Optimal Scheduling for Servicing Multiple Satellites in a Circular Constellation

Haijun Shen* and Panagiotis Tsiotras†
Georgia Institute of Technology
Atlanta, GA 30332-0150, USA

This paper studies the scheduling of servicing multiple satellites in a circular orbit. Specifically, one servicing spacecraft (SSc) is considered to be initially on the circular orbit of the satellites to be serviced. The SSc then rendezvous with each satellite of the constellation and services it until all satellites are visited. The total time is assumed to be given. A minimum-ΔV two-impulse maneuver is used for each rendezvous. The objective is to find the best sequence with the minimum total ΔV to service all satellites in the constellation. Two problems are considered. In the first case, the SSc returns to its starting circular orbital slot. In the second case, the SSc is not required to return to its starting orbital slot. These two servicing scheduling problems are formulated as combinatorial optimization problems, and are solved in a two-step process. First, the optimal time distribution problem is solved using integer programming, which yields the minimum cost maneuver for the SSc to visit the satellites in a given order. Then the optimal sequence problem is solved by a heuristic study. It is shown that integer programming is an effective scheme in solving the optimal time distribution problem. The notion of the sweep angle is introduced for each rendezvous segment as the smaller angle along the circular arc between the SSc and the next satellite to rendezvous. The Total Sweep Angle (TSA) for a servicing sequence is defined as the sum of the sweep angles for all rendezvous segments. The heuristic study shows that the best servicing sequence is always among the group of sequences that assume the minimum TSA. Specifically, for the case when the SSc returns to its starting orbital slot, the best servicing sequence is sequential (orbit-wise or counter-orbit-wise). For the case when the SSc does not return to its original orbital slot, the best sequence is sequential or partially sequential, depending on the satellite distribution on the constellation. The group of sequences with the minimum TSA are completely identified.

Introduction

Reliability and cost are two major concerns in space operations, in addition to fulfilling the mission objectives. An increasingly popular practice involves launching multiple small, simple spacecraft into low earth orbit (LEO) in order to achieve a certain task which would have otherwise been achieved by one large and complex spacecraft typically operating in geosynchronous orbit (GEO). This often reduces launching and maintenance cost. As a result, there is an increasing number of satellites orbiting in LEO. However, operating such large number of satellites brings new challenges to the space industry. One of the potential challenge lies in the area of on-orbit servicing of satellites. The satellite servicing can range from repairing faulty hardware, upgrading the operating systems, to refueling the satellites, etc. Being able to service the satellites while on orbit can dramatically cut the cost associated with replacing the ill-fated satellites with new ones.

*Ph.D. candidate, School of Aerospace Engineering, Georgia Institute of Technology. Email: haijun_shen@ae.gatech.edu. Student member AIAA.

†Associate Professor, School of Aerospace Engineering, Georgia Institute of Technology. Tel: (404) 894-9526, Fax: (404) 894-2760, Email: p.tsiotras@ae.gatech.edu. Associate Fellow AIAA.

NASA has recognized that the capability for remote resupply of a space platform with expendable fluids and other servicing will help transit space utilization into a new era of operational efficiency and cost effectiveness. However, only recently has satellite servicing been introduced as part of future spacecraft operations. A few servicing missions have been performed by NASA, most notably, the successful 1993 STS-61 mission to repair the Hubble Telescope, which received worldwide attention.

Despite the success in servicing a single spacecraft and the numerous papers studying optimal rendezvous between two spacecraft,[1] so far there has been no reported work devoted in the area of developing optimal trajectories for servicing multiple satellites in a constellation. In this paper, we consider the problem of servicing multiple satellites in a circular orbit with one SSc. The goal is to find the best sequence of satellites for which the cost of servicing the satellites (measured in terms of fuel expenditure) is minimized.

This paper is organized as follows. First, the optimal servicing problem is divided into two subproblems, the problem of servicing satellites in a given sequence (the time distribution problem) and the problem of finding the optimal sequence (optimal sequence problem). The time distribution problem is solved by integer programming.

Subsequently, a heuristic study leads to the solution of the optimal sequence problem. Finally, the optimal sequences are characterized.

Problem Statement

Consider an SSc and a constellation of satellites orbiting on a circular orbit. The task of the SSc is to service all satellites in the constellation. Therefore, the SSc is required to rendezvous with each of the satellites. After the SSc finishes servicing one satellite, it visits the next satellite until all the satellites have been serviced. Each satellite is visited only once during the servicing mission. As a practical concern, the total time to complete the mission is also specified. The goal is to find the sequence to visit all satellites in the constellation such that the total rendezvous cost (ΔV) is minimized. In addition to the sequence, the optimal time distributed to each rendezvous segment and the total ΔV are also computed.

The SSc is assumed to be initially in the same circular orbit as the constellation. This is the case when the SSc is designated to orbit along with all the satellites in the constellation. This could also be the case when the SSc has already visited a satellite in the constellation and it is ready to service the rest of the satellites. For the former case after the last satellite is serviced, the SSc is required to return to its original orbital slot. For the latter case, however, the SSc does not have to return to where it started from in the circular constellation.

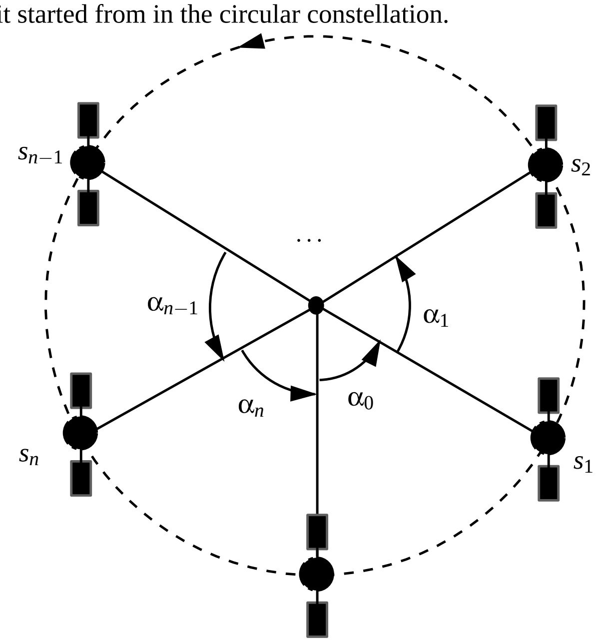

Fig. 1 A satellite constellation in a circular orbit.

Figure 1 shows $n+1$ satellites, s_0, s_1, $\cdots$, s_n, in a circular orbit, with the SSc initially at s_0. We define a cost function $C_{ij}(t_{ij})$ associated with the cost for SSc to transfer from satellite s_i and rendezvous with satellite s_j. t_{ij} represents the time of flight of this rendezvous segment. We seek solutions to the following two problems.

Problem 1. For a given total time t_f, find a sequence $Q = s_{q_1} s_{q_2} \cdots s_{q_{n-1}} s_{q_n}$ of the n satellites s_1, s_2, $\cdots$, s_n such that the total cost to service the n satellites among any other sequence is minimized. That is, the optimization problem is

$$\min \; C_{0q_1}(t_{0q_1}) + \sum_{i=1}^{n-1} C_{q_i q_{i+1}}(t_{q_i q_{i+1}}) + C_{q_n 0}(t_{q_n 0}), \quad (1)$$

subject to the constraint that the total time is given, i.e.,

$$t_{0q_1} + \sum_{i=1}^{n-1} t_{q_i q_{i+1}} + t_{q_n 0} = t_f. \quad (2)$$

The unknowns in this problem are the sequence Q and t_{0q_1}, $t_{q_i q_{i+1}}$, $i = 1, 2, \cdots, n-1$, and $t_{q_n 0}$, which are the times allotted for the rendezvous segments. The solution to this problem provides the optimal scheduling for the SSc to service all satellites and then return to its original orbital slot s_0 in the specified time t_f.

Problem 2. This problem is different from Problem 1 because the SSc is not required to return to s_0 after all satellites are visited. That is, for a fixed total time t_f, we seek a sequence $Q = s_{q_1} s_{q_2} \cdots s_{q_{n-1}} s_{q_n}$ which solves the optimization problem

$$\min \; C_{0q_1}(t_{0q_1}) + \sum_{i=1}^{n-1} C_{q_i q_{i+1}}(t_{q_i q_{i+1}}), \quad (3)$$

subject to the constraint that the total time is given, i.e.,

$$t_{0q_1} + \sum_{i=1}^{n-1} t_{q_i q_{i+1}} = t_f. \quad (4)$$

The unknowns in this problem are the sequence Q and the time intervals t_{0q_1}, $t_{q_i q_{i+1}}$, $i = 1, 2, \cdots, n-1$. The solution to this problem provides the optimal scheduling for the SSc to service all n satellites in the specified time t_f, but the SSc does not have to return to its original orbital slot s_0.

Both of these two problems can be cast as combinatorial optimization problems. However, combinatorial problems are characterized by a large and rapidly growing search space. For each of these two servicing scheduling problems, for example, there are $n!$ possible sequences (permutations of the satellites to be serviced) that are the candidates for the optimal solution. For a large number of satellites, the method of exhaustive search could become a formidable task.

The solution to the constellation servicing scheduling problem can be simplified by solving the following two subproblems: the *optimal time distribution problem* and the *optimal sequence problem*. Given a sequence of satellites and the total time, the optimal time distribution problem is to find the time allotted for each rendezvous segment and the associated minimum total cost. The optimal sequence problem deals with the determination of the best rendezvous sequence. These two subproblems are treated in the next two sections.

In all calculations in this paper, canonical units are used. That is, the distance unit is defined as the radius of the circular orbit, and the time unit is defined as the period of the circular orbit.

Optimal Time Distribution for a Rendezvous Sequence

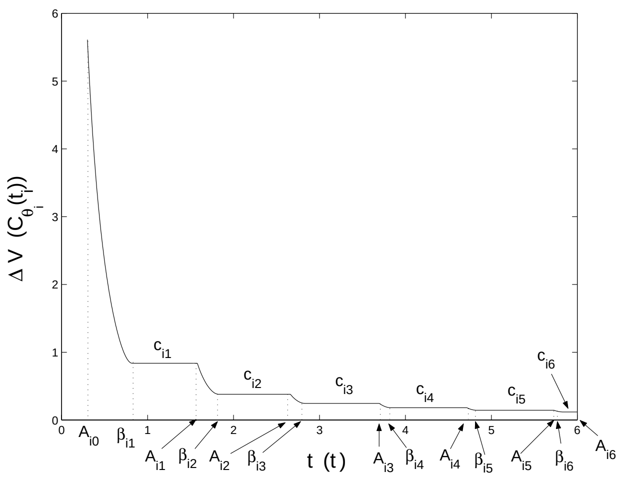

Fig. 2 Typical optimal cost vs. time-of-flight curve for two-impulse rendezvous.

It has been shown in Ref. 2 that for any two satellites s_1 and s_2 on a circular orbit, the cost for the minimum-fuel two-impulse rendezvous maneuver for s_1 to rendezvous with s_2 depends on the initial separation angle from s_1 to s_2, and the given time-of-flight. Typically, for a particular initial separation angle, the plot of the cost as a function of the time-of-flight is as shown in Fig. 2.

Suppose the SSc is given a particular sequence to service n satellites in time t_f. Then there are n consecutive rendezvous segments. Let θ_1 be the initial separation angle from the SSc to the first satellite in the sequence (the first segment), and let θ_i, $i = 2, 3, \cdots, n$ denote the initial separation angles from the $(i-1)^{\text{th}}$ satellite to the i^{th} satellite in the sequence (the i^{th} segment). Let the function $C_{\theta_i}(t_i)$ be the cost as a function of the time t_i allotted for the i^{th} rendezvous segment. Then the problem of finding the optimal time distribution can be formulated as the following.

Optimal Time Distribution Problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} C_{\theta_i}(t_i) \\ \text{subject to:} \quad & \sum_{i=1}^{n} t_i = t_f \end{aligned} \tag{5}$$

The difficulty in solving the optimal time distribution problem lies in the fact that the functions $C_{\theta_i}(t_i)$ are not differentiable with respect to t_i and the fact that these functions consist of several constant segments, as seen in Fig. 2. These two facts prevent traditional gradient-based search methods[3] from being effective.

By inspection of Fig. 2, it is seen that each cost function $C_{\theta_i}(t_i)$ is comprised of a series of constant segments which are connected by segments of smooth monotonically decreasing functions. Following the notation of Fig. 2, for the i^{th} rendezvous segment, let c_{ij}, $j = 1, 2, \cdots, j_{i,\max}$, denote the costs associated with the constant segments in the function $C_{\theta_i}(t_i)$. The upper limit for the index j, $j_{i,\max}$, depends on the maximum time-of-flight $t_{i,\max}$ allowed to be distributed to the corresponding segment. A possible choice is to set $t_{i,\max} = t_f$. Similarly, from Fig. 2, let β_{ij}, $j = 1, 2, \cdots, j_{i,\max}$ denote the times when a curve is followed by a step function, and A_{ij}, $j = 1, 2, \cdots, j_{i,\max}$ denote the time when a step function is followed by a curve. In addition, let A_{i0} denote the minimum time-of-flight allowed for the i^{th} segment. Since the cost approaches infinity when the time-of-flight approaches zero, it is wise to set A_{i0} to a positive value such that the cost required to complete the rendezvous does not become prohibitive.

Typically, $\beta_{ij} - A_{ij-1}$ is small compared to $A_{ij} - A_{ij-1}$ for any $j = 1, 2, \cdots, j_{i,\max}$, and their difference increases as the time-of-flight increases. Based on this analysis, we can approximate the cost function for each rendezvous segment as a series of step functions. As shown in Fig. 3, the original cost function is shown in dash lines and the step function approximation is shown in solid lines. It is seen that the approximation captures the cost function very well.With this approximation for the cost function for each rendezvous segment, the set $[A_{i0}, t_{i,\max}]$ is divided into $j_{i,\max}$ subdivisions, i.e., $[A_{ij-1}, A_{ij}]$, $j = 1, 2, \cdots, j_{i,\max}$. The problem of optimal time distribution is now converted to a problem of determining in which subdivision t_i should be assigned. To solve this problem, we use integer programming (IP).[4]

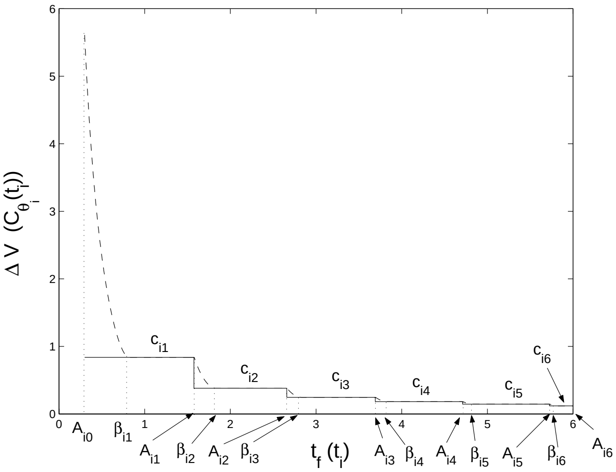

Fig. 3 Step function approximation of the cost function.

To this end, let x_{ij}, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, j_{i,\max}$ be binary variables such that

$$x_{ij} = \begin{cases} 1 & \text{if } t_i \in [A_{ij-1}, A_{ij}],\ j = 1, 2, \cdots, j_{i,\max} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Then an integer programming problem can be formulated as

$$\text{IP1:}\ \min \sum_{i=1}^{n} \sum_{j=1}^{j_{i,\max}} c_{ij} x_{ij} \tag{7}$$

subject to the following constraints

$$\sum_{j=1}^{j_{i,\max}} x_{ij} = 1,\ i = 1, 2, \cdots, n \tag{8a}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{j_{i,\max}} A_{ij} x_{ij} \geq t_f \tag{8b}$$

$$\sum_{i=1}^{n} \sum_{j=0}^{j_{i,\max}-1} A_{ij} x_{ij} \leq t_f \tag{8c}$$

The first constraint in Eq. (8a) states that each t_i can be assigned to only one of the subdivisions $[A_{ij-1}, A_{ij}]$. The second constraint in Eq. (8b) states that t_f should be smaller than the sum of the upper bounds of the subdivisions t_i are assigned to. The final constraint in Eq. (8c) states that t_f should be larger than the sum of the lower bounds of the subdivisions t_i are assigned to. Inequalities (8b) and (8c) guarantee that t_i, $i = 1, 2, \cdots, n$ can be chosen from the assigned subdivisions such that the constraint $\sum_{i=1}^{n} t_i = t_f$ can be satisfied. Thus if the problem is feasible, the solution of the integer programming problem renders the subdivision from which t_i can be chosen.

Methods for solving IP1 are well-known and include branch-and-bound, cutting planes, etc.[4] After the solution to IP1 is obtained, the unknowns t_i, $i = 1, 2, \cdots, n$ can be determined. There are three cases to consider.

Case 1. The solution to the original optimal time distribution problem is obtained if

$$\sum_{i=1}^{n} \beta_{ij_i} \leq t_f, \tag{9}$$

where j_i is such that $x_{ij_i} = 1$, $i = 1, 2, \cdots, n$. Inequality (9) implies that the optimal t_i can be chosen from $[\beta_{ij_i}, A_{ij_i}]$, $i = 1, 2, \cdots, n$ as long as $\sum_{i=1}^{n} t_i = t_f$. Clearly, the choice of the optimal t_i is not unique unless $\sum_{i=1}^{n} \beta_{ij_i} = t_f$. The simplest choice is to pick $t_i = \beta_{ij_i}$, $i = 1, 2, \cdots, n$. That is, the total time needed to complete the optimal servicing schedule is $\sum_{i=1}^{n} \beta_{ij_i}$ which is less than the given total mission time t_f. The corresponding minimum total cost is

$$\text{Total } \Delta V = \sum_{i=1}^{n} c_{ij_i} \tag{10}$$

Case 2. In case

$$\sum_{i=1}^{n} \beta_{ij_i} \geq t_f, \tag{11}$$

the optimal t_i cannot be as readily chosen as in Case 1. This is partly due to the complication associated with the possibility that some or all of the optimal t_i may be in the intervals $[A_{ij-1}, \beta_{ij}]$ where the rendezvous costs are not constant. In this case, instead of taking greater efforts in solving for the exact optimal t_i, we introduce the concept of *total-time relaxation* which uses the solutions to IP1 to yield a suboptimal solution to the time distribution problem. In essence, instead of restricting the total time to be strictly less than the given t_f, it is assumed that the total mission time can be extended to $\sum_{i=1}^{n} \beta_{ij_i}$. Thus, we can readily choose $t_i = \beta_{ij_i}$, $i = 1, 2, \cdots, n$. By doing so, we sacrifice a little on the mission time, but take advantage of the easily-obtained solution to IP1. In fact, the so-obtained time distribution yields a smaller cost ($\Delta V = \sum_{i=1}^{n} c_{ij_i}$) due to the relaxation of the total time constraint than it would have been otherwise.

The total-time relaxation is justified for problems with large given t_f. This can be seen by the observation that the length of the interval $\left|[A_{ij_i-1}, \beta_{ij_i}]\right| \ll A_{ij_i-1}$ for problems with large time-of-flight. Thus,

$$\sum_{i=1}^{n} A_{ij_i-1} \approx \sum_{i=1}^{n} \beta_{ij_i}. \tag{12}$$

Since

$$\sum_{i=1}^{n} \beta_{ij_i} \geq t_f \geq \sum_{i=1}^{n} A_{ij_i-1}, \tag{13}$$

the extension of t_f, $\sum_{i=1}^{n} \beta_{ij_i} - t_f$ is small compared to t_f.

Case 3. Suppose that in Case 2, the constraint of the total final time cannot be violated. In this case, we reformulate IP1 with a set of tighter constraints which will facilitate the choice of the time distribution. Here it is assumed that the cost function consists only of the constant segments, as shown by the solid line in Fig. 4. By doing so, it is guaranteed that the optimal t_i does not belong to intervals where the function $C_{\theta_i}(t_i)$ is not constant. That is, $t_i \in [\beta_{ij}, A_{ij}]$, $j = 1, 2, \cdots, j_{i,\max}$. Let x_{ij},

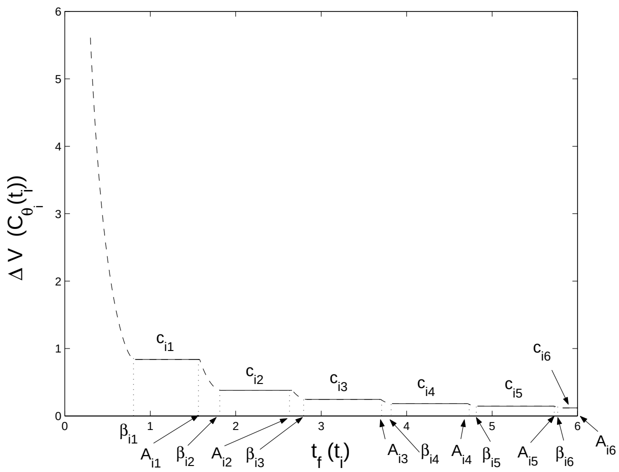

Fig. 4 Cost function with only the step part for case 3.

$i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, j_{i,\max}$ be binary variables such that

$$x_{ij} = \begin{cases} 1 & \text{if } t_i \in [\beta_{ij}, A_{ij}],\ j = 1, 2, \cdots, j_{i,\max} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Then the new IP can be formulated as follows.

$$\text{IP2:}\ \min \sum_{i=1}^{n} \sum_{j=1}^{j_{i,\max}} c_{ij} x_{ij} \tag{15}$$

subject to the following constraints

$$\sum_{j=1}^{j_{i,\max}} x_{ij} = 1,\ i = 1, 2, \cdots, n \tag{16a}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{j_{i,\max}} A_{ij} x_{ij} \geq t_f \tag{16b}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{j_{i,\max}} \beta_{ij} x_{ij} \leq t_f \tag{16c}$$

where the constraints have similar interpretations as in IP1. As in IP1, the solution to IP2 provides the intervals in which the optimal t_i can be chosen. Let j_i be the index such that $x_{ij_i} = 1$, i.e., the optimal $t_i \in [\beta_{ij_i}, A_{ij_i}]$. Since the cost is constant in the interval $[\beta_{ij_i}, A_{ij_i}]$, for each $1 \leq i \leq n$, the optimal t_i can be chosen arbitrarily as long as $t_i \in [\beta_{ij_i}, A_{ij_i}]$ and $\sum_{i=1}^{n} t_i = t_f$. The ensuing cost is $\sum_{i=1}^{n} c_{ij_i}$. One natural choice of t_i is to set $t_i = \beta_{ij_i}$, $i = 1, 2, \cdots, n$. Thus, the total time needed to complete the servicing schedule is $\sum_{i=1}^{n} \beta_{ij_i}$ which is less than the given total mission time t_f.

Optimal Sequence Problem

In the previous section, we presented a solution to the optimal rendezvous schedule of one SSc to service n satellites in a given sequence in a total mission time t_f. In this section, we study the problem of finding the optimal sequence to service a constellation of satellites in a circular orbit. Figure 1 shows a generic satellite constellation in a circular orbit with separation angles between two neighboring satellites given as α_i, $i = 0, 1, \cdots, n$. Next, we introduce the notion of the *Sweep Angle* (SA). For a given sequence of satellites, the sweep angle is defined for each rendezvous segment as follows. It is the smaller angle along the circular arc between the SSc and the satellite to be serviced. It is different from the separation angle in the sense that the latter is the angle measured from the SSc to the satellite to be serviced with the positive sense along the orbital velocity. That is, the separation angle can be between $-180°$ and $180°$ (with a sense of direction), whereas the sweep angle is always between $0°$ and $180°$ (without a sense of direction). In fact, for the i^{th} rendezvous segment for a given sequence, the sweep angle γ_i and the separation angle θ_i are related by the following relation,

$$\gamma_i = |\theta_i|,\ i = 0, 1, \cdots, n. \tag{17}$$

The *Total Sweep Angle* (TSA) is defined for a given sequence of satellites as the sum of the sweep angles of all the rendezvous segments.

In this section, the optimal sequence problem is investigated numerically. For this reason, only constellations with a small number of satellites to be serviced ($n \leq 6$) are considered. Even for a constellation with six satellites to be serviced, there are $6! = 720$ different sequences. The minimum cost is calculated for each sequence using the method described in the previous section. The total-time relaxation (Case 2 in the previous section) is used in case the integer programming IP1 does not yield the optimal solution. Extensive numerical investigations suggest that the solutions to the two servicing problems defined earlier depend strongly on the TSA of servicing sequences. For all cases tested, the optimal servicing sequence is always among the group of sequences which have the minimum TSA. Another observation is that the best sequence always appears to be totally or partially sequential. In fact, for Problem 1, where the SSc is required to return to its starting location s_0, the best sequence is one of the two sequential sequences (either orbit-wise or counter-orbit-wise). For Problem 2, where the SSc is not required to return to its starting location s_0, the best sequence may not be completely sequential. This observation is especially true when the total mission time gets larger. It is conceived that for larger number of satellites, this trend still holds.

In the following, we present four of the numerous case studies that have been conducted.

Case 1. In this case, we have one SSc and six satellites that must be serviced in a time of $t_f = 15.6$. The satellites (including the SSc) are evenly distributed along the circular orbit, i.e., the separation angle between any two neighboring satellites is 51.4°. The SSc is required to return to s_0.

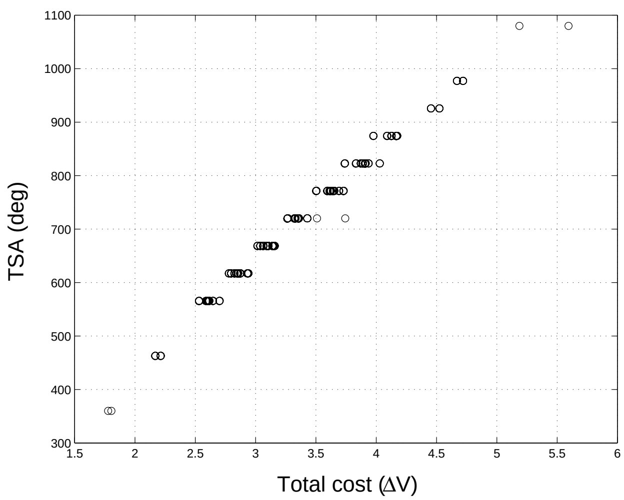

Fig. 5 TSA vs. cost for seven evenly distributed satellites (Case 1).

Figure 5 shows the plot of the TSA vs. the cost. Each circle on the plot corresponds to a satellite sequence. It is seen that the minimum cost sequence is among the group that has the smallest TSA, which is 360° in this case. Here there are only two sequences that have the minimum TSA. Figure 6 shows the cost versus the sequence index. A sequence index is used to identify a particular sequence. For n satellites, s_1, s_2, $\cdots$, s_n, the sequence index runs from 1 to $n!$, representing the $n!$ sequences. The q^{th} sequence, S_q, is defined as follows. Let $S_q = p_1 p_2 \cdots p_n$, where p_i, $i = 1, 2, \cdots, n$, are the n elements in S_q which are to be determined. To determine p_1, we let $q-1 = k_1(n-1)! + m_1$, where $0 \leq m_1 < (n-1)!$. Then $p_1 = C_1(k_1+1)$, where $C_1 = s_1 s_2 \cdots s_n$ and $C_1(k_1+1)$ is the $(k_1+1)^{\text{th}}$ element of C_1. To determine p_2, we let $m_1 = k_2(n-2)! + m_2$, where $0 \leq m_2 < (n-2)!$. Then

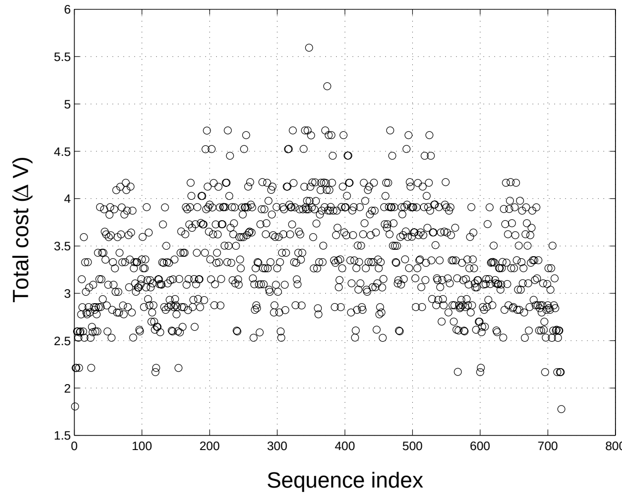

Fig. 6 Cost vs. sequence index for seven evenly distributed satellites (case 1).

$p_2 = C_2(k_2+1)$, where $C_2 = C_1 \backslash q_1$ (C_2 is obtained from C_1 by removing q_1 from C_1). Similarly, to determine p_i, $i = 3, 4, \cdots, n-1$, we let $m_{i-1} = k_i(n-i)! + m_i$, where $0 \leq m_i < (n-i)!$. Then $p_i = C_i(k_i+1)$, where $C_i = C_{i-1} \backslash p_i$. Finally, $p_n = C_{n-1} \backslash p_{n-1}$.

As seen in Fig. 6, sequence number 720 is the minimum cost sequence, which corresponds to the sequence $s_6 s_5 s_4 s_3 s_2 s_1$. We also notice that sequence number 1 is the second best sequence, which corresponds to the sequence $s_1 s_2 s_3 s_4 s_5 s_6$. However, in general, either of the two sequential order sequences can be the best sequence. Thus, it is necessary to compute the costs for both sequences and the one with the smaller cost is the best sequence.

Case 2. In this case, there are also six satellites that need to be serviced in time $t_f = 15.6$. The separation angles between neighboring satellites are:

$$\begin{aligned} \alpha_0 =& 15,\ \alpha_1 = 30^\circ,\ \alpha_2 = 15^\circ,\ \alpha_3 = 200^\circ, \\ &\alpha_4 = 50^\circ,\ \alpha_5 = 15^\circ,\ \alpha_6 = 35^\circ. \end{aligned} \tag{18}$$

The SSc is required to return to its starting location after servicing all six satellites.

Figure 7 shows the plot of the TSA vs. the cost. It is observed that the minimum cost solution is among the group that has the minimum TSA, which is 320° in this case. However, there are 32 sequences that have the minimum TSA. In Fig. 8, it is shown that sequence number 720 is again the best sequence. That is, the best servicing sequence is $s_6 s_5 s_4 s_3 s_2 s_1$. However, in general, either of the orbit-wise and counter-orbit-wise sequential sequences can be the best sequence. Thus, it is necessary to compute the costs for both sequences and the one with

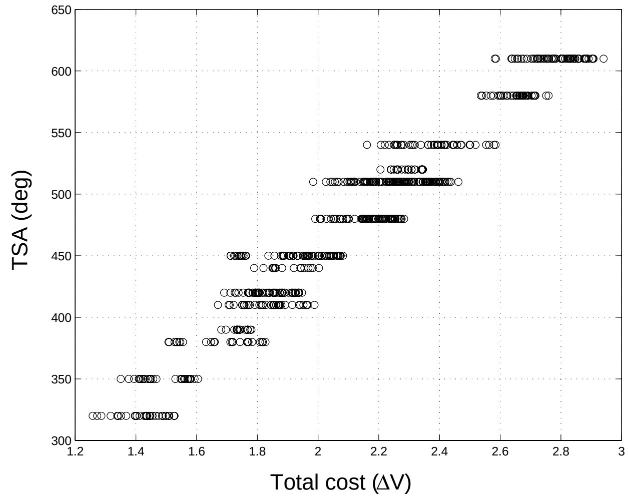

Fig. 7 TSA vs. cost for seven unevenly distributed satellites (Case 2).

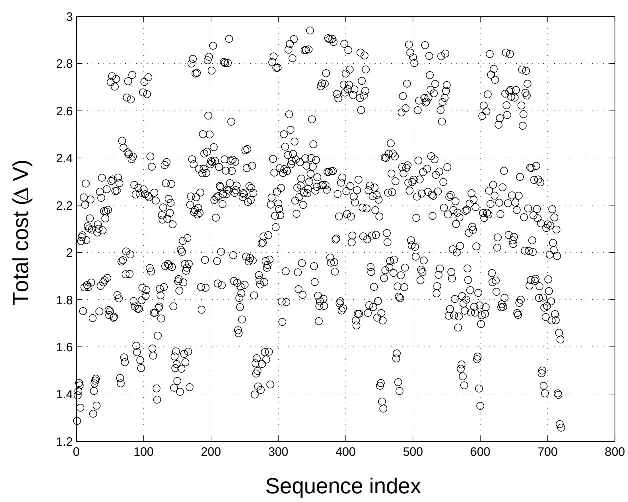

Fig. 8 Cost vs. sequence index for seven unevenly distributed satellites (Case 2).

the smaller cost is the best sequence.

Case 3. In this case, there are four satellites that need to be serviced in time $t_f = 15.6$. The four satellites and the SSc are evenly distributed along the circular orbit, i.e., the separation angle between two neighboring satellites is 72°. The SSc is not required to return to its starting location after servicing all four satellites.

Figure 9 shows the plot of the TSA vs. the cost. It is observed that the minimum cost solution is among the group that has the minimum TSA, which is 288° in this case. There are only two sequences that have the minimum TSA. In Fig. 10, where the cost versus the sequence index is shown, it is seen that the sequence number 24 is

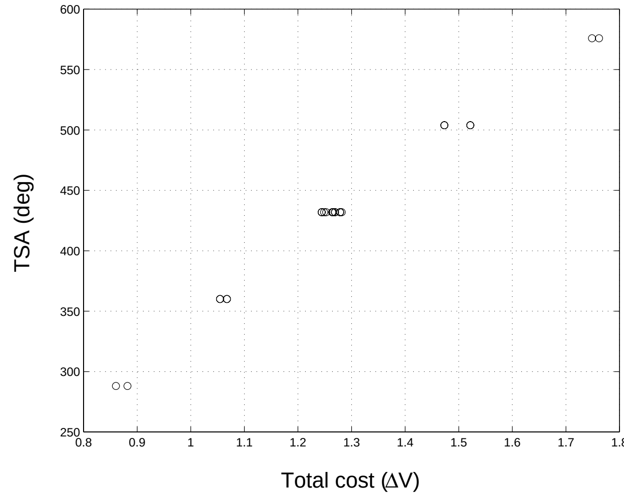

Fig. 9 TSA vs. cost for five evenly distributed satellites (Case 3).

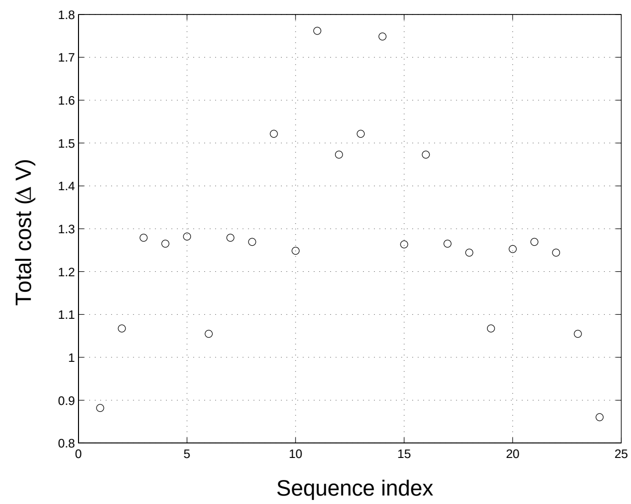

Fig. 10 Cost vs. sequence index for five evenly distributed satellites (Case 3).

the best sequence. That is, the best servicing sequence is $s_4 s_3 s_2 s_1$.

Case 4. In this case, there are five satellites that need to be serviced in time $t_f = 15.6$. The five satellites and the SSc are distributed along the circular orbit such that the separation angles between neighboring satellites are

$$\alpha_0 = 20,\ \alpha_1 = 20^\circ,\ \alpha_2 = 30^\circ,\ \alpha_3 = 200^\circ,\\ \alpha_4 = 40^\circ,\ \alpha_5 = 50^\circ. \tag{19}$$

The SSc is not required to return to its starting location after servicing all five satellites.

Figure 11 shows the plot of the TSA vs. the cost. It is observed that the minimum cost solution is among the

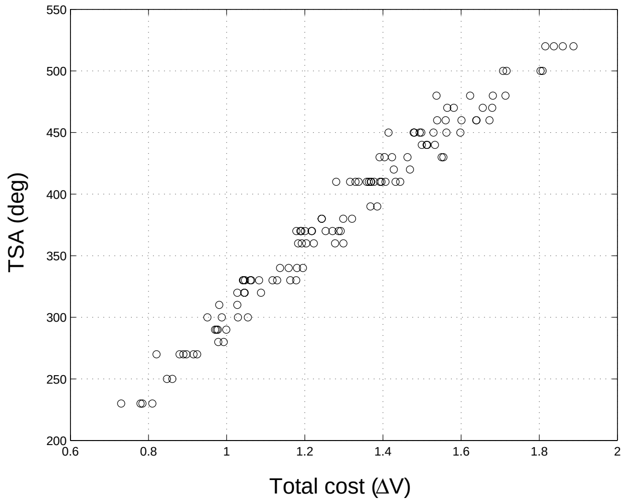

Fig. 11 TSA vs. cost for six unevenly distributed satellites (Case 4).

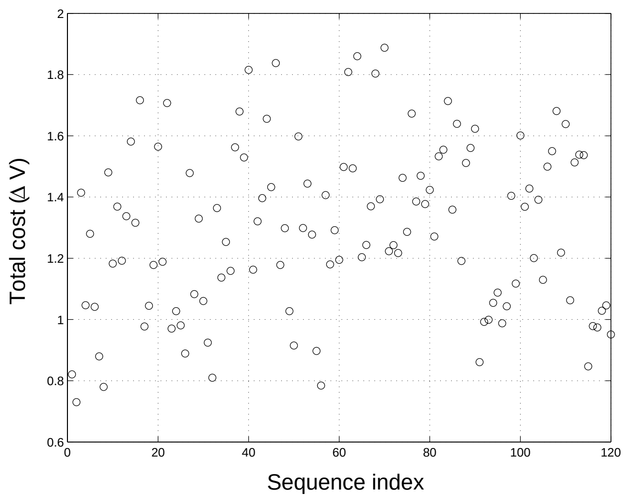

Fig. 12 Cost vs. sequence index for six unevenly distributed satellites (Case 4).

group that has the minimum TSA, which is 230° in this case. There are four sequences that have the minimum TSA. In Fig. 12, it is shown that sequence number 2 is the best sequence. That is, the best servicing sequence is $s_1s_2s_3s_5s_4$. Thus, the SSc visits s_1, s_2, and s_3 sequentially orbit-wise, and then visits s_5 and s_4 sequentially counter-orbit-wise.

Using numerical studies, thus far we have shown that the minimum cost sequence corresponds to the minimum TSA. In addition, the best sequence is totally or partially in sequential order. In this next section, we characterize the sequences with minimum TSA.

Sequences with Minimum TSA

Consider the satellite constellation depicted in Fig. 1. The SSc is initially situated at the location s_0, and s_i, $i = 1, 2, \cdots, n$ are the satellites to be serviced. Two cases are treated in the following two subsections. First, we consider the case when the SSc is required to return to its original location s_0 after visiting all n satellites. Afterwards, we deal with the case when the SSc does not have to return to s_0 after visiting all n satellites.

SSc Returns to its Original Slot.

There are two cases to consider here. First, if $\alpha_i < 180°$, for $i = 0, 1, \cdots, n$, then there are only two sequences with the minimum TSA which is 360° in this case. These two sequences are such that the SSc visits all satellites sequentially orbit-wise or counter-orbit-wise, i.e., the sequence is either $s_1 s_2 \cdots s_{n-1} s_n$ or $s_n s_{n-1} \cdots s_2 s_1$. The costs associated with both sequences need to be calculated and compared, and the smaller one yields the best sequence.

Second, if there is one angle such that $\alpha_k \geq 180°$, where $0 \leq k \leq n$, then the minimum total sweep angle is given by

$$\Gamma = 2\left(\sum_{i=0}^{k-1} \alpha_i + \sum_{i=k+1}^{n} \alpha_i\right) = 2\left(360° - \alpha_k\right) \leq 360° \tag{20}$$

There are at least two sequences which assume the minimum TSA Γ, and the two sequential sequences are among them. Next we characterize the sequences with the minimum TSA.

Let the set $S_f^k = \{s_1,\ s_2,\ \cdots,\ s_{k-1}\}$, and $S_b^k = \{s_n,\ s_{n-1},\ \cdots,\ s_{k+2}\}$, where k is the subscript associated with $\alpha_k \geq 180°$. Then the sequences with the minimum TSA can be described as follows. Given any two subsets of S_f^k and S_b^k, we can define two sequences with the minimum TSA. Specifically, let $S_{f1}^k \subseteq S_f^k$, and $S_{b1}^k \subseteq S_b^k$. One sequence is such that the SSc visits the satellites in S_{f1}^k sequentially orbit-wise, followed by a rendezvous with s_k. Then the SSc visits satellites in $S_f^k \backslash S_{f1}^k$ and S_{b1}^k sequentially counter-orbit-wise, followed by a rendezvous with s_{k+1}. Next the SSc visits the satellites in $S_b^k \backslash S_{b1}^k$ sequentially orbit-wise, and finally comes back to s_0. The other sequence is such that the SSc visits the satellites in S_{b1}^k sequentially counter-orbit-wise, followed by a rendezvous with s_{k+1}. Then the SSc visits satellites in $S_b^k \backslash S_{b1}^k$ and S_{f1}^k sequentially orbit-wise, followed by a rendezvous with s_k. Next the SSc visits the satellites in $S_f^k \backslash S_{f1}^k$ sequentially counter-orbit-wise, and finally comes back to s_0. Therefore, the total number of sequences with minimum TSA is twice the number of ways to choose the subsets of S_f^k and S_b^k, i.e., the total number of sequences with minimum TSA is

$$2\left[\sum_{i=0}^{k-1}\binom{k-1}{i}\right]\times\left[\sum_{i=0}^{n-k-1}\binom{n-k-1}{i}\right]. \qquad (21)$$

which is significantly smaller than $n!$. This heuristic approach shows that the best sequence can be chosen between the orbit-wise and counter-orbit-wise sequential sequences. In this case, the cost for both sequences can be calculated and compared, and the best sequence is the one with the smaller cost.

SSc Does Not Return to its Original Slot.

With the SSc not having to return to s_0, any satellite can be the last one in the constellation to be serviced. Therefore, the minimum total sweep angle is at least $360^\circ - \alpha_n$ if $\alpha_0 \le \alpha_n$ and $360^\circ - \alpha_0$ if $\alpha_0 \ge \alpha_n$. For the former, the sequence is $s_1 s_2 \cdots s_n$ with s_n being the last one to visit, and for the latter, the sequence is $s_n s_{n-1} \cdots s_1$ with s_1 being the last one to visit. However, in some cases, sequences with smaller TSA exist when choosing a satellite other than s_1 and s_n to be the last one to visit. To see that, assume that the satellite s_k is the last to be visited. Then the minimum TSA is given by

$$\Gamma_k = \begin{cases} 360^\circ - \alpha_{k-1} + \sum_{i=0}^{k-2}\alpha_i & \text{if } \alpha_{k-1} - \sum_{i=0}^{k-2}\alpha_i \ge \alpha_k - \sum_{i=k+1}^{n}\alpha_i \\ 360^\circ - \alpha_k + \sum_{i=k+1}^{n}\alpha_i & \text{otherwise} \end{cases} \qquad (22)$$

Thus the global minimum sweep angle is

$$\Gamma = 360^\circ - A \qquad (23)$$

where

$$A = \max\left\{ \max_{i=0,1,\cdots,n}\left\{\alpha_i - \sum_{j=0}^{i-1}\alpha_j\right\}, \max_{i=0,1,\cdots,n}\left\{\alpha_i - \sum_{j=i+1}^{n}\alpha_j\right\}\right\} \qquad (24)$$

If the index k is such that $A = \alpha_k - \sum_{j=0}^{k-1}\alpha_j$ in Eq. (24), then s_{k+1} is the last satellite to be visited. Let the set $S_f^k = \{s_1,\ s_2,\ \cdots,\ s_{k-1}\}$. Given any subset S_{f1}^k of S_f^k, we can define one sequence with the minimum total sweep angle Γ. This sequence is such that the SSc visits all satellites in S_{f1}^k sequentially orbit-wise, followed by visiting s_k. Then the SSc visits all satellites in $S_f^k \backslash S_{f1}^k$ and the satellites s_n, s_{n-1}, $\cdots$, s_{k+1} sequentially counter-orbit-wise. Thus, all sequences with the minimum TSA can be described by the same manner with all subsets of S_f^k. Of all these sequences, the heuristic study shows that the minimum cost sequence is the one such that $S_{f1}^k = S_f^k$, i.e., the best sequence is $s_1 s_2 \cdots s_k s_n s_{n-1} \cdots s_{k+1}$ which is partially sequential. For example, in Case 4 of the previous section, $n = 5$, $k = 3$, and s_4 is the last one to be visited. The best sequence is thus $s_1 s_2 s_3 s_5 s_4$.

On the other hand, if the index k is such that $A = \alpha_k - \sum_{j=k+1}^{n}\alpha_j$ in Eq. (24), then s_k is the last satellite to be visited. Let the set $S_b^k = \{s_{k+2},\ s_{k+3},\ \cdots,\ s_n\}$. Given any subset S_{b1}^k of S_b^k, we can define one sequence with the minimum total sweep angle Γ. This sequence is such that the SSc visits all satellites in S_{b1}^k sequentially counter-orbit-wise, followed by visiting s_{k+1}. Then the SSc visits all satellites in $S_b^k \backslash S_{b1}^k$ and the satellites s_1, s_2, $\cdots$, s_k sequentially orbit-wise. Thus, all sequences with the minimum TSA can be described by the same manner with all subsets of S_b^k. Of all these sequences, the heuristic study shows that the minimum cost sequence is the one such that $S_{b1}^k = S_b^k$, i.e., the best sequence is $s_n s_{n-1} \cdots s_{k+1} s_1 s_2 \cdots s_k$ which is partially sequential.

If in Eq. (24),

$$\max_{i=0,1,\cdots,n}\left\{\alpha_i - \sum_{j=0}^{i-1}\alpha_j\right\} = \max_{i=0,1,\cdots,n}\left\{\alpha_i - \sum_{j=i+1}^{n}\alpha_j\right\} \qquad (25)$$

and

$$k_1 = \arg\left\{\max_{i=0,1,\cdots,n}\left\{\alpha_i - \sum_{j=0}^{i-1}\alpha_j\right\}\right\} \neq \arg\left\{\max_{i=0,1,\cdots,n}\left\{\alpha_i - \sum_{j=i+1}^{n}\alpha_j\right\}\right\} = k_2, \qquad (26)$$

the optimal costs associated with both k_1 and k_2 need to be calculated and compared to yield the best sequence. This is the case in Case 3 in the previous section, where $n = 4$, and both sequences $s_1 s_2 s_3 s_4$ ($k_2 = 4$) and $s_4 s_3 s_2 s_1$ ($k_1 = 0$) assume the minimum total sweep angle 288°. Calculation shows that the latter yields the minimum cost.

Conclusion

The minimum-cost scheduling for one SSc to service satellites in a circular orbit is studied in this paper. The minimum-ΔV, two-impulse maneuver is used for each rendezvous between the SSc and the satellite to be serviced. The SSc is initially assumed to be on the circular orbit. Two problems are considered, corresponding to the case where the SSc returns to its starting circular orbital slot and the case where the SSc does not have to return to its starting orbital slot. The servicing scheduling problems are formulated as two combinatorial optimization problems, and are solved in two steps. First, the optimal time distribution problem is solved using integer programming, which yields the minimum cost maneuver for the SSc to visit the satellites in a given order. Then the optimal sequence problem is studied by a heuristic study. It is shown that the integer programming scheme is effective in solving the time distribution problem. The heuristic study shows that the best rendezvous sequence is always among the group of sequences that assume the minimum TSA. Specifically, the best rendezvous sequence is one of the orbit-wise and counter-orbit-wise sequential orders for the case where the SSc returns to it starting orbital slot. For the case where the SSc does not return, the best sequence is sequential or partially sequential, depending on the satellite distribution on the constellation. The sequences with the minimum TSA are completely identified.

References

[1] F. W. Gobetz and J. R. Doll. A survey of impulsive trajectories. *AIAA Journal*, 7(5):801–834, 1969.

[2] H. Shen and P. Tsiotras. Optimal two-impulse rendezvous between two circular orbits using multiple revolution Lambert's solutions. In *AIAA Guidance, Navigation, and Control Conference, Monterey, CA*, Aug. 5-8, 2002. AIAA-2002-4844.

[3] A. E. Bryson and Y.-C. Ho. *Applied Optimal Control: Optimization, Estimation, and Control*. Hemisphere Pub., Washington, D.C., 1975.

[4] G. L. Nemhauser and L. A. Wolsey. *Integer and Combinatorial Optimization*. John Wiley & Sons, New York, 1999.