51 in case of C_5, from 28.77 units to 19.21 units in case with C_6, and from 34.97 units to 22.57 units in the case of C_7. Figure 4 summarizes these observations.

CONCLUSIONS

In this paper, we have studied the peer-to-peer satellite refueling problem under the assumption that the active satellites are allowed to interchange orbital positions during their return trips. The problem is formulated as a three-index assignment problem on a tripartite constellation graph. The problem is shown to deviate from the standard three-index assignment problem (AP3) studied in the literature owing to additional constraints imposed by the nature of the problem. We have used a Greedy Random Adaptive Search Procedure (GRASP) in order to find the optimal assignments. With the help of numerical examples, it is shown that our proposed P2P refueling strategy, which allows active satellites to interchange orbital positions during their return trips, results in considerable reduction in the fuel expenditure incurred during the refueling process. It is also shown that each active satellite opts to return to an available orbital slot in the vicinity of the passive satellite with which it is involved in a fuel transaction.

Table 3: Sample Constellations.

Label	Description
C_3	12 satellites, Altitude $= 2,000$ Km, $T = 30$ f_i^-: $30, 30, 30, 10, 10, 10, 10, 10, 10, 30, 30, 30$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites
C_4	18 satellites, Altitude $= 6,000$ Km, $T = 25$ f_i^-: $25, 25, 25, 25, 25, 25, 25, 25, 25, 6, 6, 6, 6, 6, 6, 6, 6, 6$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_5	12 satellites, Altitude $= 12,000$ Km, $T = 20$ f_i^-: $25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_6	14 satellites, Altitude $= 1,400$ Km, $T = 35$ f_i^-: $25, 25, 25, 25, 25, 25, 25, 8, 8, 8, 8, 8, 8, 8$ $\bar{f}_i = 25$, $\underline{f}_i = 12$, $m_{si} = 75$ for all satellites
C_7	16 satellites, Altitude $= 30,000$ Km, $T = 15$ f_i^-: $10, 10, 10, 10, 10, 10, 10, 10, 28, 28, 28, 28, 28, 28, 28, 28$ $\bar{f}_i = 30$, $\underline{f}_i = 15$, $m_{si} = 70$ for all satellites

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